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Equilibrium global warming – a scaling perspective

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Abstract. In the field of climate-change research a lot of effort is devoted to the 'narrowing down' of uncertainties in the estimation of the (fast-feedback) Equilibrium climate sensitivity (*ECS*), the mean global warming as a result of a doubling of the CO_2 concentration in the atmosphere, in order to improve the predictability of the Earth climate system to determine required future greenhouse-gas mitigation targets. A recent update of this quantity was provided by Hansen et al. (2023), reporting a value of 4.8°C ± 1.2°C for doubled CO_2. This outcome is based on a variety of paleo-climate information to overcome limitations of the present, mainly model-based, "best estimate" of about 3°C (IPCC AR6, 2021). Applying the formal framework of feedback analysis, originating in electrical engineering and control systems, the present study sets out to explore possible consequences of this high-end *ECS* update for the long-term Earth system sensitivity (*ESS*), taking into account 'slow' feedbacks by ice sheets and trace gases in a warming world (according to the recent Hansen study for today's GHGs concentrations in the atmosphere likely leading to 10°C equilibrium global warming). As a result, principal scaling relations between variations in the fast-feedback *ECS* and slow-feedback *ESS* are derived, primarily focusing on a better mechanistic understanding of interactions *in* (besides merely improving the predictability *of*) the Earth climate system. These scaling relations may be applied to determine the equilibrium global warming eventually to be expected for a specified CO_2 amount in the atmosphere. As an illustration, implications for the current geopolitical approach, aiming at 1.5 or at most 2 degrees Celsius of global warming as required by the Paris agreement—while we already seem to be on a 10 degrees track because of warming in the pipeline—are analyzed.

1 Introduction

At present, a significant amount of effort in the field of climate-change research is largely devoted to the 'narrowing down' of uncertainties in the estimation of *Equilibrium climate sensitivity (ECS)*, defined as the global mean warming as a result of an instantaneous doubling of the CO_2 concentration in the atmosphere. The Intergovernmental Panel on Climate Change (IPCC) reported a *likely* (66% chance) range of 1.5–4.5°C, and a central value of 3°C (IPCC AR5, 2013), remaining unchanged for more than 25 years. During this period, several community assessments of *ECS* have taken place that attempted to narrow this AR5 *likely* range, based on multiple lines of evidence (For some more recent work see, for instance, Brown et al. (2017), Caldwell et al. (2018), Cox et al. (2018) or Sherwood et al. (2020)). Combining this 'evidence' from different sources, the

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latest IPCC Sixth Assessment Report (AR6) of 2021 provided an *ECS likely* range of 2.5–4°C, with a best estimate of 3°C (equaling the central value of AR5).

In a recent study of Hansen et al. (2023) an update of this quantity was given, reporting a value of 4.8°C ± 1.2 °C for doubled CO_2. This outcome is based on a variety of paleo-climate information to overcome limitations of the earlier, mainly model-based, estimates mentioned above. The rationale being the signs that ‘slow’ feedbacks by the cryosphere (ice sheets) and trace gases possibly already start to kick in (see, for instance, NASA: Global Climate Change vital-signs web portal for most recent facts and figures), potentially leading to a large-scale destabilization of the Earth climate system (according to the recent Hansen study for today’s GHGs concentrations in the atmosphere likely leading to 10°C equilibrium global warming). Hence, the fact that these effects are not accounted for by the present generation of climate models, which however stand at the basis of international policy making (aiming at 1.5 or at most 2 degrees Celsius of global warming), may be regarded as peculiar, if not worrisome. An important aspect in this respect is the uncertainty about the timescale of the cryosphere response (how *slow* is ‘slow’?). In line with the more ‘traditional’ point of view, the cryosphere response is considered to become relevant on a timescale of millennia (see Hansen, 2005, for an editorial essay on this topic). Following his reasoning however, due to the nonlinear characteristics of this process, we could very well talk about a response time of centuries or less. Because of this multiscale aspect of Climate vs Earth system sensitivity to greenhouse gas (GHG) forcing, a more qualitative assessment of the ‘long-term’ cryosphere interactions described here may be of interest.

Applying the formal framework of feedback analysis, described by Roe in his pedagogic review of 2009 and originating in electrical engineering and dynamical systems theory, the present study sets out to explore possible consequences of the high-end *ECS* update of Hansen et al. (2023) for the long-term Earth system sensitivity (*ESS*), taking into account the ‘slow’ feedbacks in a warming world. Following this approach, the (highly nonlinear) cryosphere effects do not need to be explicitly modeled to show that already something may be learned from the structural relationship between the different feedbacks involved.

