

Modelling L-band Microwave Brightness Temperature Time Series for Firn Aquifers

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Abstract. Firn aquifers are important in polar ice sheet hydrology and the associated mass and energy transport processes. Although the firn aquifer extent has been mapped using passive microwave satellite observations, models for predicting the L-band brightness temperature time series have remained elusive. This paper implements a radiative transfer model for time series L-Band V and H pol brightness temperature (TB) observations from the 3 km SMAP enhanced resolution data product. 15 The model relates the firn aquifer permittivity and properties of the dry firn layer above the aquifer to SMAP observations. Results are presented for aquifers within both the Greenland and Antarctic ice sheets. The results show that the brightness temperature is more sensitive to aquifer liquid water content changes when the water table is closer to the surface. The method provides a tool for the radiometry study of firn aquifer and a theoretical basis for potentially retrieving firn aquifer liquid water content using passive microwave data.

20 1 Introduction

Summer surface melt in the percolation facets of the Greenland Ice Sheet (GrIs) and the Antarctic Ice Sheet(AIS) is an important process [Mankoff et al. 2021, Lenaerts et al. 2019]. An extreme event in 2012, for example, produced surface melt over the entire GrIS [Nghiem et al. 2012]. Meltwater is typically expected to refreeze locally within the firn or run off into the ocean. However, under certain conditions, an ice-water mixed layer called the Perennial firn aquifer can remain unfrozen during the winter season. A major discovery of an aquifer occurred in 2011 on the southeastern coast of Greenland 25 during the Arctic Circle Traverse expedition, which involved a drilled borehole and GPR survey [Foster et al. 2013]. If drained, the Greenland aquifer could contribute up to a 0.4 mm increase in sea level [Koenig et al. 2013].

Instead of simply acting as a meltwater store, the embedded water can flow and drain the aquifer quickly [Miller et al. 2018], and studies have shown an average aquifer age of 6.5 years [Miller et al. 2020]. This indicates that the aquifer is not 30 buffering the mass loss or the rise of the sea level but perhaps amplifying its contribution. Understanding the aquifer's water content can help close the mass balance of Greenland and the meltwater routing mechanism.

The mapping of firm aquifers has been studied using airborne GPR data from Operation Ice Bridge [Miège et al. 2016, Chu et al. 2018]. The total amount of water was estimated in regions where the ice-ground reflection could be observed, from which the decay due to the aquifer layer could be estimated [Chu et al. 2018]. Such studies are limited to the available flight paths, and GPR data were gathered only once yearly in the late spring. Mapping for the horizontal extent of the firm aquifer from satellite data was first performed using Sentinel-1 C band SAR data [Brangers et al. 2020].

L-band brightness temperatures from the Soil Moisture Active and Passive (SMAP) [Entekhabi et al. 2010, Long et al. 2019] mission satellite have also been used in firm aquifer studies. Brightness temperature data shows a logistic-like decrease in brightness temperature from late summer to late spring of the next year due to increased scattering from dry firm [Miller et al. 2020]. The horizontal extent of the Greenland aquifers was then mapped based on time series signatures in enhanced resolution SMAP brightness temperature data. A saturation factor was also derived from time series data to first map the percolation zone and then identify the aquifer and subsurface ice slab [Miller et al., 2022]. However, the liquid water content inside the aquifer has not been quantitatively studied. Due to the low absorption of ice and air, thermal emission from a firm aquifer is modulated by the aquifer-firm interface and the dry firm structure above. Though modelling efforts have been made to explain the time series of the brightness temperature, a simple layered random medium model cannot explain the polarization dependence in the measurements [Bringer et al. 2017]. The modelled results showed agreement with SMAP H pol measurements, but the V pol is 40K higher than the measurements. This is because the layering structures are not able to provide coupling between V and H pol brightness.

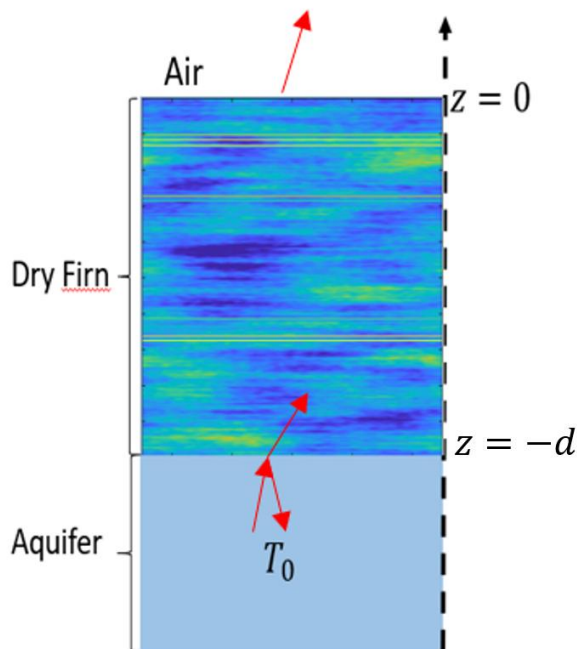
In this paper, a radiative transfer model based on 3-dimensional firm profiles [Tsang et al. 2001, Xu et al. 2023] and an effective permittivity of the aquifer[Huang et al. 2024], predicted from a full wave approach is used to explain brightness temperature time series and their polarization dependence. Changing of the water table location is a hypothesis used in this work. In modelling aquifer thermal emissions, two components need to be characterized. The first is a permittivity model for the firm aquifer, which is the key parameter that relates the firm-aquifer interface reflection to the water content of the aquifer. The second is a radiative transfer model that accounts for the reflection of thermal emission due to the firm-aquifer interface and for density variations in the dry firm. In the emission problem, the firm-aquifer interface produces upward emissions and also reflects downward going emissions from the dry firm above. Multiple scattering also occurs within the dry firm layer due to its inhomogeneity. The approach also allows the possibility of estimating the water content of the aquifer remotely.

The analysis considers 3 km enhanced resolution SMAP brightness temperature time series data for locations in Greenland and Antarctica. For Greenland, the 3 km pixels covering FA-13(66.1812,-39.0435) [Koenig et al. 2013] is studied. FA-13 provides in situ measurements of density (Sumup data set[Montgomery et al. 2018]), temperature ([Clément et al., 2020]), and water level depth (position of aquifer upper boundary). . The location lies on a 2014 survey path of the University of Kansas Accumulation Radar [CReSIS 2024]. The other location lies on the Wilkins Ice Shelf [Montgomery et al., 2020], Antarctica, where a field investigation was performed in Dec 2018. The investigation provides information on the density profile and the water table depth. Temperature and liquid water content information are not available in this location; other in

65 situ information on the Wilkins Ice Shelf is available on the website of the U.S. Antarctic Program (USAP) Data Center [Miège 2020].

Information from the in-situ measurements is applied with the forward model to simulate SMAP V and H pol brightness temperatures. Data collected from winter 2015 to summer 2016 in Greenland and during 2018 for the Wilkins Ice Shelf is analysed.

70 The next section illustrates the physical model for the thermal emission problem. First, mathematical derivations for the radiative transfer model are described, and a discussion of aquifer permittivity is then provided. Data sets used are discussed. In Section 3, The numerical results are presented together with the SMAP measurements over Greenland's east coast from winter 2015 to Spring 2016 and over Wilkins Ice Shelf in 2018. Additional discussions and conclusions are then provided in Sections 4 and 5, respectively.



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Figure 1 A two-layer model for the firn aquifer region in the early spring before surface melting starts. Properties of dry firn change with depth. Three-dimensional varying profiles characterize the firn region. Deep blue's means relatively lower density while yellow means relatively higher density. The aquifer is characterized as a mixture of ice and water. Due to the permittivity contrast of dry firn and aquifer, the interface is a reflective boundary.

80 2 Method

The model describes a firn aquifer as the two-region structure illustrated in Figure 1: dry firn on top of a saturated aquifer. The thickness of the dry firn layer is denoted as d , where $z = -d$ is the water level depth (upper aquifer boundary). The dry firn is described as a random medium having permittivity fluctuations (due to ice density variations) in both the vertical

and horizontal directions, as well as a mean value that varies with depth. The mean density profile is obtained by fitting the
 85 borehole density measurements with an exponential curve. The fluctuating profile is characterized by the following correlation
 function:

$$C(\vec{r}, \vec{r}') = \Delta^2 \exp\left(-\frac{|z - z'|}{l_z}\right) \exp\left(-\frac{|\vec{r}_\perp - \vec{r}'_\perp|^2}{l_\rho^2}\right)$$

where $\Delta = std(\rho)$ is the standard deviation of the density fluctuations, with l_z and l_ρ the vertical and horizontal correlation
 lengths. Density fluctuations in the vertical direction are modelled as exponentially correlated, while those in the horizontal
 90 direction are modelled with a Gaussian function. This model of 3D variations was previously used to model brightness
 temperatures over the accumulation zone in Greenland [Xu, 2024]; the 3-dimensional variations represented cause angular and
 polarization coupling between V and H polarizations that was found to improve the match to measured data. Previously, the
 lower boundary of the variation region was considered to have no reflection. In this case, due to the existence of a firm aquifer,
 the reflection from this boundary could not be ignored.

95 Because aquifers usually exist in the percolation zone of the ice sheet, their density properties can be very different
 from those in the accumulation zone, where the freeze-thaw process is rare. Borehole measurements indicate a high-density
 variation profile in the firm above an aquifer.

