Review of "age\_flow\_line-1.0: a fast and accurate numerical age model for a pseudo-steady flow tube of an ice sheet" by Parrenin et al. 2025

#### **General Comments**

This paper presents a 2.5D Eulerian-Lagrangian age model for ice-sheet stratigraphy, assuming steady-state geometry along a flow tube. Using the  $(\pi,\phi)$  coordinate system introduced by Parrenin et al. (2006), the model efficiently and accurately solves the transport equations. The manuscript is well within the scope of Geoscientific Model Development (GMD), and the figures are clear and helpful. Overall, I recommend this paper for publication.

We thank you very much for your careful, detailed and constructive reviewing work on our manuscript.

That said, the manuscript could more clearly articulate the novelty of this work. The introduction would benefit from a broader contextualization of ice-sheet stratigraphy and dating methods, clarifying when modeling is necessary compared to alternative approaches such as layer counting or tephra dating. When discussing existing models, it would be helpful to distinguish between different objectives, such as dating deep ice-core layers, determining ice origin for upstream corrections, modeling the age distribution across an ice sheet, or using isochrones to invert for basal parameters. A brief discussion of key challenges in ice-age modeling, such as numerical diffusion, would also be valuable. Strengthening the motivation of why a new age model is necessary and how this model fills a gap in the existing literature would further improve the framing of the study.

### We modified the following paragraph:

Modelling the age and trajectories of the ice in ice sheets is important for several reasons. This includes dating existing ice cores and correcting from upstream effects (Buchardt, 2009; Huybrechts et al., 2007; Johnsen and Dansgaard, 1992; Parrenin et al., 2007) Ice cores can also be dated by counting annual layers where these layers are thin enough (Svensson et al., 2008) or by identifying dated horizons (Bouchet et al., 2023). But modelling also provides an estimate of the thinning and horizontal displacement of ice layers. For example, moving down in the EDML ice core, there is a decreasing trend in the ice isotopic record that corresponds to a decrease in atmospheric temperature during snow deposition. This decrease is not due to temporal climatic variations, but simply to the fact that deeper down, the ice originates from further upstream, from a higher elevation and therefore colder site (Huybrecht et al., 2007). and Moreover, investigating new potential ice core drill sites (Chung et al., 2023; Obase et al., 2023; Parrenin et al., 2017; Van Liefferinge and Pattyn, 2013) usually requires some modelling since radarobserved isochrones generally do not cover the whole ice column. Age observations from ice cores (Bouchet et al., 2023; Oyabu et al., 2022) or from radio echo sounding (Cavitte et al., 2016; MacGregor et al., 2015) can also help constrain the boundary conditions of ice sheets or understand the internal processes of ice sheet models using inverse methods. For example, dated isochrones can help deduce the surface accumulation rate, the basal melting rate or the horizontal and vertical velocity profiles (Buchardt et al., 2007; Parrenin et al., 2017; Chung et al., 2023). Finally, modelling is necessary to estimate the age distribution across an ice sheet, since it is usually not covered entirely by observations. Key challenges in age modelling include numerical accuracy and efficiency, the match to observations and accounting for boundary conditions.

The application of the model to simulate the age distribution between Dome C and Little Dome C is a helpful illustration, but the purpose of this demonstration could be clearer. The paper initially suggests that the goal is to investigate the role of horizontal advection in shaping the depth-age relationship at Little Dome C, yet the model parametrization is later described as unrealistic. This raises questions about the extent to which conclusions drawn from this 'unrealistic' model run are meaningful. It would be helpful to disentangle these two aspects: Is

the aim to assess the significance of horizontal advection, or simply to demonstrate the model's capabilities? Clarifying these objectives, as well as the connection to Chung et al. (2024), would strengthen the narrative.

To clarify, we show in the current manuscript that horizontal flow is important along the DC-BELDC profile because we do not simulate the same isochrones with a 1D model (Chung et al., 2023) or with the current 2.5D model using the same set of input parameters. While in this manuscript we describe the forward 2.5D model, Chung et al. (2024) describe the inverse method around this 2.5D model where the parameters are optimized so that the model fits the ice core and isochronal age observations.

