AC answer to CC1

We would like to thank Mr. S. Singh for his comments. Our replies (regular font) for each comment (bold font) are provided below.

- The use of the inversion algorithm in this manner appears highly problematic and ad hoc. For instance, if equation (5) holds, then the shapes in equation (7) do not match.

AC. We thank Mr. S. Singh for his valuable comments. The inversion methodology presented in this study is based on cost function minimization, following a Bayesian approach, as established in previous studies (Eckhardt et al., 2008; Stohl, 2011; Kristiansen et al., 2014). Additionally, the inversion scheme developed in this study combines a-priori source information with the output of the FLEXPART dispersion model and PollyXT lidar retrievals to estimate volcanic ash emissions more accurately, ensuring that the approach is well-structured and methodologically robust. A key innovation of this study is the integration of ground-based data into the inversion scheme.

Upon carefully reviewing our calculations, we confirm that the matrix M (SRM) has correct shape of (nxm) and not (mxn), as previously stated in the text. We acknowledge this typographical mistake and have corrected it in the revised manuscript. With this correction, the shapes in Equation (5) and Equation (7) are consistent, ensuring the mathematical coherence of the formulation.

Furthermore, it is implicitly assumed in (7)-(9) that uncertainties associated with all observations are equal to 1. I do not believe this assumption to be valid.

AC. Thank you for your valuable comment. The assumption that all uncertainties associated with the observations are equal to 1 is made because uncertainty is not explicitly addressed in this work. However, we acknowledge that the uncertainties should be included in the inversion calculations and plan to incorporate them in our future work. We will make this clearer in the revised manuscript.

The absence of a regularization parameter in term (8) suggests that the regularization is implicitly taken as 1, but I find it difficult to believe that this is optimal. The authors do not provide any optimization scheme for the regularization parameter in equation (8) or any explanation of how epsilon has been selected.

AC. In this study the regularization parameter was found using a grid search. However, the regularization parameter ϵ is used to enforce smoothness in the vertical profile of emissions and is derived from a discrete second-order difference operator. While a formal optimization of ε is not performed, sensitivity experiments have been conducted to assess its influence on the solution, ensuring that the chosen value provides a stable and reliable inversion outcome.

-Additionally, no sensitivity study to a-priori emissions has been presented.

AC. Thank you for your comment. At this stage of the study, we did not perform a sensitivity study on a-priori emissions. However, we acknowledge its importance and plan to incorporate such an analysis in future work.

- I strongly recommend that the inversion algorithm be made available in a public repository for scrutiny.

AC. While the inversion methodology is detailed in the paper, the specific algorithm implementation is not available online. However, the code is available by the authors upon request.

- Since the authors have SEVIRI data, they should also perform inversion for SEVIRI data and compare the results derived from lidar and SEVIRI (In fact the same team performed the same study already and presented the work at EGU: https://meetingorganizer.copernicus.org/EGU23/EGU23-13755.html). This would serve as a basic form of validation, as has also been pointed out by respected Reviewer 2. At present, there is no validation of the results. However, considering the issues raised above, the publication of this study must be considered with utmost caution.

AC. We thank him for this suggestion. We acknowledge the importance of performing an inversion using SEVIRI data and comparing the results with those derived from lidar observations, as this would indeed serve as a valuable validation step. This aspect is already in our plans for future work, and we are currently preparing a follow-up study that will focus on this analysis in more detail.

In the present version of our work, we are primarily focused on demonstrating an inversion scheme using ground-based observations. Our goal is to establish the methodology and assess its effectiveness before extending the analysis to satellite-based retrievals. We appreciate the reference to our previous work presented at EGU and fully agree that integrating SEVIRI data along with observations from other satellite missions such as EarthCARE, would further enhance the robustness of our findings.