Response to Reviewer Comments on TC paper

Reviewers' comments

Responses to the comments

Responses to Reviewer 2

This paper concerns the modelling of L-band brightness temperatures for dry firn overlying aquifers in Greenland and Antarctica. This is an application of the Huang et al (2024) permittivity model and extension of this work from backscatter modelling in the original paper to brightness temperature in this paper, and comparison of the radiative transfer model with enhanced resolution 3km SMAP data. The paper uses sensitivity studies to examine the impact on different parameters, but stops short of using the model to retrieve aquifer liquid water content. This is a potentially exciting application of this work. The paper would benefit from clarifications and further discussions to highlight the significance of this work, as detailed below.

Specific comments:

1. Representation of the dry firn layer. In this model framework, the dry firn layer, up to ~15m deep, is represented by a single layer that contains density fluctuations around the mean. Figures 4 and 5 show (as expected) vertical variation in density from 350-800 kg / m3. It would be useful to gain some insight as to how good the assumption of a single layer is. It seems some of the observed fluctuations in brightness temperatures are rather extreme e.g. at a depth of 4m in Figure 5, it looks like there may be an ice lens between two lower density layers of around 500 kg / m3. Existing multi-layer radiative transfer theory models (SMRT, LS-MEMLS, DMRT) could be used to test the impact of these layers compared with a single layer approach and thereby demonstrate whether the experimental set-up here is appropriate.

   Thank you for the comments. I would like to clarify that our solution to the radiative transfer function is not using a single layer with uniform properties but is using a layer that has varying properties along the depth. We are assuming a constant density fluctuation parameter because this is the simplest case that can fit to the SMAP data. The scattering properties due to the variation is actually changing along the depth since that the mean density is changing with depth.

In the multi-layer radiative transfer theory models, the phase matrices are considering the scattering due to the snow particles, the scattering is significant at higher frequencies such as C, X and Ku bands. At L band, due to the smallness of snow particles compared to the L band wave length, scattering from snow particle is small. The dry firn is considered as inhomogenous permittivity region.

The phase matrix in our radiative transfer model is obtained by Born's approximation using the statistical properties of inhomogeneous permittivity profile. The phase matrix not only depends on the amplitude of variation but also the mean value. This indicates that although the density variation is the same, change in the background medium would also affect the phase matrix properties. In the formulation, the parameters in the radiative transfer equations change with respect to depth z this is why the final expression is so complicated with integration with z.

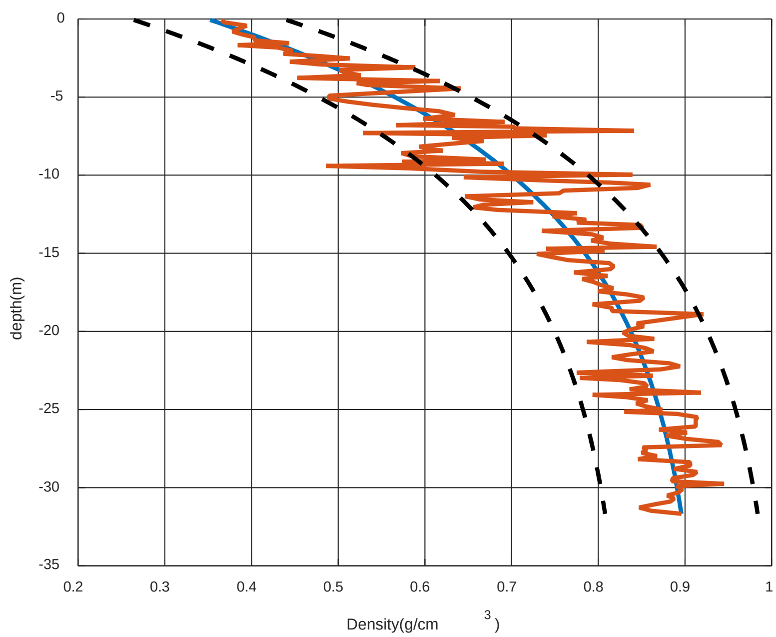
We agree that there might be ice lens at that depth. But the borehole measurement is a measured profile from the area, which is just a single realization of the density variation. While on the other hand, radiometer is measuring the overall effects within the resolution cell, that is many realizations. Radiative transfer equation is handling the averaged results from many realizations. Thus, we are treating the ice lens as part of the density fluctuation profile.