### We have modified section 4.2 as follows:

#### 4.2 Modelling of the DC-BELDC flow line

We have performed a simulation where both vertical and horizontal ice flow is taken into account. We show that ithorizontal flow has a non negligible effect on the age modelling of the Beyond EPICABELDC ice core, since our modelled isochrones do not have the same geometry than the ones simulated by the 1D model (Chung et al., 2023) when using the same set of input parameters. Indeed, while the 1D model simulates isochrones very close to the observed ones (Chung et al., 2023), the

isochrones simulated by the 2.5D model significantly deviate from them (Figure 7). In our forward simulation using parameters inverted by the 1D model, with particles originateing sometimes >1520 km from their current position. Our simulation is provided as a proof of concept for the model we have developed. However, it is not appropriate to use the result of the 1D inverse model by Chung et al. (2023) to determine the parameters of the 2.5D model. Furthermore, we can see in Figure 7, that modelled isochrones (in black) do not agree well with the observed isochrones (in white). For a more realistic simulation, the

parameters of the model should be optimized so that modelled age fits the radar and ice core age observations. This is the

purpose of the companion paper of Chung et al. (2024) who developed this inverse methodology.

The discussion is thorough but could better address model validation, limitations, and potential applications. The role of horizontal flow is highlighted, yet discrepancies between modeled and observed isochrones are only briefly mentioned. Expanding on possible sources of error and how the model compares to other 2.5D approaches (e.g., Buchardt et al. 2007) would be beneficial. Additionally, while the authors acknowledge simplifications in accumulation and basal melt assumptions, a more critical discussion of potential biases would strengthen the manuscript.

As for model validation, we compare our results with the 1D model of Chung et al. (2023). Limitations is already covered in sections 4.3 and 4.4 but going beyond that would require to compare with other models which would be a significant work which could be the subject of a future manuscript. Potential applications and comparison to other modelling effort is now covered in a new section 4.5 pasted below:

## 4.5 Possible applications and comparison with other modelling efforts¶

This flow tube model could be applied to several nearly steady flow lines of the current Greenland and Antarctic ice sheets, especially those that have ice core information.

The Vostok flow line in East Antarctica was originally modelled by Ritz (1992) but only the age along the Vostok ice core was calculated by a Lagrangian tracer scheme. This work was later extended by Parrenin et al. (2001; 2004) who optimized some parameters of the model to fit the ice core age observations, but still without accounting for the isochronal information. Another flow tube model was developed and applied to the Vostok flow line by Salamatin et al. (2009), accounting for both the ice core and isochronal age observations. It would be valuable to apply the current numerical model to this Vostok flow line and compare the results with these previous modelling efforts.

The EDML ice core in East Antarctica is situated on a ridge and horizontal flow also needs to be taken into account. Huybrecht et al. (2007) applied a large scale ice sheet model of Antarctica with a local high resolution and high order model nested inside around EDML. It would be valuable to see how our numerical model, with its very high resolution and numerical accuracy and its fitting to isochronal observations, compares with this previous transient and large scale model.

The NEEM and NorthGRIP ice cores in Greenland are also situated on a ridge coming from Summit. Isochrones around GRIP were simulated using a flow tube model by Buchardt and Dahl-Jensen (2007) and fitted to radar observations. The basal melting was hence deduced. Applying our numerical model to this flow line, we could compare it to the previous work and see if its high resolution and accuracy can improve the modelling scenario.

Another interesting flow line in Greenland is the one going from Summit to the EastGRIP ice core. The age of the ice along this profile was modelled by Gerber et al. (2021) using a similar model to that of the NorthGRIP flow line, based on a Dansgaard-Johnsen velocity profile (Johnsen and Dansgaard, 1992). Again, our numerical model could be compared to this previous modelling effort.