2.1 Radiative transfer model in the dry zone region

The emission can be described in the dry zone region by the coupled radiative transfer equations; geometry of the
 100 model is presented in Figure 1:

$$\begin{aligned} \cos\theta \frac{d}{dz} \vec{T}_u(\theta, z) &= -\vec{\kappa}_e(\theta, z) \vec{T}_u(\theta, z) + \kappa_a(z) T_0(z) + \int_0^{\frac{\pi}{2}} d\theta' \sin\theta' [\vec{P}(\theta, \theta', z) \vec{T}_u(\theta', z) + \vec{P}(\theta, \pi - \theta', z) \vec{T}_d(\theta', z)] \\ -\cos\theta \frac{d}{dz} \vec{T}_d(\theta, z) &= -\vec{\kappa}_e(\theta, z) \vec{T}_d(\theta, z) + \kappa_a(z) T_0(z) + \int_0^{\frac{\pi}{2}} d\theta' \sin\theta' [\vec{P}(\pi - \theta, \theta', z) \vec{T}_u(\theta', z) + \vec{P}(\pi - \theta, \pi - \theta', z) \vec{T}_d(\theta', z)] \end{aligned}$$

where $\vec{T}_u(\theta, z)$ and $\vec{T}_d(\theta, z)$ are the upward and downward brightness temperatures, $\vec{\kappa}_e(\theta, z)$ is the extinction coefficient matrix,
 $\kappa_a(z)$ is the absorption coefficient. $\vec{P}(\theta, \theta', z)$ is the phase matrix that couples the brightness temperature in θ' to the direction of θ .
 105 The parameters in the radiative transfer equations are functions of depth. This is because the temperature and density profiles vary
 according to the depth. The radiative transfer equations are subject to the boundary conditions:

$$\begin{aligned} \vec{T}_d(\theta, z = 0) &= \vec{R}_{10}(\theta) \vec{T}_u(\theta, z = 0) \\ \vec{T}_u(\theta, z = -d) &= \vec{R}_{12}(\theta) \vec{T}_d(\theta, z = -d) + (1 - \vec{R}_{12}(\theta)) T_{aquifer} \end{aligned}$$

where $\vec{R}_{10}(\theta)$ and $\vec{R}_{12}(\theta)$ are diagonal reflectivity matrices for the dry firm-air interface (the upper boundary of the dry firm
 110 region) and the dry firm-aquifer interface (the low boundary of the dry firm region). At the dry firm-aquifer interface, the

brightness temperature emitted from the aquifer is transmitted into the dry firm region, as indicated by the term.

$(1 - \overline{R}_{12}(\theta)) T_{aquifer}$ where $T_{aquifer}$ is the aquifer's physical temperature.

115 The radiative transfer equations are solved using an iterative approach. The differential equations are converted into integral equations that express the upward and downward brightness temperature in terms of integrals. Details on how to obtain the expressions are documented in [Xu 2024], and major steps are listed in the Appendix.

The integral expression and the 0th solution are provided in the appendix. The nth-order solutions are obtained by substituting the n-1 th-order term inside the scattering expressions as:

$$\begin{aligned}
 \overline{T}_u^n(\theta, z) &= \vec{S}^n(\theta, z) \\
 &+ \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp \left(-2 \sec \theta \int_{-d}^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \right]^{-1} \vec{r}_{12}(\theta) \overline{W}^n(\theta, -d) \exp \left(-\sec \theta \int_{-d}^z \overline{\kappa}_e(\theta, z'') dz'' \right) \\
 120 \quad &+ \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp \left(-2 \sec \theta \int_{-d}^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \right]^{-1} \\
 &\quad \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp \left(-\sec \theta \int_{-d}^z \overline{\kappa}_e(\theta, z'') dz'' \right) \exp \left(-\sec \theta \int_{-d}^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \vec{S}^n(\theta, 0)
 \end{aligned}$$

$$\begin{aligned}
 \overline{T}_d^n(\theta, z) &= \overline{W}^n(\theta, z) \\
 &+ \left[I - \vec{r}_{12}(\theta) \vec{r}_{10}(\theta) \exp \left(-2 \sec \theta \int_{-d}^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \right]^{-1} \exp \left(-\sec \theta \int_z^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \vec{r}_{10}(\theta) \vec{S}^n(\theta, 0) \\
 &+ \left[I - \vec{r}_{12}(\theta) \vec{r}_{10}(\theta) \exp \left(-2 \sec \theta \int_{-d}^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \right]^{-1} \exp \left(-\sec \theta \int_z^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \vec{r}_{12}(\theta) \\
 125 \quad &\vec{r}_{10}(\theta) \exp \left(-\sec \theta \int_{-d}^0 \overline{\kappa}_e(\theta, z'') dz'' \right) \overline{W}^n(\theta, -d)
 \end{aligned}$$

Where

$$\begin{aligned}
 S^n(\theta, z') &= \sec \theta \int_{-d}^z dz' \exp \left(-\sec \theta \int_{z'}^z \overline{\kappa}_e(\theta, z'') dz'' \right) \int_0^{\frac{\pi}{2}} d\theta' \sin \theta' [\vec{P}(\theta, \theta', z') \vec{T}_u^{n-1}(\theta', z') \\
 &\quad + \vec{P}(\theta, \pi - \theta', z') \vec{T}_d^{n-1}(\theta', z')]
 \end{aligned}$$

And

$$\begin{aligned}
 130 \quad W^n(\theta, z) &= \sec \theta \int_z^0 dz' \exp \left(-\sec \theta \int_z^{z'} \overline{\kappa}_e(\theta, z'') dz'' \right) \int_0^{\frac{\pi}{2}} d\theta' \sin \theta' [\vec{P}(\pi - \theta, \theta') \vec{T}_u^{n-1}(\theta', z) \\
 &\quad + \vec{P}(\pi - \theta, \pi - \theta') \vec{T}_d^{n-1}(\theta', z)]
 \end{aligned}$$

The solution of the downward and upward brightness temperatures is the sum of the multiple orders:

$$\vec{T}_u(\theta, z) = \overline{T}_u^0(\theta, z) + \overline{T}_u^1(\theta, z) + \overline{T}_u^2(\theta, z) + \dots$$

$$\vec{T}_d(\theta, z) = \overline{T}_d^0(\theta, z) + \overline{T}_d^1(\theta, z) + \overline{T}_d^2(\theta, z) + \dots$$

135 The iteration continues until the increment of the next order is smaller than 0.3K.

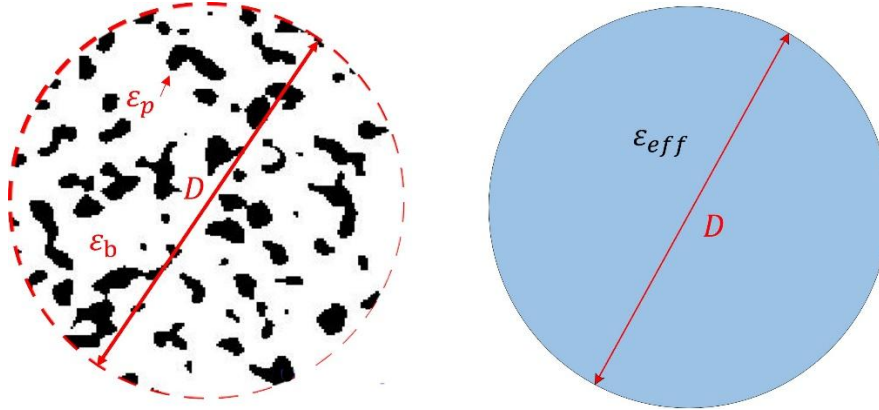
The brightness temperature that is measured by a radiometer at the observation angle θ_{ob} is then

$$\vec{T}(\theta_{ob}) = (1 - r_{10}(\theta))\vec{T}_u(\theta, z = 0)$$

2.2 Full-wave simulations for the aquifer permittivity

A key aspect of the model is the relative permittivity of the aquifer layer, for which a full wave approach is used. Full wave simulation refers to the numerical solutions to Maxwell equations. A firm is a complex, porous structure formed as meltwater percolates down to the deeper part of the firm and fills the space between the ice. In Huang et al. 2024, a computer-generated bi-continuous medium is used in Monte Carlo full-wave simulations to study the effective permittivity of an aquifer. A brief review of the method is presented here.

Consider Figure 2, in which two problems are solved to evaluate the effective permittivity. The random media to be evaluated is truncated into a sphere, as in Figure 2(A). Due to the randomness of the media, several samples of the bi-continuous modelled firm aquifer structure are generated. The scattered waves and absorption for the random structure samples are calculated to evaluate the scattering and absorption coefficients, Q_{scat}^A and Q_a^A respectively. In Figure 2, the white and black areas represent ice and water, respectively.



150 **Figure 2 Problem A(left) and Problem B(right) to be solved to evaluate effective permittivity.**

Notice that the scattering and absorption from problem A are evaluated based on the average scattering and absorption from the computer-generated samples, which are obtained by the following:

$$Q_{abs}^A = \left\langle \frac{k}{\pi(D/2)^2} \int \epsilon''(\vec{r}) |E_{int}(\vec{r})|^2 dV \right\rangle$$

$$Q_{scat}^A = \frac{1}{\pi \left(\frac{D}{2}\right)^2} \int_0^{4\pi} |\langle f_{pp}(\theta_s, \phi_s) \rangle|^2 + |\langle f_{qp}(\theta_s, \phi_s) \rangle|^2 d\Omega$$

155 Where the angular bracket $\langle \rangle$ stands for a Monte Carlo ensemble average over the samples. $E_{int}(\vec{r})$ is the internal field inside the mixture. f_{pp} and f_{qp} are the scattering amplitudes for p-polarized incident waves, with the scattered waves being p and q-

polarized. In the linear polarization basis, p and q can both represent H and V. For the mixture of water and ice in the aquifer, in the L band, water has much higher dielectric loss. The absorption loss due to ice can thus be ignored.

In problem B, the scattering and absorption cross-section (Q_{SCAT}^B and Q_{abs}^B) of a homogeneous sphere are solved. The sphere's diameter in Problem B is the same as the artificial boundary in Problem A. The calculations are based on the analytical Mie scattering theory. Q_{scat}^B and Q_{abs}^B are compared with Q_{scat}^A and Q_{abs}^A . The real and imaginary parts of problem B are adjusted until $Q_{scat}^B = Q_{scat}^A$, $Q_{abs}^B = Q_{abs}^A$. The effective permittivity of the mixture is considered to be the value of the adjusted permittivity.

In this research, the aquifer's liquid water contents range from 5% to 25%. The values are obtained through full-wave simulation using bi-continuous media with $\langle \zeta \rangle = 11000$ and $b=5$, which correspond to an effective particle size of 1mm. A detailed discussion of the simulation process can be found in Huang et al. 2024. In the dry firn, the density profile is converted into the permittivity profile from the work of Matzler 1996 and Tiuri 1984.

Results from the radiative transfer model combined with the aquifer effective permittivity were computed for the three sites of interest using the available in-situ data. The region above the aquifer is assumed to be dry snow.