If we calculate the effect of the ice lens in the radiative transfer model, we are assuming such a high-density layer extended across the resolution cell of the radiometer, which means an extent of 3km.

2. Fitted versus physically-derived parameters. It would be useful to highlight which of the parameters applied in the model have been derived directly from the measurements and which have been fitted to the SMAP observations. As far as I understand, the mean densities, temperature profiles and aquifer depths were from observations, whereas the density fluctuations, liquid water content, and vertical and horizontal correlation length were all fitted to the SMAP observations – is this correct? I would recommend parameters for all sites (Tables 3, 4, 5) be combined into a single table, and highlight (perhaps in bold) which ones were directly from the observations. It would be useful to overlay the mean density and a shaded region for fluctuations on to the density profiles to demonstrate how representative the model is.

Yes, your understanding is correct. The density variation parameters are fitting values for all 3 cases. Liquid water content for Greenland is derived from in situ measurements. LWC for Wilkins ice shelf is a fitting parameter. The temperature profile for Wilkins Ice shelf is derived from a exponential like function with the constrains at the surface and water table. Water table is 0C, surface is from MODIS temperature data. Water table location is also a fitting value for the time series except for the one in April. The values in April are from GPR measurements.

We will include a table to show all the fitting parameters used in the model when revising the manuscript. The figure of density is shown below, we use 2 dash lines to represent the region where the density is off from the mean by 1 std.



3. Difference in parameters between two Greenland sites. These two sites are 5-6km apart (different values given in different parts of the paper), leading to the assumption that some of the parameters (density, temperature) are transferable. However, the aquifers have different liquid water assumptions and consequently effective permittivity. What is the reason for the 10% difference? The SMAP data presented show rather different signatures, but are likely adjacent or near adjacent pixels within the same radiometer footprint. A scatterplot comparison between the two may be informative - what in the footprint is causing the difference in TB, given that it's likely coming from the radar? Is

there something specific about the topography that could cause differences in the aquifer water content?

We are referring to the FA-13 measurements from the borehole measurements. I want to apologize for the typo in table 2. FA-13 is using 20% of LWC rather than 10%. The permittivity values are the same if you look at table 2 and table 3. So the 2 are using the same liquid water content.
We want to clarify that the data we used is the enhanced resolution SMAP brightness temperature data with 3km resolution rather than the initial 36km resolution. The selection of the position is to make sure that it is not in the same resolution cell of FA-13.
The difference in TB signature can be explained by the different final water table location as from the observation of radar sounder as the place near FA-13 has a water table location at 15m below surface.

4. Aquifer liquid water assumption. How were the values used derived? Are they realistic for the regions?

   The values used in Greenland is derived from Dr. Lora Koenig's paper: Initial in situ measurements of perennial meltwater storage in the Greenland firn aquifer, GEOPHYSICAL RESEARCH LETTERS, VOL. 41, 81–85, doi:10.1002/2013GL058083, 2014.

   The value in Wilkins Ice shelf is a fitting parameter that can explain the time series data. We believe the values are realistic as they are shown in the measurements.

5. Spatial extent of application. This model assumes dry firn above aquifers, which may form under 'certain conditions' (line 24). What are the conditions that cause this situation and how frequently / on what scale do they occur. This is important to understand how significant this problem is. Although potential contribution to sea level rise is given as 0.4mm for total aquifer drainage, it would be useful to understand whether this a few large aquifers or many small ones.