The code is well-structured and documented, and I was able to run the DC-BELDC example without issues. Expanding the GitHub README with guidance on adapting the code to other regions would be beneficial. While Section 2.4 provides useful information for users, its placement disrupts the readability of the manuscript. Consider moving it to the GitHub repository for better accessibility.

### We added the following sentences in the README:

If you want to set up a new flow tube experiment, we suggest to copy an existing experiment directory such as DC-BELDC. Then you can incrementally modify the parameters.yml parameter file and the .txt data files.

As for section 2.4, we considered removing it but we reckon that it is OK to have such a technical section on the code itself for a GMD manuscript which is not only about scientific conclusions, but also tool development.

## **Specific Comments**

Line 12: Specify the coordinate system more clearly—replace "innovative" with "logarithmic flux coordinate system."

"logarithmic flux" added.

Lines 21-24: The sentence is too long; consider breaking it down or simplifying.

We broke it down using semi-columns.

Lines 26-28: This sentence is hard to follow. Consider restructuring for clarity.

We broke this sentence in two sentences for clarity.

Line 29: Add a sentence elaborating on why this is important or provide an example of how upstream effects impact the paleo-record.

We modified this paragraph as follows:

Modelling the age and trajectories of the ice in ice sheets is important for several reasons. This includes dating existing ice cores and correcting from upstream effects (Buchardt, 2009; Huybrechts et al., 2007; Johnsen and Dansgaard, 1992; Parrenin et al., 2007). Ice cores can also be dated by counting annual layers where these layers are thin enough (Svensson et al., 2008) or by identifying dated horizons (Bouchet et al., 2023). But modelling also provides an estimate of the thinning and horizontal displacement of ice layers. For example, moving down in the EDML ice core, there is a decreasing trend in the ice isotopic record that corresponds to a decrease in atmospheric temperature during snow deposition. This decrease is not due to temporal climatic variations, but simply to the fact that deeper down, the ice originates from further upstream, from a higher elevation and therefore colder site (Huybrecht et al., 2007). and Moreover, investigating new potential ice core drill sites (Chung et al., 2023; Obase et al., 2023; Parrenin et al., 2017; Van Liefferinge and Pattyn, 2013) usually requires some modelling since radarobserved isochrones generally do not cover the whole ice column. Age observations from ice cores (Bouchet et al., 2023; Oyabu et al., 2022) or from radio echo sounding (Cavitte et al., 2016; MacGregor et al., 2015) can also help constrain the boundary conditions of ice sheets or understand the internal processes of ice sheet models using inverse methods. For example, dated isochrones can help deduce the surface accumulation rate, the basal melting rate or the horizontal and vertical velocity profiles (Buchardt et al., 2007; Parrenin et al., 2017; Chung et al., 2023). Finally, modelling is necessary to estimate the age distribution across an ice sheet, since it is usually not covered entirely by observations. Key challenges in age modelling include numerical accuracy and efficiency, the match to observations and accounting for boundary conditions.

Line 30: Clarify that modeling is particularly important where annual layer counting is not possible.

See above the modified paragraph.

Lines 32-35: This sentence is difficult to understand. Aren't you constraining the boundary conditions of ice sheet models and understand internal processes of the ice sheet? It would also be good to specify what is being inverted for and give an example with citation

We added the following sentence:

boundary conditions of ice sheets or understand the internal processes of ice sheet models using inverse methods. For example, dated isochrones can help deduce the surface accumulation rate, the basal melting rate or the horizontal and vertical velocity profiles (Buchardt et al., 2007; Parrenin et al., 2017; Chung et al., 2023). Finally, modelling is necessary to estimate the age distribution across an ice sheet, since it is usually not covered entirely by observations. Key challenges in age modelling include numerical accuracy and efficiency, the match to observations and accounting for boundary conditions.

Line 36: "types". Implemented into what?

"types" corrected. "implemented" changed for "developed".

Line 37: Specify what they are used for.