2.3 Investigation sites and data

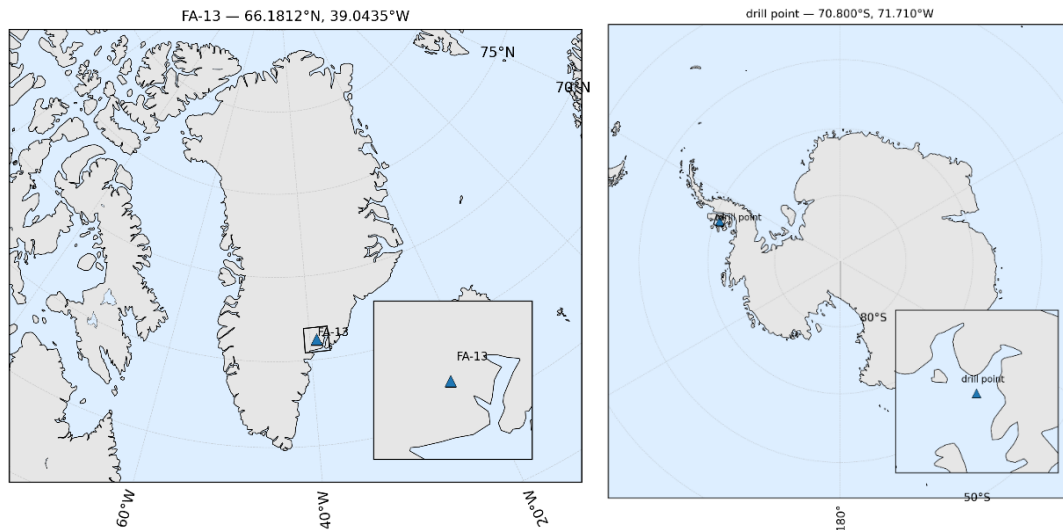


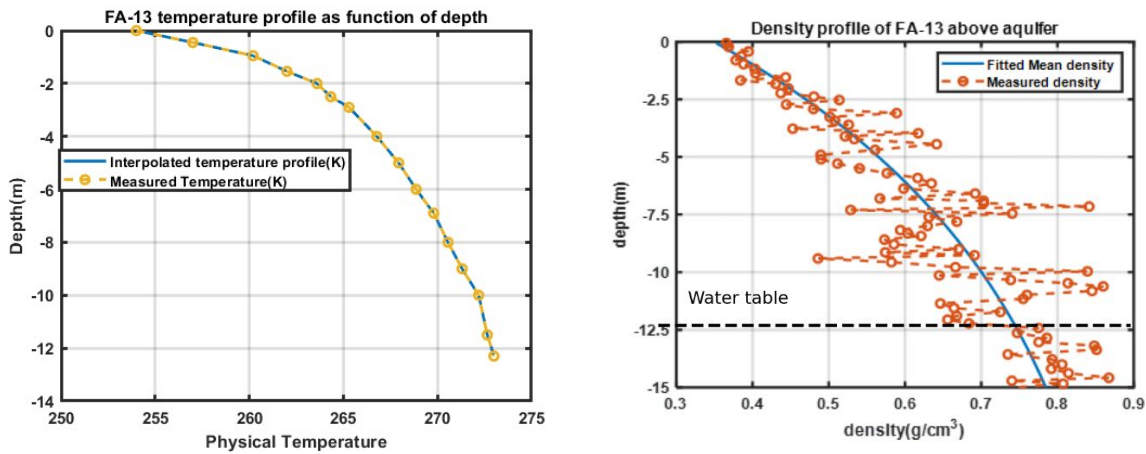
Figure 3 Borehole locations The borehole is near the southeast coast of Greenland, and the borehole study on Antarctica is on the Wilkins Ice shelf.

The borehole locations are presented in **Error! Reference source not found.** for Greenland and Antarctica, respectively. Both two locations are close to the ocean coast.

In situ measurements of physical temperature and firn density are plotted in Figures 4-5 for the Greenland and Antarctic sites, respectively.

For the borehole of FA-13 in Greenland, the water table location (upper boundary of the aquifer) is available from the GPR echogram from the Accumulation Radar survey 2014. The FA-13 borehole was drilled and studied in 2013. Density

180 measurement and water table location are available from the Wilkins Ice Shelf's borehole (Figure 5). The physical temperature in the borehole is not publicly available. A fitting method is used to obtain the temperature profile and the details are discussed later. As the radiative transfer model needs a temperature profile as input, a fitted temperature profile is generated using the surface temperature and water-ice mixture profile. The density and temperature profiles are plotted in Figures 4 and 5.



185 **Figure 4** Temperature(left) and density profiles of FA13 from [Koenig et al. 2013, and Montgomery et al. 2018]. The measurements are taken on a scale of 0.5 meters.

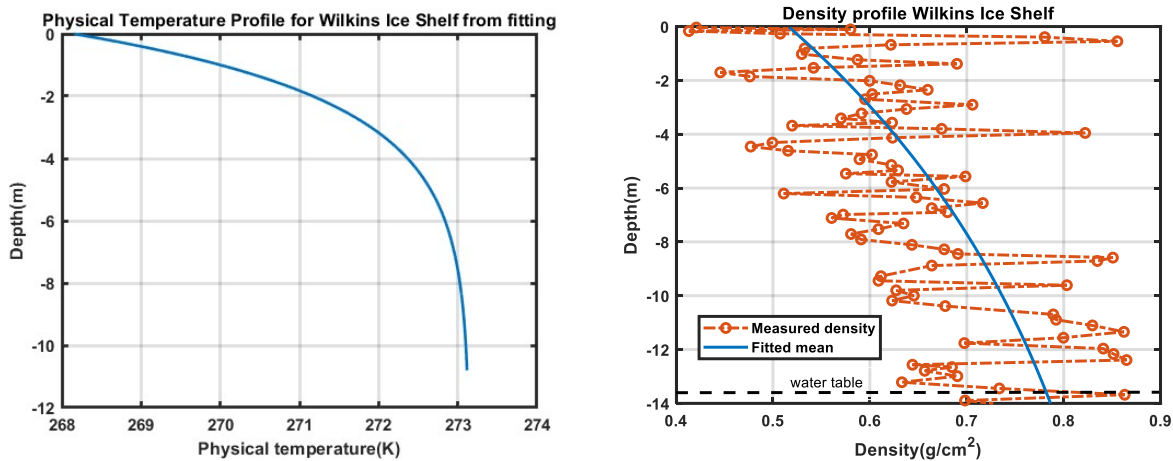


Figure 5 Density profile from the borehole in Wilkins Ice Shelf and the fitted temperature connecting surface and water table. The temperature profile is composed of the surface temperature and the water table depth.

190 The temperature profiles are obtained by the following steps. For FA-13, the temperature profile smoothly increases from 254K at the firm-air interface to 273.15K at the dry firm aquifer interface. This indicates that the top layer contains no liquid water before reaching the water table since the temperatures are below the melting point. For the Wilkins location, the function $T(z) = A - Bexp(Cz)$ is used to connect the surface and aquifer temperatures. In the forward simulation, the physical temperature in the dry firm region is obtained as follows: Assume that the mean surface temperature is at -5°C and the

195 temperature profile follows an exponential-like function as $T(z) = 273.15 - 5\exp(bz)$. The b value can be obtained by forcing $T(z = -13.5) - 273.15 < 10^{-3}$. The results show that setting b, being one-fifth of the water table depth, which is $b=2.65\text{m}$, would fit the need. The surface temperature in the forward simulation will be obtained from MODIS surface temperature product rather than assuming -5°C .

200 For the density profiles, the mean density is obtained by fitting a function of the shape $C - D\exp(-Ez)$ to the measured data. The density profiles show properties that are very different from those in the inner part of Greenland or Antarctica. Instead of having a slowly increasing mean density, as shown in the borehole measurements where the mean profile reaches the critical density of 0.55g/cm^3 at about 15m down from the surface, the mean density profiles in the aquifer regions increase dramatically. The density can be as high as 0.8g/cm^3 at a depth of 12m, as shown in the case of FA13. The magnitude of density fluctuations is also much higher than those in the accumulation zone of Greenland and Antarctica. In terms of the
 205 density variations, the density fluctuates dramatically from the mean value due to the presence of refrozen ice structures in these regions.

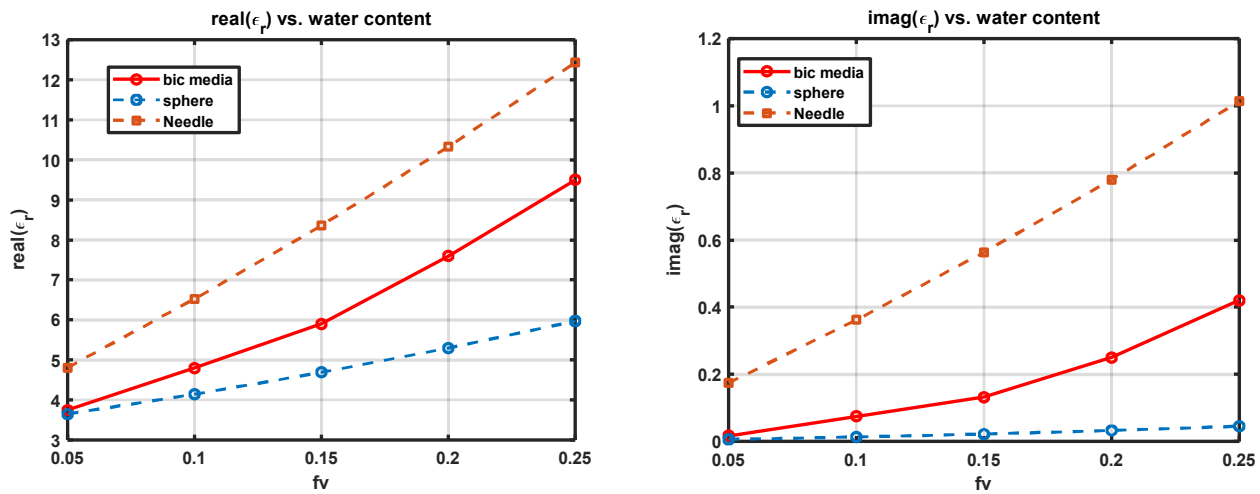
The water level for all the boreholes used in this research is presented in **Error! Reference source not found.**. The positions of FA-13 and Wilkins Ice Shelf are from the borehole measurements.

Table 1 Water Table levels for all three locations

Positions	FA13(66.1812N, 39.0435W)	Wilkins Ice Shelf(-70.80S, -71.71W)
Water table depth from surface(m)	12.5	13.6

210 **Error! Reference source not found.** provides the effective permittivity of the aquifer (ice water mixture) as a function of liquid water content. The three curves correspond to different shapes of inclusions, such as spheres, bicontinuous media(shapeless), and needles. Maxwell-Garnett mixing formula is used to evaluate the sphere and needle cases, while the bicontinuous media is evaluated using the method in section 2.2. As shown in the plots, both the real and imaginary parts of effective permittivity values increase with respect to the increase in liquid water content. However, the speed of the increase
 215 is very different.