As a result, principal scaling relations between variations in the fast-feedback *ECS* and slow-feedback *ESS* are derived, primarily focusing on a better mechanistic understanding of interactions *in* (besides merely improving the predictability *of*) the Earth climate system. These scaling relations may be applied to determine the equilibrium global warming eventually to be expected for a specified CO_2 amount in the atmosphere.

As an illustration, implications for the current geopolitical approach, aiming at 1.5 or at most 2 degrees Celsius of global warming as required by the Paris agreement—while we already seem to be on a 10 degrees track because of warming in the pipeline—are analyzed.

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2 Method

As stated in the introduction, our main objective is to support a better mechanistic understanding of the various mutual interactions in the Earth climate system by investigating the role of climatic feedback, and *ECS* in particular, from a broader, dynamical systems perspective. To achieve this, we examine the structural relationship between the different feedbacks involved, making a distinction between 'fast' (climate) and 'slow' (cryosphere and other remaining) processes following a standard feedback-analysis approach. By explicitly separating the role of climatic feedback from the overall system, a 'reference system' (i.e., a system without the feedback) may be obtained, which constitutes the central aspect of feedback analysis as described by Roe in his pedagogic review of 2009.

As a first step, in Section 2.1 the classical notions of climate sensitivity and feedback are introduced and the underlying principal parameters (feedback factor f and gain g) are defined. This will serve as the methodological basis for the combined feedback analysis of 'fast' (climate) and 'slow' (cryosphere and other) processes.

In Section 2.2 the concept of climate sensitivity is viewed from a dynamical systems perspective. The simple Hasselmann model is introduced, approximating the transfer from radiative forcing to the global mean surface temperature change by a first-order differential equation. The principle parameters of interest are the system's heat capacity C and the radiative damping coefficient λ back to space, the ratio of which defines the dynamical response time τ of the system. Solving the equation for the equilibrium condition for an instantaneous doubling of the CO_2 concentration yields the equilibrium climate sensitivity *ECS*. By explicitly considering the radiative transfer in the absence of climate feedbacks (radiative blackbody damping only), the reference system is obtained which enables us to derive some simple relations between λ and *ECS* in the transfer domain, and the climate feedback factor f and gain g as introduced above in the feedback domain.

Subsequently, in Section 2.3 the effects of the remaining 'slow' feedbacks are incorporated by adding an additional term to the climate feedback gain g as described above. For a given climate sensitivity *ECS* this yields an estimate of the corresponding Earth system sensitivity *ESS*. In terms of both f and g, using a combined feedback analysis it is shown that this scaling relation can be rewritten in a two-stage serial form. This form proves to be very useful for the calculation of *ESS* values on top of *ECS* ranges for different values of the cryosphere feedback gain. These scaling relations may be applied to determine the equilibrium global warming eventually to be expected for a specified CO_2 amount in the atmosphere.

2.1 Climate sensitivity

Climate sensitivity (S) is defined as the equilibrium global mean surface temperature change (ΔT_{eq}) in response to a specified unit radiative forcing (F) according to (Hansen et al., 2012):

$$S = \Delta T_{eq}/F \qquad (1)$$

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This quantity S depends on climate feedbacks, making a distinction between the 'fast-feedback' Charney sensitivity (Charney, 1979) related to fast hydrological responses (water vapor, cloud and sea ice) and the 'long-term equilibrium' sensitivity, related to slow processes (e.g. surface-albedo feedbacks governed by changes of ice-sheet area and vegetation cover). These two components map onto *ECS*, the 'Equilibrium climate sensitivity', and *ESS*, the 'Earth system sensitivity'. To further elaborate on this climate-feedback perspective, the equilibrium global mean surface temperature change (ΔT_{eq}) is written as (in accordance with Hansen et al., 2008):

$$\begin{aligned} \Delta T_{eq} &= f\,\Delta T_0 \\ &= \Delta T_0 + \Delta T_{feedbacks} \\ &= \Delta T_0 + \Delta T_1 + \Delta T_2 + \ldots, \end{aligned} \tag{2a}$$

where ΔT_0 is the global mean surface temperature change in the absence of climate feedbacks (radiative blackbody damping only), f is the net feedback factor and the ΔT_i are increments due to specific feedbacks. As an alternative to the feedback factor f, Hansen introduced the gain g to clarify the role of climate-feedback processes:

$$\begin{aligned} g &= \Delta T_{feedbacks}/\Delta T_{eq} \\ &= (\Delta T_1 + \Delta T_2 + \ldots)/\Delta T_{eq} \\ &= g_1 + g_2 + \ldots \end{aligned} \tag{2b}$$

g_i is positive for an amplifying feedback and negative for a feedback that diminishes the response. The additive nature of the g_i, unlike f_i, is a very useful characteristic of the gain. Evidently

$$f = 1/(1 - g) \qquad \text{or, conversely,} \qquad g = 1 - 1/f \tag{3}$$

Note that some studies use a different (or even reversed) definition of gain and feedback factor (see, for instance, the review of Roe (2009) as cited in the introduction).

