$$f = A \cdot x, \tag{D2}$$

$$\begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} = \frac{1}{n} \begin{pmatrix} \overbrace{1 \cdots 1}^{n} & \overbrace{0 \cdots 0}^{n} \\ \underbrace{0 \cdots 0}_{n} & \underbrace{1 \cdots 1}_{n} \end{pmatrix} \begin{pmatrix} u_1 \\ \vdots \\ u_n \\ v_1 \\ \vdots \\ v_n \end{pmatrix}. \tag{D3}$$

Thus, the covariance matrix of the transformed variable $\Sigma^f = \begin{pmatrix} \sigma_{\bar{u}}^2 & \sigma_{\bar{u}\bar{v}} \\ \sigma_{\bar{u}\bar{v}} & \sigma_{\bar{v}}^2 \end{pmatrix}$ can be obtained in the following way:

$$\begin{aligned} \Sigma^f &= A \cdot \Sigma^x \cdot A^T \\ &= \frac{1}{n^2} \begin{pmatrix} 1 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 \end{pmatrix} \\ &\begin{pmatrix} \sigma_{u_1}^2 & \cdots & \sigma_{u_1 u_n} & \sigma_{u_1 v_1} & \cdots & \sigma_{u_1 v_n} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \sigma_{u_n u_1} & \cdots & \sigma_{u_n}^2 & \sigma_{u_n v_1} & \cdots & \sigma_{u_n v_n} \\ \sigma_{v_1 u_1} & \cdots & \sigma_{v_1 u_n} & \sigma_{v_1}^2 & \cdots & \sigma_{v_1 v_n} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ \sigma_{v_n u_1} & \cdots & \sigma_{v_n u_n} & \sigma_{v_n v_1} & \cdots & \sigma_{v_n}^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{pmatrix} \\ &= \frac{1}{n^2} \begin{pmatrix} \sigma_{u_1}^2 + \cdots + \sigma_{u_1 u_n} + \cdots + \sigma_{u_n u_1} + \cdots + \sigma_n^2 \\ \sigma_{u_1 v_1} + \cdots + \sigma_{u_1 v_n} + \cdots + \sigma_{u_n v_1} + \cdots + \sigma_{u_n v_n} \\ \sigma_{v_1 u_1} + \cdots + \sigma_{v_1 u_n} + \cdots + \sigma_{v_n u_1} + \cdots + \sigma_{v_n u_n} \\ \sigma_{v_1}^2 + \cdots + \sigma_{v_1 v_n} + \cdots + \sigma_{v_n v_1} + \cdots + \sigma_{v_n}^2 \end{pmatrix} \\ &= \frac{1}{n^2} \begin{pmatrix} \mathrm{sum}\left(\Sigma_{uu}^x\right) & \mathrm{sum}\left(\Sigma_{uv}^x\right) \\ \mathrm{sum}\left(\Sigma_{uv}^x\right) & \mathrm{sum}\left(\Sigma_{vv}^x\right) \end{pmatrix}. \end{aligned} \tag{D4}$$

Once the daily sea surface current components $\begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix}$ and their covariance matrix $\begin{pmatrix} \sigma_{\bar{u}}^2 & \sigma_{\bar{u}\bar{v}} \\ \sigma_{\bar{u}\bar{v}} & \sigma_{\bar{v}}^2 \end{pmatrix}$ are obtained, it is possible to change from Cartesian to polar coordinates:

$$\begin{cases} r = \sqrt{\bar{u}^2 + \bar{v}^2} \\ \theta = \arctan\left(\frac{\bar{v}}{\bar{u}}\right) \end{cases}. \tag{D5}$$

The transformation now is not linear, and one must resort to the Jacobian matrix:

$$J = \begin{pmatrix} \frac{\partial r}{\partial \bar{u}} & \frac{\partial r}{\partial \bar{v}} \\ \frac{\partial \theta}{\partial \bar{u}} & \frac{\partial \theta}{\partial \bar{v}} \end{pmatrix} = \begin{pmatrix} \frac{\bar{u}}{\sqrt{\bar{u}^2+\bar{v}^2}} & \frac{\bar{v}}{\sqrt{\bar{u}^2+\bar{v}^2}} \\ -\frac{\bar{v}}{\bar{u}^2+\bar{v}^2} & \frac{\bar{u}}{\bar{u}^2+\bar{v}^2} \end{pmatrix} \tag{D6}$$

to obtain the new covariance matrix (θ and σ_θ are in radians):