Medium with sphere inclusions has the lowest permittivity values in both the real and imaginary parts. The increments are also the smallest in all three cases. For the real and imaginary parts, the values increase from 3.65 to 5.97 and from 0.006 to 0.045 as water content increases from 0.05 to 0.25. Needle inclusion has the largest values and the largest increments. The values increase from 4.8 to 12.4 and 0.17 to 1.01 for the real and imaginary parts, respectively. The bicontinuous media is
 220 between the two, with an increase from 3.75 to 9.5 and 0.017 to 0.42 for the real and imaginary parts, respectively. This indicates that effective permittivity depends highly on the shape of the inclusion. A good geometry description of the mixture in permittivity modelling is important in modelling the emission.



225 Figure 6 Effective permittivity values through the full wave bi-continuous media and spheres

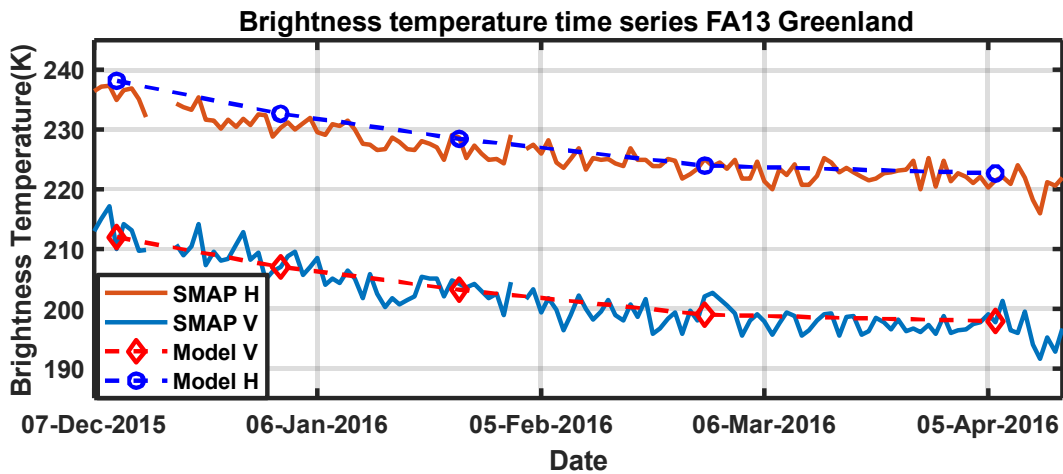
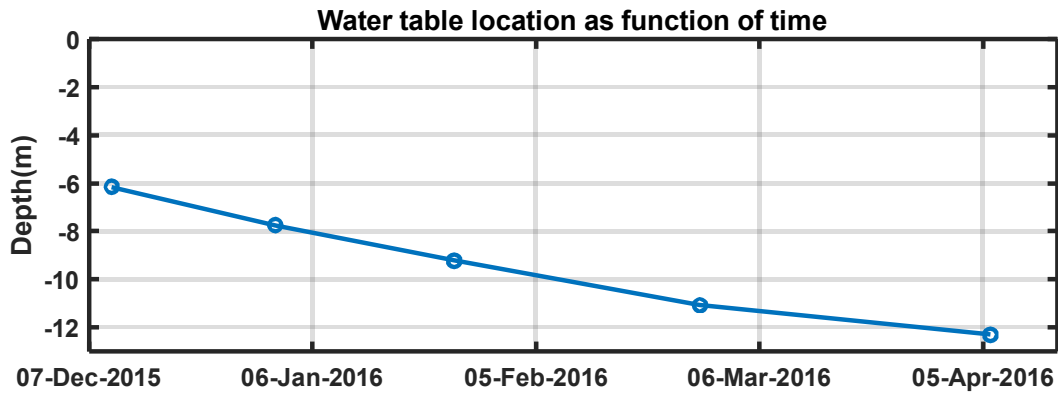
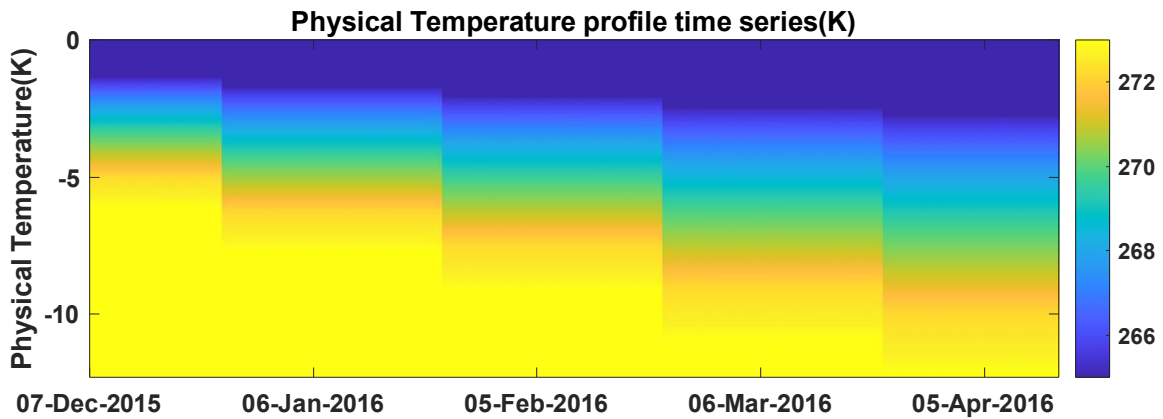
3. Results

3.1 Forward simulation of SMAP brightness temperatures

Time series results for FA13 are provided in Figure 7. The numerical simulation is performed for five points in the time series from Dec 7th, 2015, to early April 2016. Melting events occur in the later days of April. Thus, the later part of the time series is not discussed. The parameters we used for the five-time series points are listed in Table 2.

The temperature profiles and water table location are provided in the top two plots in Figure 7. At the beginning of the time series, early December 2015, the water table is assumed to be half of the depth of April 2016. As spring of 2016 came, the water table gradually decreased to its lowest location of 12.3m from the surface. Notice that the change of water table is a hypothesis as it is observed that the water table of mountain glacier aquifers would decrease seasonally [van den Akker et al, 2025]. Further research is needed to quantify the seasonal changes in the water table. Regarding the temperature profile, at each time point, the temperature profile in April is “squeezed” to fit the temperature at the dry firn aquifer interface. The temperature–depth profile was rescaled by applying a uniform scaling factor to the depth axis, while preserving the temperature values. The scaling factor was determined such that the deepest point of the profile coincides with the water table depth.

As in **Error! Reference source not found.**, the mean density profile is used as input for the iterative approach. The density fluctuation properties are used as a tuning parameter. Radiative transfer theory uses the statistical properties as the inputs, profile from measurement only represent the given location but cannot provide a statistical information for the radiometer resolution area. For this case, a density standard deviation of 0.088g/cm³ is used with vertical and horizontal.



245 Figure 7 Time series for FA13 from winter 2015 to summer 2016. The match-up stopped in early April 2016

Table 2 FA-13 parameters for the 5 points of time series in forward simulations of TB For Greenland. TB reduction evaluates the difference between emissivity=1, direct emission from aquifer and the TB modelled time series. This is to evaluate how much brightness is reduced due to the dry firn above the aquifer.

		Early Dec 2015	Jan	3Feb	March	Early April					
Dry firn thickness(d)(m)		6	7.75	9.2	10.76	12					
Aquifer ϵ_r (10%lwc)		7.26+0.25i	7.26+0.25i	7.26+0.25i	7.26+0.25i	7.26+0.25i					
firn permittivity adjacent to the aquifer		2.25	2.36	2.47	2.59	2.65					
Dry firn density variations		$Std(\rho) = 0.088g/cm^3, l_z = 8.5cm, l_\rho = 20cm$									
		V	H	V	H	V	H	V	H	V	H
TB modeled time series		234.97	209.75	230.7	206.2	227.43	203.44	224	201	222.8	199.5
TB reduction: 273-Model(from line above)		34.8	61	40.3	66	44.5	70	49	74	50.2	74.5
TB reduction	Firn+snow Air interface	21.8	34	28	43	33.8	49.6	39.5	56	40.7	57.5
	Snow -air interface	13	27	12.3	23	10.7	20.4	9.5	18	9.5	17