2.2 Equilibrium climate sensitivity *ECS*

To simulate and explore the dynamic response of the Earth climate system to variations in radiative forcing, simple models in structure similar to the one proposed by Hasselmann (1976) may be useful. At the core of these models stands a first-order differential equation, linking changes in radiative forcing ΔR to the global mean surface temperature change ΔT_s:

$$C\frac{d\Delta T_s}{dt} = -\lambda\,\Delta T_s + \Delta R \tag{4}$$

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with C the system's (mainly ocean) heat capacity and λ the radiative damping coefficient back to space.

For the present feedback analysis of climate sensitivity, defined as the equilibrium mean surface temperature change in response to an instantaneous doubling of the CO_2 concentration, a control-systems approach might be useful to determine the dynamic characteristics of this step response. From this perspective, Eq. (4) can be considered as the 'planetary thermostat equation' which after Laplace transformation with the differential operator s yields the following simple transfer function:

$$H_r(s) = \frac{\Delta T_s(s)}{\Delta R(s)} = \frac{1}{(Cs + \lambda)} \quad (5)$$

In Fig. 1 the corresponding block diagram is presented as a 'negative-feedback controller' with a feedback gain of λ, the radiative damping coefficient**.** According to the control diagram, the net incoming energy flux is integrated and stored as additional heat in the system, raising its temperature at a rate determined by the system's heat capacity C. Subsequently, governed by the radiative damping coefficient λ, the outgoing energy flux is increased until a new thermal equilibrium is obtained. To illustrate the limiting role of the possibly increasing net fast climate feedbacks on the outgoing energy flux for higher temperatures, the influence on the effective radiative damping coefficient λ is expressed as a nonlinear function of ΔT_s.

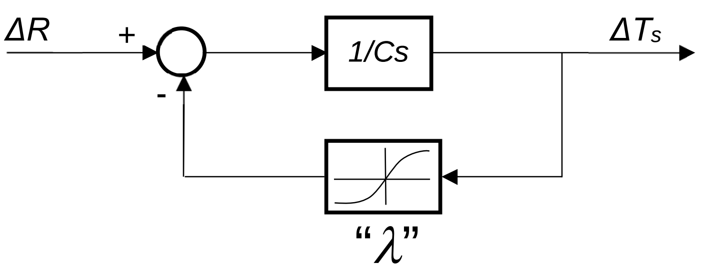

Figure 1. Transfer from radiative forcing ΔR to global mean surface temperature change ΔT_s as a 'negative-feedback controller'. According to the diagram, the net incoming energy flux is integrated and stored as additional heat in the system, raising its temperature at a rate determined by the system's heat capacity C. Subsequently, governed by the radiative damping coefficient λ, the outgoing energy flux is increased until a new thermal equilibrium is obtained. In this equilibrium condition, the global mean surface temperature change ΔT_s is given by $\Delta R/\lambda$.

An important parameter characterizing the transient response of this first-order system is given by the ratio C/λ, defined as the time constant or response time τ (see, for instance, Buchdahl, 1999). Given the large value of the system's (mainly ocean) heat capacity this may lead to a climate response time in the order of magnitude between 50 and 100 years, as described in Hansen (2005). This large heat capacity also plays a central role in the 'reddening' of the stochastic (white-noise) radiative forcing of the ocean by chaotic weather systems. Considering the transfer function for this case in the frequency domain by

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substituting $s = j\omega$ in Eq. (5) results in a 'low-pass' filter equation with a critical cut-off frequency $\omega_c = 1/\tau = \lambda/C$, providing a simple theoretical basis for the analysis of 'natural variability' of the climate system at different (up to multidecadal) timescales, as was originally studied by Hasselmann (1976). In Cox et al. (2018) this stochastic case of white-noise forcing was used to provide a theoretical basis for their search of an emergent constraint on *ECS*, based on the instrumental record of observed variations of the global-mean temperature for the past century.

In equilibrium, as can be directly seen by setting the differential operator $s = 0$ in Eq. (5), the (stationary) relation between radiative forcing ΔR and global mean surface temperature change ΔT_s is given by $1/\lambda$, the DC gain of the above Hasselmann model, which is independent of the system's heat capacity *C*.