   Previous researchers have performed the study on the spatial distribution of firn aquifers using SMAP radiometer data, please refer to the work: "Mapping Firn Saturation Over Greenland Using NASA's Soil Moisture Active Passive Satellite" J.Z. Miller JSTARTS 2020. The conditions for the firn aquifers to form is still unclear, but classification through time series signature is possible.

6. Generalisation of modelling. Borehole / GPR were used to parameterize the model – how could modelling be approached without these data i.e. extended to other locations? This also ties in with point 10 below: what would you need to know to be able to do the aquifer liquid water retrievals.

   This point would be answered together with point 10.

7. Credit to earlier work. In particular, this work leverages earlier permittivity work by Huang et al. (2024). Figures 2 and 6 in this paper are rather similar to figures in the Huang paper, which is understandable, but please be explicit about how the work in this paper builds on the work of Huang, particularly placing into context here the treatment of effective permittivity, which is studied much more comprehensively in Huang. There are also numerous other radiative transfer models available – please be explicit about how the one used here is different, and why it is used rather than any other. I would really like to have seen the impact of different permittivity model assumptions on TB – this would be helpful to the community and would again help demonstrate the impact of this work.

   Thank you for your comments.

   Part 1: For the radiative transfer models. We are using a new radiative transfer model that (2)treats the dry frin above firn aquifer as a layer with inhomogeneous permittivity values due to the density variations. (2) The random variations of density profile(permittivity) is not only vertically changing but also horizontally changing. The horizontally changing effect is not included in the multi-layer versions of radiative transfer models. (3)This model is a low frequency model as the scattering from snow particles are not evaluated. The scattering is mainly because of the permittivity inhomogeneity. We have used this model to explain the SMOS angular and polarization dependence observed over Dome-C antarctica.
   Part 2: For the permittivity values from different models. Here the values are listed comparing the shallow water table case and deep water table case:

	Needle $\epsilon_r = 10 + 0.8i$ *TB(K)*		*Bic-model* $\epsilon_r = 7.6 + 0.25i$ *TB(K)*		*Spheres* $\epsilon_r = 5.2 + 0.02i$ *TB(K)*	
	V	*H*	*V*	*H*	*V*	*H*
Water table at	*227*	*196.1*	*231*	*200*	*236.8*	*204.9*

z=-6						
Water table at z=-12.5	*215.6*	*186.1*	*217.3*	*187.4*	*218.5*	*189*

The reason for using this model is because this model is similar to the real physical case of the firn structure.

8. Apparent sensitivity to aquifer wetness in Figure 10. Please explain the physical mechanism for the decrease in TB with increasing wetness here. This is counterintuitive to the increase in permittivity to near blackbody behaviour for wet snow.

   Physically speaking, in the simulation the temperature of firn aquifer is fixed at 0 degree. The change of wetness would affect the permittivity of the aquifer, as wetness increases, permittivity increases. Emissivity from such a medium would decrease as e=(1-r), where r=|R|^2 is the reflectivity and R is the Fresnel reflection coefficient. R is a function of permittivity values across the boundary. When the ice permittivity does not change, the increase permittivity of aquifer would increase the reflectivity and reduce the emissivity. This means the total emission from aquifer is reduced.

   The wet snow phenomenon can be explained by the 2 layer model in radiative transfer theory. Different from the aquifer case, the warm wet layer is above the cold dry layer. Wet snow is different since the wet snow layer is at the top of the dry layer. When wetness increases, with the increase of permittivity ,the absorption of wet snow layer increases, which means it blocks the emission from the colder dry snow layer below it. Although the emissivity at the top boundary is decreased, the increase of the imaginary part of the wet snow makes the emission from this wet warm layer contributes more than the reduction. The radiometer tends to see more contribution from the relatively warmer wet snow due to the increased absorption.