Sentence changed to:

developed implemented. For example, large scale transient models have been used (<u>Lhomme</u> et al., 2005a; Sutter et al., 2019, <u>Born and Robinson</u>, 2021) to estimate the <u>stratigraphy</u> of the <u>Greenland and Antarctic ice sheets</u>. The advantages of these

Line 37: Born & Robinson (2021) could also be relevant here.

#### Reference added.

Lines 57-62: Explain the motivation for developing a new age model—what improvement does it offer over existing models?

## We added the following sentence:

scheme that uses an analytical derivation of trajectories and a grid which tracks these trajectories. This model offers improved numerical accuracy and efficiency over existing models and is therefore appropriate for inverse methods where many forward simulations are necessary. In Section 2, we first present the analytical and numerical foundations of the model and its

Line 58: Wasn't this already introduced in Parrenin et al. (2006)?

The coordinate system was published in Parrenin et al. (2006) but only in a restrictive case where the velocity profiles are spatially homogeneous. Parrenin and Hindmarsh (2007) then generalized the approach. We added the first reference as well.

Line 65: Consider specifying "steady-state flow-tube." Indicate that the flow line starts at an ice-sheet dome and ends at the margin (since x is later defined as "distance from the dome").

#### Sentence modified to:

We first consider a steady flow line-tube of an ice sheet, which starts at a dome and ends at the ice sheet margin. This means that we assume that factors such as the geometry of athe flow linetube (e.g. ice thickness) and the vertical shape function do not change in time. In this model, we will consider non-steady accumulation rate and melting rate, but this will be discussed

Line 70: Specify whether z is defined as positive (height above bed/sea level) or negative (depth below surface).

## We now specify:

we allow the flow tube width and relative density to vary vertically. We write the equations in the (x,z) coordinates where x, the horizontal coordinate along the direction of ice flow, is the distance from the dome and z is the vertical coordinate pointing upward. We suppose that the horizontal direction of the flow does not depend on the vertical position and is time-independent,

Line 84: "Passing below depth z"—note that depth was previously defined as "d", but I'm not sure that 'd' is actually used. Ensure consistency.

d is actually not used so we removed the definition.

#### We now write:

We now define fluxes used to derive the stream function. The partial horizontal flux  $q_H(x,z)$  is defined as the horizontal flux passing below depthelevation z:

Line 113: How do you justify that basal melt rate and surface accumulation have the same temporal variation? There is no direct response of basal melt rate to accumulation changes.

Well, we do not justify this since it is just a mathematical simplification but which has no physical justification. This is explained in the discussion, section 4.3:

Second, the same temporal multiplicative factor is applied to both surface accumulation and basal melting. There are no physical reason to assume surface accumulation and basal melting have varied in concert, this is just a mathematical convenience. But because basal melting is generally small in front of compared to surface accumulation, this assumption should not be too dramatic if the right real average-in-time basal melting is given as input to the model. Moreover, this temporal factor assumes that the surface accumulation spatial pattern has remained stable in time. This assumption relies on a stable snow precipitation process and a stable snow re-deposition by wind, which might not always be the case (Cavitte et al., 2018).

We now refer to the discussion when introducing this assumption.

Line 115: Change to "temporal average."

Done.

Line 122: For a general model description, reword as: "using a relative density profile informed by ice core observations or firn models in the simulated area."

Done, thank you for the suggestion.

Line 132-133: 'a point slightly downstream of the dome' could be more specific. How far downstream?

We changed these sentences as follow:

boundary should be known. As the  $\pi$  scale is logarithmic, there is a singularity at the dome. Therefore, the horizontal  $\pi$  scale starts at a point slightly downstream of the dome, where an age can be prescribed, for example using a dome solution (which assumes purely vertical movement). If one is interested in a flank ice core, the upstream boundary can be chosen such that the upstream condition does not affect the age modelling of the ice core, i.e., the ice in the ice core comes from the surface boundary and not from the upstream boundary.

Line 138: "linear-by-parts function."