In Fig. 2, the situation is presented in the absence of climate feedbacks (radiative blackbody damping only), from now on referred to as the 'reference system' as introduced by Roe (2009). The theoretical value of the radiative damping coefficient for this reference case, defined as λ_0, amounts to 4 W m^{-2} K^{-1}, which can be derived from a linearization of the Stefan-Boltzmann law of blackbody emission, (see, for instance, NRC (2003) for a derivation, and elaborated in the *practical considerations* in the next section).

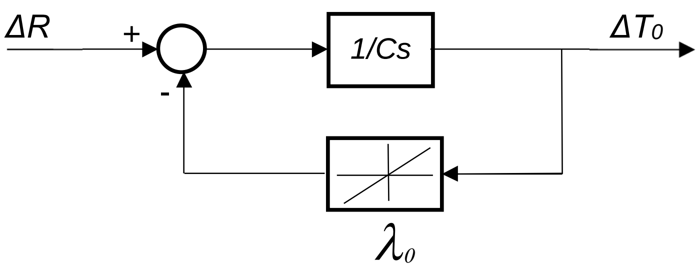

Figure 2. Introducing the 'reference system': transfer from radiative forcing to global mean surface temperature change in the absence of climate feedbacks (radiative black-body damping only). In thermal equilibrium, the global mean surface temperature change ΔT_0 is given by $\Delta R/\lambda_0$.

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Combining this relation with $\Delta R/\lambda$ as derived above in the presence of climate feedbacks yields a ratio $\Delta T_s/\Delta T_0$, defined as the feedback factor *f*, of (Eq. (2a)):

$$f = \Delta T_s/\Delta T_0 = (\Delta R/\lambda)/(\Delta R/\lambda_0) = \lambda_0/\lambda \qquad \text{or, conversely,} \qquad \lambda = \lambda_0/f \tag{6}$$

With respect to the feedback gain *g* this yields (right-hand part of Eq. (3)):

$$g = 1 - 1/f = 1 - \lambda/\lambda_0 \qquad \text{or} \qquad \lambda = \lambda_0.(1 - g) \tag{7}$$

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As can be seen from Fig. 1, a special case arises for a forcing ΔR of λ_0 W m^{-2}. This forcing more or less equals the radiative forcing of approx. 4 W m^{-2} for a doubling of atmospheric CO_2 concentration, defined as ΔR_{2x}. According to the control diagram, for this case the global mean surface temperature change ΔT_s in thermal equilibrium is given by λ_0/λ, equaling the feedback factor f as defined above. Hence, the equilibrium mean surface temperature change for a doubling of atmospheric CO_2 (being the definition of *ECS*) can be considered as a practical, first estimate of the climate feedback factor f:

$$f = \lambda_0/\lambda \approx \Delta R_{2x}/\lambda = \Delta T_{2x} = ECS \qquad \text{and} \qquad g = 1 - 1/f \approx 1 - 1/ECS \tag{8}$$

As an example, for the current IPCC best estimate of ECS = 3°C this corresponds to a positive climate feedback gain g of 1 – 1/3 = 2/3. According to the right-hand part of Eq. (7), this reduces the radiative damping coefficient λ of the 'planetary thermostat' of Eq. (4) by a factor 3, by the reciprocal dependency increasing both the DC gain $1/\lambda$ and response time C/λ of the first-order reference system by a factor 3.

2.3 Earth system sensitivity *ESS*

To obtain an estimate for the 'long-term' Earth system sensitivity, the effect of slow feedbacks may be incorporated by adding a long-term equilibrium component to the climate fast-feedback gain as described above.
Fig. 3 shows the combined effect of these two feedbacks (in the diagram denoted as 'g_1' and 'g_2') in the form of a 'positive-feedback amplifier', taking the 'no-feedback' reference system output ΔT_0 of Fig. 2 as an input.