9. Temporal resolution of simulations. Five data points are presented for each site, to represent the seasonal evolution over a year. What are the temporal resolution of observations available for the in situ data and how were these particular points chosen? Is it possible to include other data points? It is not clear how this varied in time, and how the aquifer depth was inferred from these observations.

We want to clarify that the temporal change of firn aquifer is not available from in situ measurements, so depth is a fitting parameter. FA-15 has this kind of information but it is not applicable to our model since a wet firn layer appears between the dry firn and aquifer. The time points are basically chosen one from each month. I think to include more data points, we would need to assume a decreasing rate of the water table, starting from the summer when the surface is wet.

The point we want make is that the brightness time series change can be explained by the change of firn aquifer water table solely or together with the aquifer liquid water content change as shown in the sensitivity analysis. This is one of the possible explanations for the brightness temperature change.

10. Aquifer liquid water content retrievals. As the application of this paper is for aquifer liquid water content retrievals, it would be hugely beneficial to attempt this in the paper, and use the sensitivity study to indicate how well the parameters need to be known. In lieu of this, please could you suggest a methodology for liquid water retrieval, particularly how some of these unmeasured parameters may be estimated. As it stands, this paper does not support the last sentence in the abstract.

Thanks for the suggestion. Actually the next stage of the work would be the retrieval of liquid water content of the aquifer. However, I could not proceed due to founding issue. I would like to continue the work when new funding comes in.

Here is a way I think I can make use of to retrieve the liquid water content of the firn aquifer. (1) We will use the Sum-up data set for the density information that overlaps with the detected aquifer region from SMAP (2) For the temperature profile, we would use a fitting model $T(z) = 273 - C\exp(Dz)$ *with the surface temperature from MODIS measurements and the water table .(3) We would use the brightness temperature from the nearby percolation facet(where no aquifer or ice slab exist) to bracket the density variation properties of the dry firn. (4) We create a cost function to retrieve the water content of aquifer and water table depth at the same time, based on the SMAP brightness temperature.*

11. Structure of material. Site data from other studies should be in the methodology section, along with the map of the sites. These are other people's work with no new analysis in this study.

Thank you for the suggestion. We will move this part to the methodology section with the map of the sites in the manuscript when revising the paper.

12. Code and data availability. The authors are strongly encouraged to make the code publicly available. 'Code available upon request' does not conform to FAIR principles and has meant that it is much more difficult to interpret how the authors have undertaken their research. 'Upon request' is more or less unjustifiable and incompatible with how research is conducted in present times. Links download the data need to be provided in this section.

Thank you for the suggestion. The code has been uploaded to the author's personal github website. Links to the data are also provided: https://github.com/Jokerleonxv/firn-aquifer

We will add the links to the data used in this paper when revising the manuscript.

Technical comments:

Line 37: 'logistic-like': should this be logarithmic?

Line 44: Explain the polarization signal that is not captured by a single layer model, and why the single layer model here is different.

Section 2.1 A figure showing the geometry and nomenclature would be useful here.

Figure 3: needs to be in the context of a larger map (and in the methodology)

Figure 5: Show water table location

Line 215. Explain how this fitting was done.

Line 216. Figure 6 does not show this.

Line 258. Where does this assumption come from? Is it reasonable?

Line 260. Explain the mechanism for squeezing the temperature profile.

Line 261. Why tune the density fluctuations? Why not use the observations?

Table 2. Explain what is meant by ‘TB reductions’ i.e how these are calculated. How are boundary effects separated, given multiple scattering?

Figure 8, caption. How and why is the temperature gradient changed ‘to have a slower changing speed’ – please clarify what this means.

Line 288. ‘As indicated in…’ – please rephrase to clarify what this sentence means.

Line 314. ‘higher aquifer water content’ in Wilkins is inconsistent with values indicated in simulation parameter tables.

Line 383. The vertical correlation length lz controls the layer-like behaviour, not the horizontal.

Thank you, we will address these issues when revising the manuscript