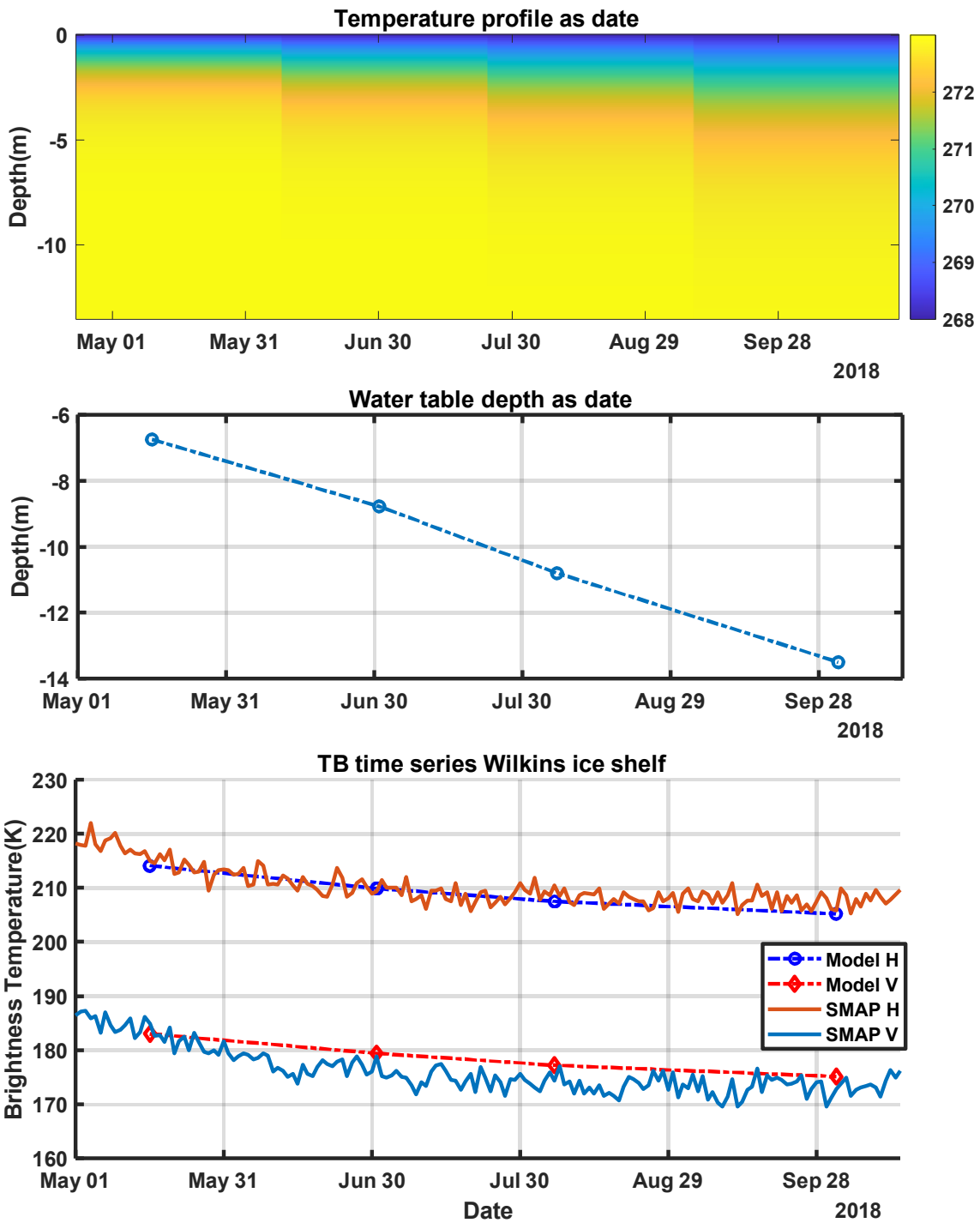


Figure 8 Brightness temperature time series data and forward modelled results over the borehole pixel in Wilkins Ice shelf, 2018.

255 correlation lengths of 8.5cm and 20cm, respectively. The statistical properties of the density fluctuations are considered to be homogeneous from the top to the bottom of the dry firn.

The firn aquifer permittivity is set as $7.6+0.25i$, corresponding to 20% of liquid water content for the water and ice mixture in the firn aquifer. The results are obtained from the full wave simulations using bi-continuous media, as in **Error! Reference source not found.** In the time series simulations, the permittivity of the region below the water table is assumed to be the same for all the data points where forward simulation is performed.

Table 3 Density fluctuation parameters and the aquifer permittivity for the forward match-up results of FA-13 in Figure 7

	Value
$std(\rho)$	$0.088g/cm^3$
l_z	$8.5cm$
l_ρ	$20cm$
$\epsilon_r^{aquifer}$	$7.6 + 0.25i$

With the given density parameters, the simulated time series matches the measured brightness temperatures for both V and H polarizations. The overall standard deviation is around 3K. A detailed analysis of the contribution to the brightness temperature reduction in terms of reflectivity is provided in Table 2. In the analysis, the depth of the water table decreases from 6m in early December from the surface to 12.3m in early April. As indicated in the table, reduction from the aquifer interface decreases. This can be explained by the change of firn permittivity adjacent to the aquifer. The permittivity of dry firn above the aquifer changes from 2.25 to 2.65. The permittivity difference between the aquifer and the dry firn is thus decreased. A major reduction in the emission is due to the density fluctuations in the dry firn. As the depth increases, dry firn volume increases, leading to an increase in reflection numbers due to density fluctuations. The brightness temperature reductions can be explained by the increase in the dry firn volume. Notice that when modelling the time series, the mean density profile is fixed. The change of the water table would ‘truncate’ the mean density profile that holds the variations.

The SMAP brightness temperature time series of the Wilkins Ice Shelf’s aquifer from May to mid-October is then analysed. Based on the in situ measurement from the borehole and the GPR survey over those areas, the water table is 13.5m from the snow-air interface. As shown in Figure 8, the measured brightness temperature over the Wilkins ice shelf is much lower than the observations over Greenland. The typical brightness temperature of H pol in Greenland is usually above 180K, while for the observations in Wilkins Ice Shelf, the H pol measurements are around 173K. The lower brightness temperature could be due to the higher density fluctuations than that in FA-13.

To simulate the brightness temperature for the time series points, the water table location is adjusted as shown in Figure 8. The properties are used to characterize the density fluctuation properties to match up both the V and H pol brightness temperatures: $std(\rho) = 0.13g/cm^3$, $l_z = 4cm$, $l_\rho = 16cm$. The forward simulated results are plotted together with the

SMAP measurements. The simulated results reflect the decrease in brightness temperature, assuming the changes are purely due to the changes in the water table.

285

Table 4 Density fluctuation parameters and the aquifer permittivity for the forward match-up results in Figure 10 for the Wilkins Ice shelf

	Value
$std(\rho)$	$0.13g/cm^3$
l_z	$4cm$
l_p	$16cm$
$\epsilon_r^{aquifer}$	$7.26 + 0.25i$

3.2 Parameter Sensitivity Analysis

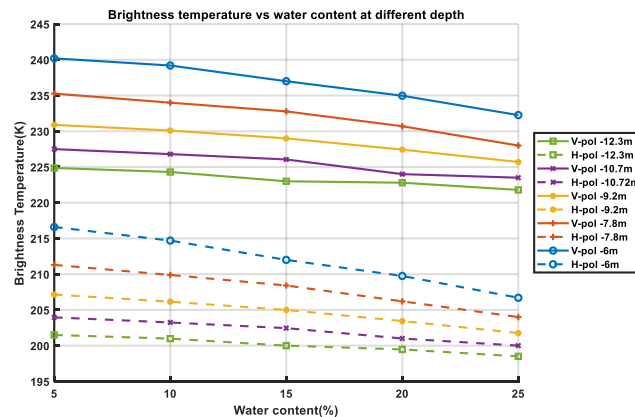
In this section, the FA-13 site is used as an example to analyse the sensitivity of brightness temperature to liquid water content in the firn aquifer. Brightness temperatures at 40 degrees are calculated for different aquifer water table depths and liquid water content.

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Brightness temperatures as a function of liquid water content in the aquifer are plotted in Figure 8. Each subfigure corresponds to a different water table depth from the surface, starting from 6.15 m (half of the maximum depth) to 12.3 m (maximum depth) below the firn-air interface. When performing the simulations, the volume scattering from the dry firn is characterized using $std(\rho) = 0.088g/cm^3$, $l_z = 0.085m$, $l_p = 0.2m$, which is the same as the parameters used for the time series match-up.

295

All brightness temperature plots show a decreasing trend with the increased liquid water content inside the aquifer. However, the impact of liquid water content decreases as the water table depth increases. When the water table is 6.15 m below the surface, the brightness temperatures decrease by 8K in V pol and 10K in H pol as the liquid water content increases from 5% to 25%.



300

Figure 9 Brightness temperature as a function of liquid water content in firn aquifer with water table at different depths.

Then, for water table depth at 7.68, 9.23, and 10.22 m, the brightness temperature decreased by 7.2K, 5.4K, and 4K for V-pol, and 7.3K, 5.4K, and 4K for H-pol, respectively. The changes in brightness temperature finally become 3K and 3.3K when the aquifer water table depth is 12.3 m. This phenomenon happens due to two factors: 1. The decrease of permittivity contrast between the dry firm and aquifer. The permittivity of the aquifer is fixed as the liquid water content is fixed. However, the firm permittivity increases when depth increases due to the increase in mean density. The second factor is the increase of volume scattering in the dry firm region—the total volume scattering increases as more scatterers are included due to the increase in depth.

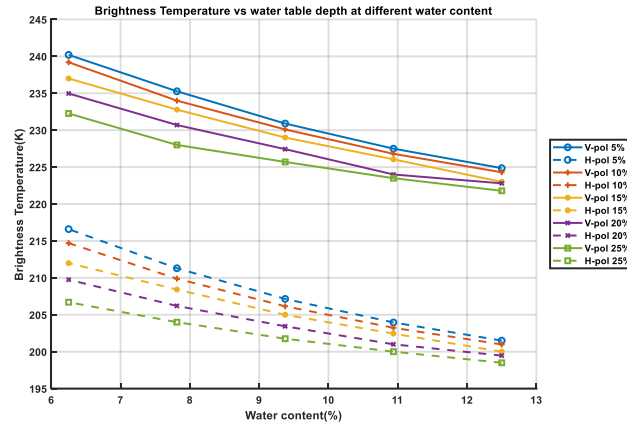
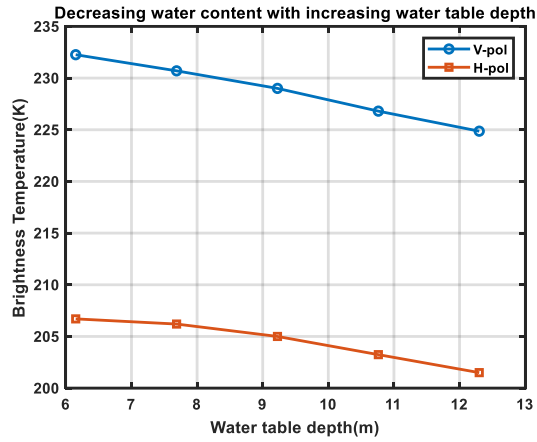
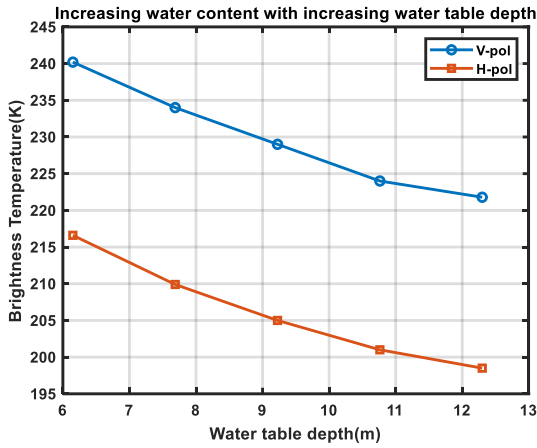


Figure 10 Brightness temperature changes as a function of water table depth at different fixed liquid water contents

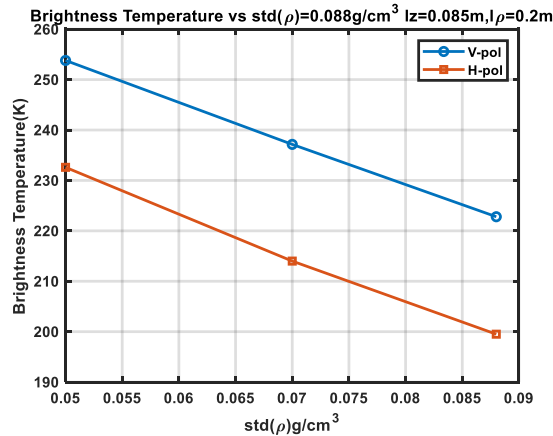
Brightness temperature changes as a function of depth are plotted in Figure 10 for fixed aquifer liquid water content. For all the cases, as the water table depth changes from 6.15 to 12.3 m, the decrease in brightness temperatures is usually more than 10K for both V and H pol. The most significant drop in brightness temperature happens for liquid water content of 5%. The brightness temperature drops from 240K to 225K for V-pol and 216.6K to 201.5K. At 25% liquid water content in the aquifer, the brightness temperature drops about 10K when the water table changes from 6.15 to 12.3 m. The dynamic range decreases as the liquid water content increases. This is due to the increase of multiple reflections happening between the top and bottom interfaces of the dry firm region.