Corrected.

Line 141: Delete "that" after "1/a."

Deleted, thanks.

Lines 158-166 & 174-178: Consider removing these paragraphs—it sounds more suited for a "README" in the GitHub repository rather than the paper.

We reckon it is OK to have such a section in a GMD manuscript, which is not only about scientific conclusions, but also about tool availability.

Line 179: Clarify—do you mean "flow tube"?

We get the age along a flow line but for that we model a flow tube, so in our opinion both formulations are correct.

Lines 180-181: Provide more context about Beyond Epica for readers unfamiliar with the project. For example that modeling here is necessary for ice core dating due to small layer

thicknesses. It would also be beneficial to already here explain the motivation for this modeling effort.

#### We modified this section as follow:

To demonstrate the performance and ability of our age model, we apply it to the <u>East Antarctica</u> flow line between <del>Dome CDC</del> and <u>Little Dome Cthe BELDC</u> drill site (<u>East Antarctica</u>). Beyond EPICA is a European project that aims to drill a continuous ice core record back to 1.2 million years at least, which makes this flow line particularly interesting. Numerical modelling is necessary to estimate the age of the ice deeper than the deepest visible radar horizons and to estimate the trajectories of ice particles. The <u>DC-BELDC</u> flow tube has already been determined in the companion paper Chung et al. (2024). It is ~40 km

Lines 185-186: Clarify why you use "mechanical ice thickness" for the bottom boundary conditions. Do you mean that you are using basal conditions from Chung et al. (2023)? Why not use observed ice thickness from your radar survey?

# Sentence change to:

thickness and uses the comparison with the observed ice thickness to infer basal conditions. For consistency and the sake of comparison, it is therefore the mechanical ice thickness that we use for our bottom boundary condition here, instead of the observed ice thickness. The aim of our simulation here is therefore to estimate whether horizontal advection is an important

Lines 186-187: The aim of the simulation should be stated earlier in the paragraph rather than after explaining model parameterization. The purpose is unclear—you state that you investigate whether horizontal advection affects depth-age relationships at LDC but then call the setup unrealistic. Separate these two points and clarify the goal here versus in Chung et al. (2024).

#### We changed the formulation of this paragraph which was indeed a bit awkward:

observed ice thickness. The aim of our simulation here is therefore to estimate whether horizontal advection is an important mechanism to take into account in the modelling of the age along this flow line, by comparing the result of 1D and 2.5D models with the same set of parameters. This 2.5D simulation is however not fully optimized purely for demonstration purposes and should not be seen as realistic. This is because we are using the results of the 1D inverse model for a 2.5D flow tube model, instead of optimizing the 2.5D model directly onto the observed isochrones and ice core datasets. The optimization of this 2.5D model is the scope of the companion paper Chung et al. (2024).

### Line 191: Where do these boundary conditions come from?

#### We added the following sentence:

The boundary conditions of the model are plotted in Figure 4. The accumulation and basal melting rates are taken from the 1D inverse model (Chung et al., 2023), while the flow tube width is calculated from the back-tracking of adjacents flow lines from BELDC (Chung et al., 2024). The meshes of the model are plotted in Figure 5. The horizontal flux shape function ω is plotted

Line 193: BELDC has not been defined previously.

### This is now defined in the introduction.

Line 201, Fig. 8: Indicate where the ice divide is located. This would make the statement "the ice particles may originate >20 km upstream along the divide" clearer.

### We now write in the figure caption:

**Figure 8:** Trajectories of ice particles (black lines) along the DC-BELDC flow line which is situated along a divide. The positions of the EDC and BELDC deep drill sites are plotted in dashed red. The black dashed line represents the trajectory originating from the surface upstream corner. The observed bedrock is in thick black and the mechanical bedrock in violet. This figure was automatically generated by the age flow line-1.0 software.

Line 221: Cite as "1D inverse model by Chung et al. (2023)."

Done.

Lines 230-231: Missing citations for this statement —add references.