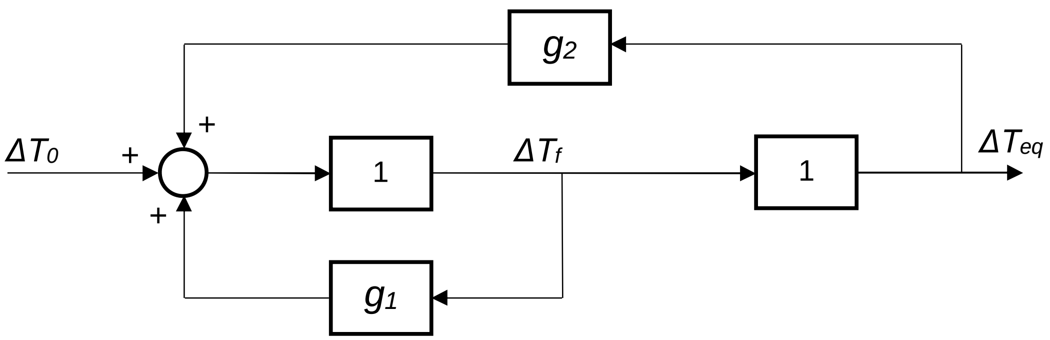

Figure 3. Amplifier diagram for the combined amplifying effects of two positive feedbacks g_1 and g_2. The first part of the diagram describes the amplification by the feedback g_1 from ΔT_0, the global mean surface temperature change in the absence of feedbacks, to ΔT_f, the global mean temperature change including the feedback. In the second part the amplifying effect of the second feedback g_2 is added, resulting in the equilibrium global mean surface temperature change ΔT_{eq}.

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In terms of climate sensitivity, this yields the following 'scaling scheme' from *ECS* to *ESS*:

1.	for a given climate sensitivity *ECS* the fast-feedback contribution to the gain g is calculated according to Eq. (8)
2.	from this the combined gain g_c is calculated by adding the fast-feedback and long-term equilibrium components: $g_c = g_1 + g_2$
3.	the combined earth-system feedback factor f_c is derived from g_c according to Eq. (3)

which results in an estimate of the Earth system sensitivity *ESS*.

An alternative way to achieve this scaling is based on a direct, nonlinear expression for the combined earth-system feedback factor f_c (see, for instance, Buchdahl, 1999):

$$f_c = \frac{f_1.f_2}{f_1+f_2-f_1.f_2} \tag{9}$$

with f_1 the net feedback factor of the 'fast' feedbacks and f_2 the feedback factor of the 'slow' second-order feedback processes. To obtain an analytical derivation of this expression, we take the basic relation between feedback gain g and feedback factor f of Eq. (3) as a starting point:

$$f = \frac{1}{1-g} \quad \text{or, conversely,} \quad g = \frac{f-1}{f} \tag{10}$$

For two different gains g_1, g_2 this yields:

$$g_1 = \frac{f_1-1}{f_1} \quad \text{and} \quad g_2 = \frac{f_2-1}{f_2} \tag{11}$$

As stated in Section 2.1, unlike f, a very useful characteristic of the gain g is its additive nature with respect to the individual feedbacks: $g = g_1 + g_2 + \ldots$

In our case of two gains g_1 and g_2 this yields for the combined gain g_c:

$$g_c = g_1 + g_2 = \frac{f_1-1}{f_1} + \frac{f_2-1}{f_2} = \frac{f_1.f_2 - f_2 + f_1.f_2 - f_1}{f_1.f_2} = \frac{2.f_1.f_2 - (f_1+f_2)}{f_1.f_2} \tag{12}$$

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and the corresponding combined feedback factor:

$$f_c = \frac{1}{1 - g_c} \tag{13}$$

Finally, substituting Eq. (12) for g_c in Eq. (13) and rearranging terms yields:

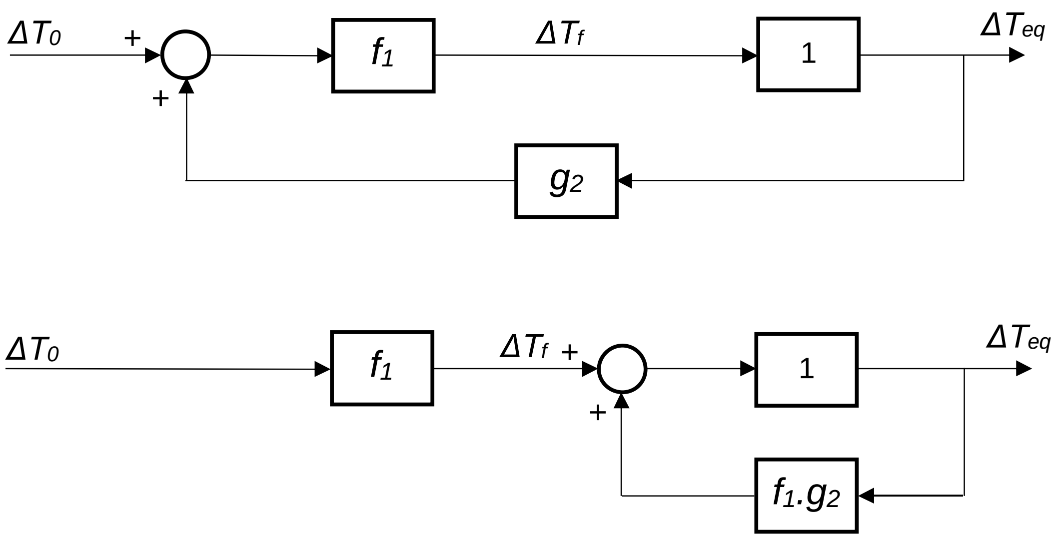
$$f_c = \frac{1}{1 - \dfrac{2.f_1.f_2 - (f_1+f_2)}{f_1.f_2}} = \frac{f_1.f_2}{f_1+f_2 - f_1.f_2} \tag{14}$$