The combined effects of water content and water table depths are presented in this section with the following two combinations: 1. Increasing liquid water content in the aquifer as the water table deepens 2. Decreasing the liquid water content in the aquifer as the water table deepens. It is unclear how the liquid water content would change according to the seasonal changes. These results illustrate the two possible trends of liquid water content from winter to spring. The simulation results would provide a physical intuition for interpreting the time series data.



325 **Figure 11** Brightness temperature changes as a function of depth with the liquid water content in an increasing and decreasing order. For the left-hand side figure, liquid water content increases from 5% to 25% from 6.2 to 12.3m water table depth. For the right-hand side figure, liquid water content decreases from 25% to 5% as the water table depth increases.

Figure 11 plots brightness temperatures for the cases when the liquid water content increases and decreases with respect to the water table depth. The left figure shows the case where liquid water content increases as water table depth decreases. The figure indicates that the decrease in water table depth, when accompanied by an increase in water content, results in a more pronounced reduction in brightness temperature than that caused by the decrease in water table depth alone. 330 The overall effect contributed to a 20K reduction in V-pol and an 18K reduction in H-pol. On the other hand, for the righthand side figure, the decreasing water content counters the effects of the increasing depth. The overall effects decrease the change of brightness temperature with respect to depth. The V-pol decreases by 7k, and the H-pol decreases by only about 5K in total. The parameters of $std(\rho)$, l_z and l_ρ are the parameters that control the properties of volume scattering in the dry firn.



335 **Figure 12** Brightness temperature change with respect to $std(\rho)$ V-pol and H-pol both decrease as the value of $std(\rho)$ Increases.

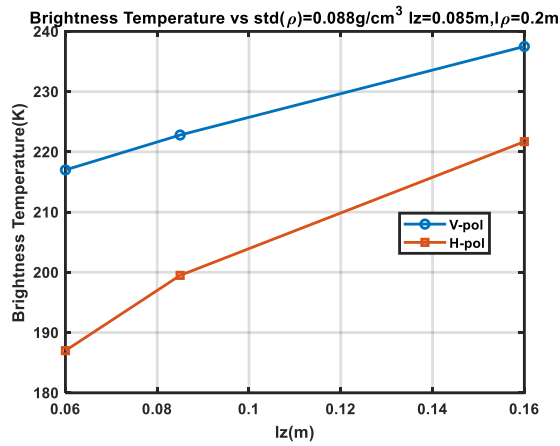


Figure 13 Brightness temperature changes with respect to l_z . With smaller l_z , the reduction to brightness temperature gets stronger.

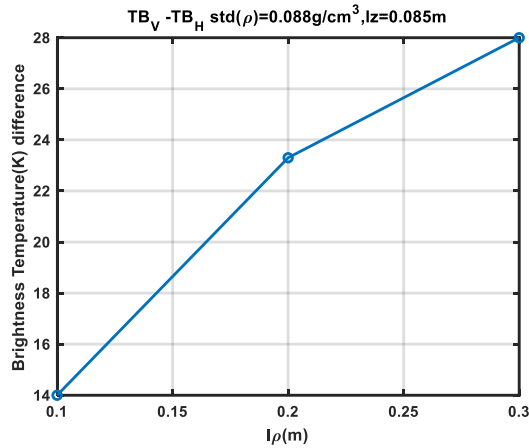


Figure 14 The brightness temperature difference between V and H pol with respect to horizontal correlation length.

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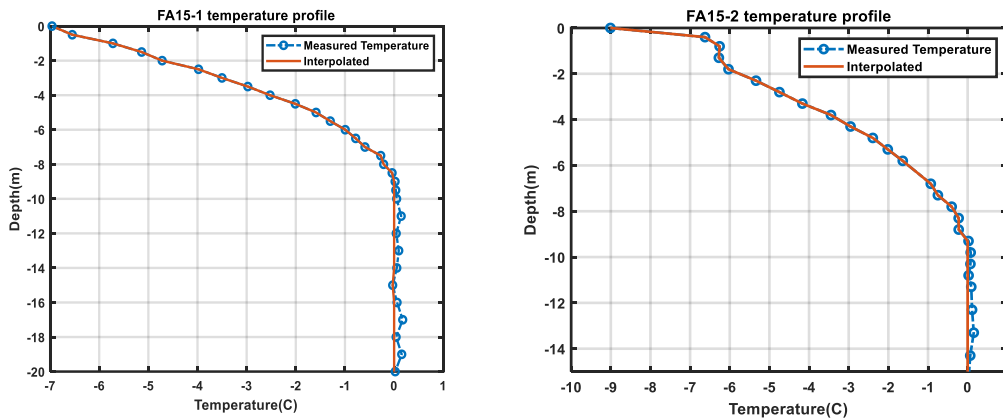
The parameters of $std(\rho)$ and l_z are determining the strength of each reflection and the number of reflections in a unit length in the z-direction. As shown in Figure 13 and Figure 14, the increase of $std(\rho)$ and the reduction of l_z would reduce the brightness temperature in both V and H pol. The horizontal correlation length controls the difference between V-pol and H-pol emission, as shown in Figure 14. The increase of l_ρ would make the media behave more likely to layers as longer horizontal correlation length means the media is homogeneous in a longer horizontal area (layers has $l_\rho = \infty$), making the V-pol and H-pol more different from each other. Smaller l_ρ values allow the V-pol and H-pol to be coupled to each other.

4. Discussion

The radiative transfer model provided in this paper has shown its ability to predict the time series brightness
350 temperature changes from winter to early spring before surface melting starts for both the V and H pol time series brightness
temperatures. This model potentially provides an explanation to the difference between V and H polarized brightness
temperature data. For future work on the retrieval of liquid water content for the top region of the aquifer, this model enables
the use of both V and H pol measurements.

Comparing the time series results from aquifer, it can be observed that the brightness temperatures from Wilkins Ice
355 Shelf are about 10K lower than the TB observed over southeast Greenland. Our analysis provides one of the possible
explanations dry firn above the aquifer of the Wilkins Ice Shelf has a higher density variation. The selection of higher density
variation than Greenland can be justified by the comparison of density profiles in the first section.

In Figure 7, the time series is simulated based on the assumption of unchanged liquid water content in the firn aquifer
and density parameter with a changing depth of the firn aquifer. In Figure 11 The simulated brightness temperatures are based
360 on the same density parameters but include changes in the liquid water content. If the liquid water content in the aquifer has a
decreasing trend, the brightness temperature from December to April is much less than the case when the liquid water content
increases with time and the change in Figure 7. This shows that the change in the liquid water with respect to time can also
influence the time series brightness temperature at the L-band.



365 **Figure 15 Temperature(left) of FA15-1 and FA15-2 [O.Miller et al. 2017]. The water table locations are -19.8m and -14.4m respectively. Unlike what is observed in FA-13, where the physical temperature becomes 0°C at the interface of the dry firn and aquifer, the physical temperature profiles in these two boreholes reach 0°C before reaching the aquifer water table**

The model used in this paper has assumed a dry condition above the aquifer; when there is wetness above, current
model cannot handle it. Figure 15 shows the temperature profiles measured in the boreholes of FA 15-1 and -2. This is different
370 from the case in FA-13, where the temperature profiles reach 0°C at the position of the water table, the temperature profiles in
these two boreholes reach 0°C at a much shallower position than the water table. As a result of this, there is a “warm” region.
The high temperatures in these regions allow the existence of liquid water. These regions are called unsaturated zones in the

work of [Miller et al. 2017]. The thermal emission from these regions is much more complicated since the liquid water content in these regions will affect the contribution from the aquifer. With higher liquid water, radiometers may not see emissions from the aquifer. Further studies are needed to understand the loss factor of the unsaturated zone, where the media is a three-phase mixture of air, ice, and water. A method of separating the regions with and without unsaturated zones is needed to apply the physical model presented in this paper.

One of the features that could affect the emission is the mean density profile of the firm. The mean profile affects both the reflections due to the dry firm and the interface of dry firm and aquifer. To evaluate its effects, several different mean density profiles need to be used in the simulation. However, the in-situ data is very limited. Another approach is to use the simulated profiles. The Community Firm Model [Stevens et 2021] has been updated lately to better deal with the percolation facets of the polar ice sheet. However, the simulated mean density profile reaches the ice density at a much shallower depth than the measured mean density, which does not correctly reflect the truth. The modelled results need to be further improved before using it to analyse the mean density profile effects.

As discussed earlier in the paper, the variation properties of the dry firm affect the emission. These properties need to be characterized. A possible approach is to use the data from higher frequency microwave sensors, such as the AMSR-2 multi-frequency radiometer data at C, X, Ku, and even Ka Bands.

The uncertainty in the surface temperature could affect the parameters used in the forward simulation of the brightness temperature. For example, the surface temperature is changed by 5K in the FA-13 in December. Keeping the same density properties, by changing the water table depth 0.5m higher, the model predicted TB would be within 1K of the observed results. The question of how the seasonal temperature change affects the water table location of aquifer is a more complicated scientific problem beyond the scope of this paper. The question shall be answered in future studies.