$$\begin{aligned} \begin{pmatrix} \sigma_r^2 & \sigma_{r\theta} \\ \sigma_{r\theta} & \sigma_\theta^2 \end{pmatrix} &= J \begin{pmatrix} \sigma_{\bar{u}}^2 & \sigma_{\bar{u}\bar{v}} \\ \sigma_{\bar{u}\bar{v}} & \sigma_{\bar{v}}^2 \end{pmatrix} J^T \\ &= \begin{pmatrix} \frac{\bar{u}}{\sqrt{\bar{u}^2+\bar{v}^2}} & \frac{\bar{v}}{\sqrt{\bar{u}^2+\bar{v}^2}} \\ -\frac{\bar{v}}{\bar{u}^2+\bar{v}^2} & \frac{\bar{u}}{\bar{u}^2+\bar{v}^2} \end{pmatrix} \begin{pmatrix} \sigma_{\bar{u}}^2 & \sigma_{\bar{u}\bar{v}} \\ \sigma_{\bar{u}\bar{v}} & \sigma_{\bar{v}}^2 \end{pmatrix} \begin{pmatrix} \frac{\bar{u}}{\sqrt{\bar{u}^2+\bar{v}^2}} & -\frac{\bar{v}}{\bar{u}^2+\bar{v}^2} \\ \frac{\bar{v}}{\sqrt{\bar{u}^2+\bar{v}^2}} & \frac{\bar{u}}{\bar{u}^2+\bar{v}^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\bar{u}^2\sigma_{\bar{u}}^2 + 2\bar{u}\bar{v}\sigma_{\bar{u}\bar{v}} + \bar{v}^2\sigma_{\bar{v}}^2}{\bar{u}^2+\bar{v}^2} & \frac{-\bar{u}\bar{v}\sigma_{\bar{u}}^2 + \left(\bar{u}^2-\bar{v}^2\right)^2\sigma_{\bar{u}\bar{v}} + \bar{u}\bar{v}\sigma_{\bar{v}}^2}{\left(\bar{u}^2+\bar{v}^2\right)^{3/2}} \\ \frac{-\bar{u}\bar{v}\sigma_{\bar{u}}^2 + \left(\bar{u}^2-\bar{v}^2\right)^2\sigma_{\bar{u}\bar{v}} + \bar{u}\bar{v}\sigma_{\bar{v}}^2}{\left(\bar{u}^2+\bar{v}^2\right)^{3/2}} & \frac{\bar{v}^2\sigma_{\bar{u}}^2 - 2\bar{u}\bar{v}\sigma_{\bar{u}\bar{v}} + \bar{u}^2\sigma_{\bar{v}}^2}{\left(\bar{u}^2+\bar{v}^2\right)^2} \end{pmatrix}. \end{aligned} \tag{D7}$$

The Ekman angle standard deviation is then taken as follows:

$$\sigma_{\theta_{\mathrm{E}}} = \min\left(\sigma_\theta, \pi\right). \tag{D8}$$

Code and data availability. The exact version of the model used to produce the results used in this paper is archived on Zenodo (https://doi.org/10.5281/zenodo.14562025, Flora et al., 2024), as are the input data and scripts used to run the model for all the simulations presented in this paper. In particular, `twd_model.f90` is the code implementing the DET model, `twg_model.f90` is the code implementing the GAU model, `tws_model.f90` is the code implementing the STO model with $\nu = 2$ and `twb_model.f90` is the code implementing the STO model with $\nu = 1/2$. Additionally, `twb_FRRD_model.f90`, `twb_FRRS_model.f90`, `twb_datassD_model.f90` and `twb_datassS_model.f90` are the codes implementing the STO model with $\nu = 1/2$ in the FRR and observation-based perturbation applications (applied to the deterministic or stochastic signals).

The HFR sea surface current data of the Gulf of Trieste are freely available from the European HFR node (https://doi.org/10.57762/8RRE-0Z07, OGS et al., 2023). The WRF forecasted wind field is obtainable upon request from ARPA FVG (2024) (https://www.arpa.fvg.it, CRMA).

Author contributions. SF performed the major part of the research and the writing of the paper. LU and AW contributed to the research and the writing.

Competing interests. The contact author has declared that none of the authors has any competing interests.

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Acknowledgements. The forecasts, analyses and related services are based on data and products of the Centro Regionale di Modellistica Ambientale (CRMA), which is a sector of the Agenzia regionale per la protezione dell'ambiente del Friuli Venezia Giulia (ARPA) FVG (https://www.arpa.fvg.it, last access: 29 October 2024). The Piran HFR station has been operated by the Slovenian Environment Agency since July 2022. Between 2015 and July 2022, it was operated by the National Institute of Biology. ChatGPT was used for text refinement of Sect. 5 on an earlier version of the paper.

Review statement. This paper was edited by Vassilios Vervatis and reviewed by two anonymous referees.