Sentence changed to:

in the past, which is still unclear. Greenland and West Antarctica may have had a relatively stable geometry since the last glacial inception but they have probably encountered more important changes during the last interglacial since they are more sensitive to climatic variations in the ocean and in the atmosphere (Quiquet et al., 2013; Wolff et al., 2025).

Line 234: "In front of"—do you mean "compared to"?

Yes, corrected.

Line 235: what do you mean with "right" here? How do you know what's right without direct observations.

Changed "right" to "real".

Line 254: Sentence is incomplete—revise for clarity.

Yes, sorry, copy/paste problem which is now corrected.

Figure 6: Explain the cause of discrepancies between observed and modeled bedrock.

In this legend, we now specify:

**Figure 6:** The  $\omega$  horizontal flux shape function along the DC-BELDC flow line. The positions of the EDC and BELDC deep drill sites are plotted in dashed red. The observed bedrock is in thick black and the mechanical bedrock (the elevation where the extrapolated velocity becomes zero) in violet. This figure was automatically generated by the age\_flow\_line-1.0 software.

Line 345: The citation of this thesis seems out of place. Does most of the content of this paper rely on it? Consider revising.

In this thesis, we generalized the analytical developments done in Parrenin et al. (2006) and Parrenin and Hindmarsh (2007), so we think it is a useful citation.

Figure 1: Clarify whether z points upwards or downwards.

z points upwards, as we now write in section 2.1. This is consistent with the figure.

Figure 3: A map overview of where this is located in Antarctica could be useful. Instead of surface elevation, I would consider the surface flow velocities a more relevant context for this work.

We modified the figure according to your suggestions:

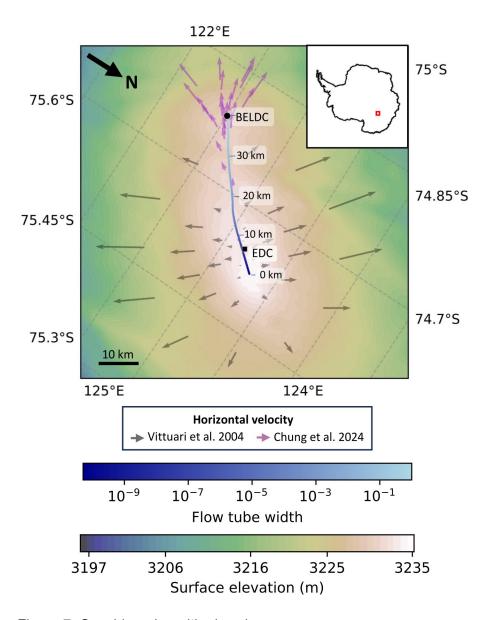


Figure 7: Consider a logarithmic colormap

We considered a logarithmic colormap, but kept the current colormap for two reasons. First, the age at surface is zero and therefore a logarithmic colormap would have a singularity at surface. Second, we find it useful to visualize that the young layers occupy most of the thickness.

Figure 10: I don't see a green line.

Yes, sorry, corrected to orange.

**Technical comments** 

Line 66: e.g.

Corrected.

Line 195: In Fig. 5.

Corrected.

Line 320, 322, 328, 335: missing DOIs.

# 322 unfortunately does not have a DOI. The 3 others were corrected.

Site and Ice Core Naming Consistency: The distinction between Dome C and Little Dome C as sites, and EDC/BELDC as ice core names, may be confusing for readers unfamiliar with the terminology. Consider using a single consistent name per site throughout the text. Right now it is a mix of DC, EDC, Dome C and LDC, Little Dome C, BELDC, and Beyond EPICA. Alternatively, explicitly define all terms early in the paper.

### Thanks for the comment.

We replaced "LDC" and "Little Dome C" with "BELDC" throughout the manuscript. We replaced "Beyond EPICA ice core" with "BELDC ice core" throughout the manuscript. "DC" is now defined once and used instead of Dome C throughout the manuscript.