which is the highly nonlinear, neither additive nor multiplicative relation of Eq. (9) as reported in Buchdahl (1999). In terms of both f and g, this combined expression for f_c can be rewritten in the following two-stage serial form:

$$f_c = f_1 . \frac{f_2}{f_1+f_2 - f_1.f_2} = f_1 . \frac{1}{1 - f_1.(1 - 1/f_2)} = f_1 . \frac{1}{1 - f_1.g_2} \tag{15}$$

As can be seen from the right-hand part of this expression, the first-stage feedback factor f_1 serves as an additional amplification of the second-stage feedback gain g_2, showing the complex cascading nature of this basic two-stage example as illustrated below in Fig. 4.

(a)



(b)

Figure 4. Equivalent representation of the amplifier diagram of Fig. 3 in terms of both the feedback factor f and feedback gain g. As a first step **(a)** the feedback gain g_1 of Fig. 3 is replaced by the corresponding feedback factor f_1. Subsequently, this is rearranged in the equivalent two-stage serial form from input to output **(b)**.

Substituting *ESS* and *ECS* as estimates for f_c resp. f_1 (first part of Eq. (8)), this may be rewritten as:

$$ESS \approx ECS. \frac{1}{1 - ECS.g_2} \quad \text{or} \quad ESS/ECS \approx \frac{1}{1 - ECS.g_2} \tag{16a}$$

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Note that in this equation the scale factor between *ECS* and *ESS* is dependent of *ECS*, expressing the nonlinear nature of this scaling relation.

As a function of the radiative damping coefficient λ, an alternative formulation is given by (applying Eq. (6)):

$$ESS \approx ECS.\frac{1}{1-\lambda_0/\lambda.g_2} \quad \text{or} \quad ESS/ECS \approx \frac{1}{1-\lambda_0/\lambda.g_2} \qquad (16b)$$

Practical considerations

As described in Section 2.2, the theoretical value of the radiative damping coefficient in the absence of climate feedbacks, defined as λ_0, amounts to 4 W m^{-2} K^{-1}, which can be derived from a linearization of the Stefan-Boltzmann law of blackbody emission. In thermal equilibrium, for the 4 W m^{-2} radiative perturbation that a doubling of CO_2 produces this corresponds to a global mean surface temperature change ΔT_0 of $4/\lambda_0 \approx 1$°C.

In practice, as described in Roe (2009), the finite absorptivity of the atmosphere in the longwave band implies that in global climate models this value is increased to about 1.2°C, with a corresponding value of λ_0 of $4/1.2 \approx 3.3$ W m^{-2} K^{-1}.

Applying this 'practical' value of λ_0 to Eq. (8) yields with respect to the climate feedback factor f :

$$f = \lambda_0/\lambda \approx 4/1.2/\lambda = \Delta T_{2x}/1.2 = ECS/1.2 \quad \text{and} \quad g = 1 - 1/f \approx 1 - 1.2/ECS \qquad (17)$$

which, besides the introduction of an additional 'correctional' scalar of 1.2, obviously has no further implications for the algebraic expression of the scaling relations derived above.

In the following, the scaling relations will be applied to determine the equilibrium global warming eventually to be expected for a specified CO_2 amount in the atmosphere under the present Holocene conditions. Of specific interest is the recently updated estimate of *ECS* by Hansen et al. (2023), reporting a value of 4.8°C ± 1.2°C, in contrast to the latest IPCC AR6 (2021) range of 2.5–4°C, with a best estimate of 3°C for doubled CO_2 (Section 3.1). Further, in Section 3.2 implications for the current geopolitical approach, reducing current GHG emissions to achieve 1.5 or at most 2 degrees Celsius of global warming as required by the Paris agreement, are analyzed.