Table 5 Model results by changing surface temperature

	V	H
Use April surface temperature	238.2	212
Use 5K lower surface temperature and move water table 0.5m higher	237.8	211.7

395 5. Conclusions

A physical model for the L-band microwave thermal emission of a firm aquifer region was reported. The radiative transfer model formulation is solved iteratively to include scattering contributions within the inhomogeneous dry firm region. The reflections of thermal emission are considered due to the firm-aquifer interface, air-firm interface, and density fluctuations in the dry firm region. A full wave approach also was used to simulate the effective permittivity of the aquifer. The 3D structure

400 of the modeled density fluctuations allowed both V and H-polarized SMAP measurements to be reproduced using a single set of parameters and to reproduce SMAP time series data over the Greenland coast and Wilkins Ice Shelf, Antarctica.

Sensitivity analyses further show that the L-band brightness temperature varies more strongly with aquifer liquid water content when the water table is closer to the surface. This is because the permittivity difference in the aquifer and the dry firm relatively more contributes to the brightness temperature reduction. This indicates a possible better result in the future
 405 retrieval work compared to deeper aquifer cases. It is also shown that both the firm aquifer depth and the liquid water content in the firm aquifer would affect the time series brightness temperature at the L-band. The method provides a tool for the radiometry study of firm aquifer. The brightness temperatures due to different combinations of density properties and firm aquifer properties can be estimated through the model. The method can also be extended to explain the passive microwave measurements in other frequencies. The model also provides a theoretical basis for potentially retrieving firm aquifer liquid
 410 water content using passive microwave data. Look up tables can be generated and used in the future retrieval.

In this paper, we analyze the brightness temperature with a radiative transfer model. Liquid water and the change of water table content (thus increase total volume scattering) may drive the observed time series brightness temperature signatures. Other alternate or other drivers are also likely. Greenland and Antarctic sites may have different drivers. More research work needs to be done.

415

Appendix A. Iterative solution of radiative transfer equations

The first step is to convert the equations from differential-integral form into pure integral forms. For the upward brightness temperature, the variable z is changed into z' , then the term $\exp\left(-\sec\theta\int_{z'}^0\vec{\kappa}_e(\theta,z'')dz''\right)$ is multiplied to both sides of the equation. For upward-going brightness temperatures, integration performed on z' ranges from $z' = -d$ to $z' = z$. The
 420 equation is thus changed into:

$$\int_{-d}^z \cos\theta_i \frac{d}{dz'} \vec{T}_u(\theta, z') \exp\left(-\sec\theta\int_{z'}^0 \vec{\kappa}_e(\theta, z'') dz''\right) =$$

$$\cos\theta_i \vec{T}_u(\theta, z) \exp\left(-\sec\theta\int_z^0 \vec{\kappa}_e(\theta, z'') dz''\right)$$

$$- \cos\theta_i \vec{T}_u(\theta, z' = -d) \exp\left(-\sec\theta\int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right)$$

$$- \int_{-d}^z dz' \cos\theta_i \vec{T}_u(\theta, z') [\sec\theta \vec{\kappa}_e(\theta, z')] \exp\left(-\sec\theta\int_{z'}^0 \vec{\kappa}_e(\theta, z'') dz''\right)$$

425 Integrate the left-hand side by parts and cancel out the same terms on both sides of the equation. The formatted equation becomes the following:

$$\begin{aligned}\vec{T}_u(\theta, z) &= \vec{T}_u(\theta, z' = -d) \exp\left(-\sec \theta \int_{-d}^z \vec{\kappa}_e(\theta, z'') dz''\right) \\ &\quad + \sec \theta \int_{-d}^z \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{z'}^z \vec{\kappa}_e(\theta, z'') dz''\right) dz' \\ &\quad + \vec{S}(\theta, z)\end{aligned}$$

430 Where $\vec{S}_u(\theta, z)$ is the term that accounts for the scattering:

$$S(\theta, z') = \sec \theta \int_{-d}^z dz' \exp\left(-\sec \theta \int_{z'}^z \vec{\kappa}_e(\theta, z'') dz''\right) \int_0^{\pi/2} d\theta' \sin \theta' [\vec{P}(\theta, \theta', z') \vec{T}_u(\theta', z') + \vec{P}(\theta, \pi - \theta', z') \vec{T}_d(\theta', z')]$$

Using a similar technique, the downward brightness temperature in the dry firn region can be obtained by multiplying the term. $\exp\left(-\sec \theta \int_{-d}^{z'} \vec{\kappa}_e(\theta, z'') dz''\right)$ and integrate z' from $z' = z$ to $z' = 0$. After some math, the downward brightness temperature can be simplified into the following:

$$\begin{aligned}435 \quad \vec{T}_d(\theta, z) &= \vec{T}_d(\theta, 0) \exp\left(-\sec \theta \int_z^0 \vec{\kappa}_e(\theta, z'') dz''\right) \\ &\quad + \sec \theta \int_z^0 \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_z^{z'} \vec{\kappa}_e(\theta, z'') dz''\right) dz' \\ &\quad + \vec{W}(\theta, z)\end{aligned}$$

Where:

$$W(\theta, z) =$$

$$440 \quad \sec \theta \int_z^0 dz' \exp\left(-\sec \theta \int_z^{z'} \vec{\kappa}_e(\theta, z'') dz''\right) \int_0^{\pi/2} d\theta' \sin \theta' [\vec{P}(\pi - \theta, \theta') \vec{T}_u(\theta', z) + \vec{P}(\pi - \theta, \pi - \theta') \vec{T}_d(\theta', z)]$$

account for the scattering contribution to the downward directions.

Looking at the expressions for upward and downward going brightness temperatures, the boundary values on the right-hand side of the equations, $\vec{T}_u(\theta, z = -d)$ and $\vec{T}_d(\theta, z = 0)$, are still unknown. Since the targeted unknowns in the radiative transfer equations are $\vec{T}_u(\theta, z)$ and $\vec{T}_d(\theta, z)$, The boundary values need to be canceled using the boundary condition equations.

445 By setting $z = 0$ in T_u and $z = -d$ at $z = -d$, and use the boundary conditions to cancel out the terms $T_u(\theta, z = 0)$ and $T_d(\theta, z = -d)$. The terms can be specified as:

$$\begin{aligned}\vec{T}_u(\theta, -d) &= \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp\left(-2\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right]^{-1} \\ &\quad \left\{ (1 - \vec{r}_{12}(\theta)) T_2 + \vec{r}_{12}(\theta) \sec \theta \int_{-d}^0 \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{-d}^{z'} \vec{\kappa}_e(\theta, z'') dz''\right) dz' \right. \\ 450 \quad &+ \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \sec \theta \exp\left(-\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \int_{-d}^0 \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{z'}^0 \vec{\kappa}_e(\theta, z'') dz''\right) dz' \end{aligned}$$

$$\begin{aligned}
& + \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp\left(-\sec \theta \int_{-d}^0 \kappa_e(\theta, z'') dz''\right) \vec{S}(\theta, 0) \\
& + \vec{r}_{12}(\theta) \vec{W}(\theta, -d) \}
\end{aligned}$$

$$\begin{aligned}
455 \quad \vec{T}_d(\theta, 0) &= \left[I - \vec{r}_{12}(\theta) \vec{r}_{10}(\theta) \exp\left(-2\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right]^{-1} \\
& \left\{ \vec{r}_{10}(\theta) (1 - \vec{r}_{12}(\theta)) T_2 \exp\left(-\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right. \\
& \quad \left. + \vec{r}_{10}(\theta) \sec \theta \int_{-d}^0 \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{z'}^0 \vec{\kappa}_e(\theta, z'') dz''\right) dz' \right. \\
& \quad \left. + \sec \theta \vec{r}_{12}(\theta) \vec{r}_{10}(\theta) \exp\left(-\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \int_{-d}^0 \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{-d}^{z'} \vec{\kappa}_e(\theta, z'') dz''\right) dz' \right. \\
& \quad \left. + \vec{r}_{10}(\theta) \vec{S}(\theta, 0) \right. \\
& \quad \left. + \vec{r}_{12}(\theta) \vec{r}_{10}(\theta) \exp\left(-\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \vec{W}(\theta, -d) \right\}
\end{aligned}$$

460 Substitute back to the upward and downward expressions,

The final expressions for the upward and downward brightness temperatures are:

$$\begin{aligned}
\vec{T}_u(\theta, z) &= \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp\left(-2\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right]^{-1} \exp\left(-\sec \theta \int_{-d}^z \vec{\kappa}_e(\theta, z'') dz''\right) (1 - \vec{r}_{12}(\theta)) T_2 \\
& \quad + \sec \theta \int_{-d}^z \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{z'}^z \vec{\kappa}_e(\theta, z'') dz''\right) dz' \\
& \quad + \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp\left(-2\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right]^{-1} \exp\left(-\sec \theta \int_{-d}^z \vec{\kappa}_e(\theta, z'') dz''\right) \vec{r}_{12}(\theta) \sec \theta \\
465 \quad & \int_{-d}^0 \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{-d}^{z'} \vec{\kappa}_e(\theta, z'') dz''\right) dz' \\
& \quad + \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp\left(-2\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right]^{-1} \exp\left(-\sec \theta \int_{-d}^z \vec{\kappa}_e(\theta, z'') dz''\right) \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \sec \theta \\
& \quad \exp\left(-\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \int_{-d}^0 \kappa_a(z') T_0(z') \exp\left(-\sec \theta \int_{z'}^0 \vec{\kappa}_e(\theta, z'') dz''\right) dz' \\
& \quad + \vec{S}(\theta, z) \\
& \quad + \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp\left(-2\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right]^{-1} \vec{r}_{12}(\theta) \vec{W}(\theta, -d) \exp\left(-\sec \theta \int_{-d}^z \vec{\kappa}_e(\theta, z'') dz''\right) \\
470 \quad & + \left[I - \vec{r}_{10}(\theta) \vec{r}_{12}(\theta) \exp\left(-2\sec \theta \int_{-d}^0 \vec{\kappa}_e(\theta, z'') dz''\right) \right]^{-1}
\end{aligned}$$

$$\vec{r}_{10}(\theta)\vec{r}_{12}(\theta)\exp\left(-\sec\theta\int_{-d}^z\overline{\kappa}_e(\theta,z'')dz''\right)\exp\left(-\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right)\vec{S}(\theta,0)$$

