Review: Love number computation within the Ice-sheet and Sea-level System Model (ISSM V4.24) – Caron et al.

Summary:

The manuscript of Caron et al. presents the theory for computing Love numbers for Maxwell, Burgers, and Extended Burgers rheologies. In this discussion they present a general overview of the parallelization strategy and demonstrate their optimization strategy. Furthermore, they compare their Love numbers to those of Spada et al. (2011), a benchmarking paper in the GIA community, and find good agreement for a Maxwell rheology. This body of work and the associated code is an important contribution to GIA modeling and coupled GIA/ice sheet modeling. The code may also have utility for other deformation problems given suitable modifications. Overall, this manuscript is well written and in fairly good shape. Below I outline a few major items and several minor items that would help improve the quality of the manuscript. None of these should take very long to implement and thus, I suggest this manuscript be accepted following minor revisions.

Major requests:

- Abstract: Please add the something about the Love number approach being for radial 1D (i.e., spherical symmetric earth models) so the limitations of this approach is more clear to the general audience. You could also add a sentence on the benefits of this approach (e.g., that it is computationally quick compared to codes with 3D viscosity structures) to balance out the negative.
- The dot used to show the divergence is at the bottom "." as opposed to the middle ".". Please fix this in equation 2, 18
- The dot product in equation 12 is at the bottom "." and should be in the middle "."
- Please make sure all variables and super-/subscripts are defined within the text. Table 1 is a helpful guide, but it doesn't contain everything and it makes for an unpleasant read to hunt for variable definitions that could easily be stated in the text. For example: eq 11-14: SH degree n and SH order m are not explained nor are the bounds for their summations; eq 19 what is E; eq $29 \text{explain} \tilde{\rho}$ in text please; etc
- Line 166-171: There is a disagreement in the ordering of the text and y_i in comparison to the equations 11-14. For a savvy reader this will not be a problem, but for others this could be confusing. I suggest first presenting equations 11, 12, 13, and 14 in order. So, introduce the full displacement vector field, the full traction vector field, etc. Then introduce "y_i" and state the order of the vector elements following the convention of Alterman et al.
- Equation 17 Please state what N(\theta,\phi,t) is (i.e., how is it different from N^{m}_{n}, the normalization factor or the number of layers N_{L}) and also consider a different variable to avoid confusion
- Is there a standard you followed for using \sim , ', and f_{i} for the different factors in 38. It would be helpful if there was and if you could please explain it.
- Figure 7: Please add a bounding box to left figure to show the location of the right figure.
- Figure 8: Please add something to the caption describing where the h love numbers are plotting.
- Multi-panel figures: Please choose a consistent method for referencing multi panel figures. For example, Figure 8 uses (a) and (b), but none of the other figures use letters.
- Multi-panel Figures: Please ensure there is a description of each panel in the caption. For example, Figure 2 does not explain both plots. Second please modify the captions so that it is easier to find where the descriptions of each panel is. Bold and italic font might be helpful, particularly when using "Right", "Top Row", etc.
- Section 5.2: As described, it is unclear how experiment 1 & 2 are changing the mesh discretization. Both are described as "increasing the radial grid resolution". The term layer is also used, but in this context, it is unclear if the authors are referring to the layers shown in Figure 1 or if they mean a spherical shell. A figure showing the change to the mesh for each experiment would be helpful.

Minor requests:

• Line 28: "... model disparities related to solid Earth structure and mantle Rheologies." Please add "1D" to the solid Earth structure to be clearer.

- Line 75: missing a period at the end of sentence "... Kierulf et al., 2022)."
- Line 79: Table 4 is introduced before Table 1, 2, and 3. Consider revising the text of table locations/labels so things appear in the order in which they are discussed.
- Line 83: Since you are discussing changes in Earth's shape would \dot{J}_2 be more appropriate than just J 2?
- Line 103: "... Heaviside function forcing ..." -> "... Heaviside forcing function ...", maybe easier to read?
- Line 161: "... and material ..." -> "... and then material ..."
- Line 205: What is meant by "resolved"?
- Equations 24, 25, 26: The 0 on the right-hand side should be written as a vector and not a scalar.
- Table 3: Please add units to your times. Currently one must assume they are in seconds like the precision you report in the table caption.
- Line 440: delete "with"

Review Criteria:

- Scientific Significance: 1
- Scientific Quality: 1
- Scientific Reproducibility: 1
- Presentation: 1

Review Questions:

- 1. Does the paper address relevant scientific modelling questions within the scope of GMD? Does the paper present a model, advances in modelling science, or a modelling protocol that is suitable for addressing relevant scientific questions within the scope of EGU? Yes & Yes
- 2. Does the paper present novel concepts, ideas, tools, or data? Yes
- 3. Does the paper represent a sufficiently substantial advance in modelling science? Yes
- 4. Are the methods and assumptions valid and clearly outlined? Mostly Yes
- 5. Are the results sufficient to support the interpretations and conclusions? Yes
- 6. Is the description sufficiently complete and precise to allow their reproduction by fellow scientists (traceability of results)? In the case of model description papers, it should in theory be possible for an independent scientist to construct a model that, while not necessarily numerically identical, will produce scientifically equivalent results. Model development papers should be similarly reproducible. For MIP and benchmarking papers, it should be possible for the protocol to be precisely reproduced for an independent model. Descriptions of numerical advances should be precisely reproducible. Most yes, but some issues are discussed above.
- 7. Do the authors give proper credit to related work and clearly indicate their own new/original contribution?

 Yes
- 8. Does the title clearly reflect the contents of the paper? The model name and number should be included in papers that deal with only one model. Yes
- 9. Does the abstract provide a concise and complete summary? Yes
- 10. Is the overall presentation well structured and clear? Mostly Yes
- 11. Is the language fluent and precise? Yes
- 12. Are mathematical formulae, symbols, abbreviations, and units correctly defined and used? Mostly yes, but there is room for improvement (See above comments).
- 13. Should any parts of the paper (text, formulae, figures, tables) be clarified, reduced, combined, or eliminated? Yes, see comments
- 14. Are the number and quality of references appropriate? Yes
- 15. Is the amount and quality of supplementary material appropriate? For model description papers, authors are strongly encouraged to submit supplementary material containing the model code and a user manual. For development, technical, and benchmarking papers, the submission of code to perform calculations described in the text is strongly encouraged. Yes