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3 Results

3.1 Earth system sensitivity *ESS*

At present, as elaborated in the recent Hansen et al. (2023) study, the radiative forcing increase due to all human greenhouse gases emissions in the atmosphere is already equivalent to 4.6 W m^{-2}, exceeding the 4 W m^{-2} of a CO_2 doubling scenario. To obtain an estimate of the corresponding Earth system sensitivity *ESS*, it is assumed that for this CO_2 doubling scenario the total climate forcing would include complete deglaciation of Antarctica and Greenland, resulting in an additional surface albedo forcing (slow feedback) of 2 W/m^2, and 25% amplification by non-CO_2 GHG feedbacks. For an initial CO_2 doubling forcing of 4 W m^{-2} this yields a total forcing of (4+2)*1.25 = 7.5 W m^{-2}. Together with an *ECS* of 4.8°C, this results in an *ESS* of (7.5/4)*4.8°C = 9°C.

As demonstrated in the feedback analysis in Section 2.3, the scale factor of 7.5/4 between *ESS* and *ECS* is in principle only valid for the particular value of *ECS* under consideration, in this case the recent 'Hansen value' of 4.8°C. For other values the scaling relation Eq. (16a) should be applied. Of special interest is the IPCC AR6 best estimate of 3°C, and the corresponding *likely* range of 2.5–4°C. Given the reciprocal relation between *ECS* and λ, the same 'validity restriction' with respect to the scale factor between *ESS* and *ECS* holds for the radiative damping coefficient, as described in Eq. (16b). This relation is especially useful to obtain an estimate of *ESS* given a particular value of λ, for instance as provided by the CMIP5 archive of climate models in Appendix A.

In Fig. 5 the estimated long-term Earth system sensitivity *ESS* is plotted as a function of the Equilibrium climate sensitivity *ECS*. In the figure, on the horizontal axis both the IPCC AR6 (2021) and Hansen (2023) *ECS* best estimates and uncertainty ranges are presented. Using the non-linear scaling relation of Eq. (16a) (red curve), the Hansen *ECS* range is mapped onto the corresponding *ESS* range (vertical bar on the right axis). The linear scale factor (dashed red line) derived above for *ECS* = 4.8°C is used as a calibration value, intersecting the gain curve at *ESS* = 9°C.

Furthermore, individual scaling results for the CMIP5 models are added as a scatter plot by applying the right-hand part of Eq. (16b) to the different values of λ_i and ECS_i provided in Appendix A.

3.2 Equilibrium global warming *EGW*

In Fig. 6 the estimated long-term Equilibrium global warming *EGW* is plotted for a radiative forcing equivalent to achieve an equilibrium warming of 2°C, the maximum allowable value in accordance with the Paris Agreement. The figure is obtained by a linear scaling of Fig. 5 with a factor 2/3, reducing the maximum vertical scale value from 15°C to 10°C. For the IPCC AR6 *ECS* 'best estimate' of 3°C (left axis Fig. 5), this would indeed result in an *EGW* of 2/3*3°C = 2°C. At the left axis, the currently already realized warming of 1.2°C is presented, to which the additional warming commitment according to the Hansen analysis (right axis) is added to achieve an estimate for the future warming 'in the pipeline' (6°C in total). Again, as in Fig. 5, scaling results for individual CMIP5 models are added as a scatter plot.

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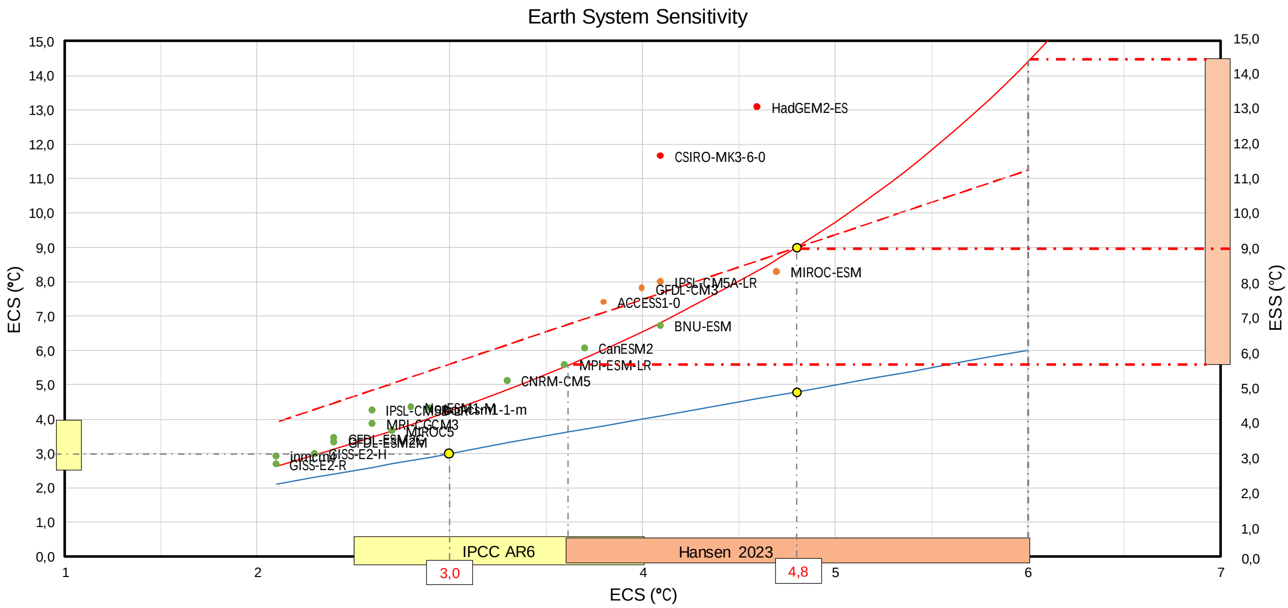