For the downward component:

$$\begin{aligned} \vec{T}_d(\theta,z) &= \left[I - \vec{r}_{12}(\theta)\vec{r}_{10}(\theta)\exp\left(-2\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right) \right]^{-1} \exp\left(-\sec\theta\int_z^0\overline{\kappa}_e(\theta,z'')dz''\right)\vec{r}_{10}(\theta) \\ & (1 - \vec{r}_{12}(\theta))T_2\exp\left(-\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right) \\ 475 \quad & + \sec\theta\int_z^0\kappa_a(z')T_0(z')\exp\left(-\sec\theta\int_z^{z'}\overline{\kappa}_e(\theta,z'')dz''\right)dz' \\ & + \left[I - \vec{r}_{12}(\theta)\vec{r}_{10}(\theta)\exp\left(-2\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right) \right]^{-1} \exp\left(-\sec\theta\int_z^0\overline{\kappa}_e(\theta,z'')dz''\right)\vec{r}_{10}(\theta) \\ & \sec\theta\int_{-d}^0\kappa_a(z')T_0(z')\exp\left(-\sec\theta\int_{z'}^0\overline{\kappa}_e(\theta,z'')dz''\right)dz' \\ & + \left[I - \vec{r}_{12}(\theta)\vec{r}_{10}(\theta)\exp\left(-2\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right) \right]^{-1} \exp\left(-\sec\theta\int_z^0\overline{\kappa}_e(\theta,z'')dz''\right)\vec{r}_{12}(\theta) \\ & \vec{r}_{10}(\theta)\exp\left(-\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right)\sec\theta\int_{-d}^0\kappa_a(z')T_0(z')\exp\left(-\sec\theta\int_{-d}^{z'}\overline{\kappa}_e(\theta,z'')dz''\right)dz' \\ 480 \quad & + \vec{W}(\theta,z) \\ & + \left[I - \vec{r}_{12}(\theta)\vec{r}_{10}(\theta)\exp\left(-2\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right) \right]^{-1} \exp\left(-\sec\theta\int_z^0\overline{\kappa}_e(\theta,z'')dz''\right)\vec{r}_{10}(\theta)\vec{S}(\theta,0) \\ & + \left[I - \vec{r}_{12}(\theta)\vec{r}_{10}(\theta)\exp\left(-2\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right) \right]^{-1} \exp\left(-\sec\theta\int_z^0\overline{\kappa}_e(\theta,z'')dz''\right)\vec{r}_{12}(\theta) \\ & \vec{r}_{10}(\theta)\exp\left(-\sec\theta\int_{-d}^0\overline{\kappa}_e(\theta,z'')dz''\right)\vec{W}(\theta,-d) \end{aligned}$$

Each expression includes terms without scattering (the three terms at the beginning of each expression) and terms that
 485 involve scattering (the remaining four terms). The first and second terms are the direct contribution of the aquifer and dry firn
 emissions. The third term is the downwelling dry firn emission reflected from the aquifer interface. The fourth term is the
 scattering source term, while the fifth is scattering reflected at the boundary once. The last terms are scattering sources due to
 double reflections at upper and lower boundaries.

The terms without scattering are considered as the 0th-order solution in the iterative approach.

490 **Validations of the Iterative Approach for Homogenous Dry Firn**

In this case, the dry firn layer has a uniform mean density profile so that the iterative approach can be compared to
 the eigenvalue approach[Tsang et al. 2001]. The physical temperatures and density properties are all constant. The physical

temperatures of the aquifer and dry fin are set as 273.15K and 265K, respectively. Assuming the aquifer has a liquid water content of 20%, through the full wave simulation approach, the corresponding permittivity value is $7.6+0.25i$ [Huang et al. 2024]. The mean density profile of the dry fin is set as 1.55g/cm^3 , corresponding to a permittivity of 1.65 for the real part. For the imaginary part, the value of 0.01 is chosen to accelerate the convergence of the simulation. The density variation is characterized by a standard deviation of 0.05g/cm^3 with a vertical and horizontal correlation length of 5cm and 50cm, respectively. For both methods, 64 quadrature angles are selected to solve the same emission problem. In the iterative approach, contributions from each order are recorded to check the convergence of the solution. In Figure 3, up to the 9th order iteration, results are plotted for the V and H brightness temperature at 40 degrees as an example to illustrate the convergence. Brightness temperature as a function of observation angles is plotted in Figure 4 to compare the solutions.

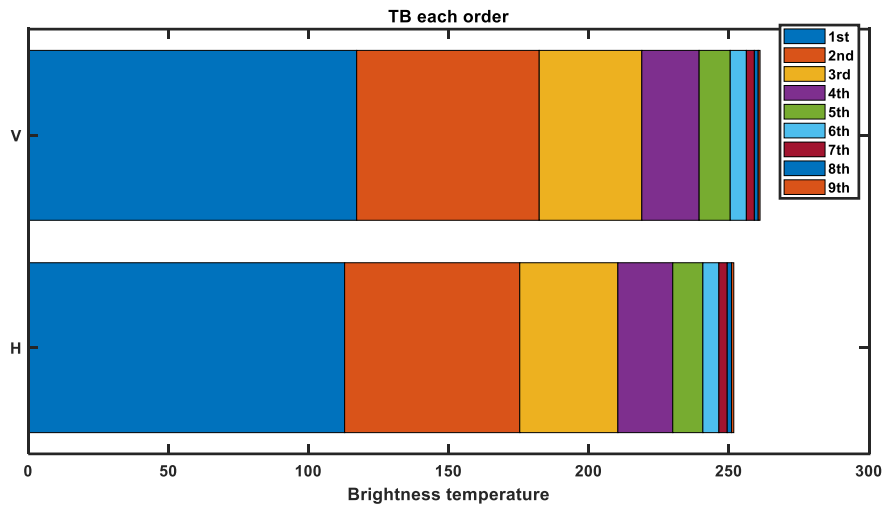


Figure 16 Contribution of brightness temperature from each order for 40 degrees. The first several orders are shown for this simple case. In this particular case, contributions beyond 2nd order have little contribution.

For this problem, the iteration approach calculates up to 30 orders. As indicated in Figure 3, the first five orders are the major contributors to the solution. Contributions from the 8th and 9th orders are much less significant. Only 1.3K and 0.65K are provided for these two orders in V pol 0.8K and 1.5K for H pol. The contributions from the iterations will be much lower as the order goes up. Brightness temperature predictions as a function of observation angles are provided in Figure 4. As it is shown, for both V and H polarizations, the two methods show good agreement with each other. The V polarization results first increase as the observation is close to the Brewster angle and then go down. At the same time, the H pol decreases monotonically.

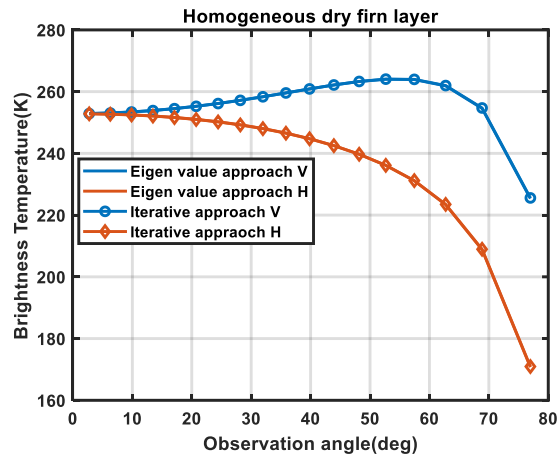


Figure 17 A single homogenous dry firn layer results from eigen value method and iterative approach. The results from the two methods agree very well with each other.

515 Appendix B. Accumulation Radar echogram near FA-13

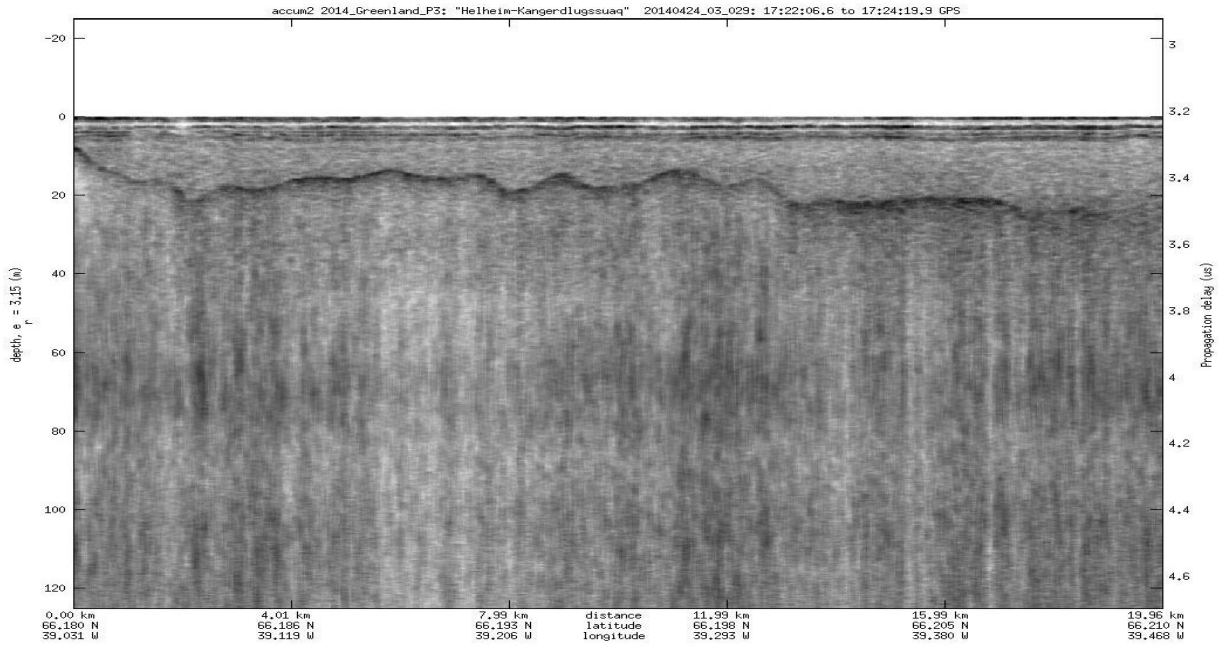


Figure 18 Accumulation Radar data in 2014 near FA-13

The accumulation radar echogram is provided in Figure 13, where the second location of (66.19N, 39.1865W) is in the middle left of the figure. According to the image, the area near the second location has a water table depth of around 15-16m. An average of 15.5m of water table depth is used for the simulation.

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Data and code availability

615 All the data used in the paper is provided in the references.

The code is published on first author’s github website: <https://github.com/Jokerleonxv/firn-aquifer>

Author contribution

620 Haokui Xu performs the major work. Leung Tsang and Brooke Medely provided suggestions on formulating the work and suggestions on the results. Julie Miller provided suggestions on the sensitivity analysis. Joel Johnson provided suggestions on the goal of the paper and the language.

Competing interests

The authors declare no competing interests.