Figure 5. Earth system sensitivity (*ESS*) as a function of *ECS*. On the horizontal axis (horizontal bars) the IPCC AR6 and Hansen (2023) *ECS* best estimates and uncertainty ranges are presented. Using the non-linear scaling relation of Eq. (16a) (red curve), the Hansen *ECS* range is mapped onto the corresponding *ESS* range (vertical bar on the right axis). The linear scale factor (dashed red line) derived above for *ECS* = 4.8°C is used as a calibration value, intersecting the gain curve at *ESS* = 9°C. Individual scaling results for CMIP5 models are added as a scatter plot by applying the right-hand part of Eq. (16b) to the different values of λ_i and ECS_i provided in Appendix A.

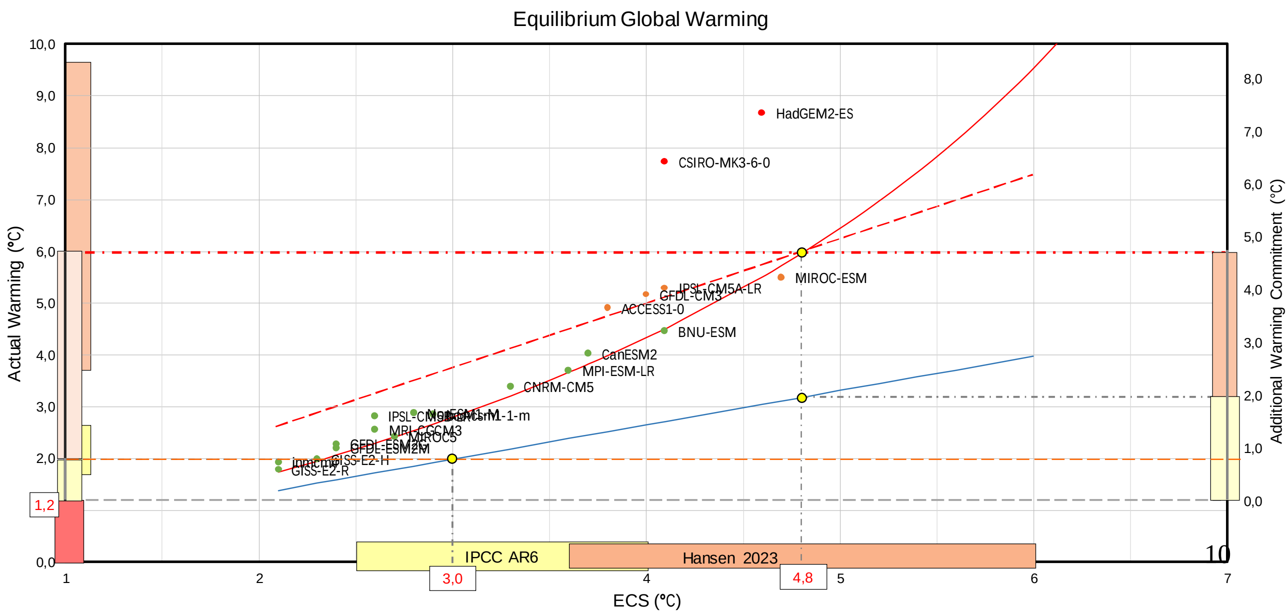

Figure 6. Equilibrium global warming (*EGW*) as a function of *ECS* for a radiative forcing equivalent to achieve an equilibrium warming of 2°C, in accordance with the Paris Agreement. The figure is obtained by a linear scaling of Fig. 5 with a factor 2/3, reducing the maximum vertical scale value from 15°C to 10°C. At the left axis (vertical bars), the currently already realized warming of 1.2°C is presented, to which the additional warming commitment according to the Hansen analysis (right axis) is added to achieve an estimate for the future warming 'in the pipeline' (6°C in total). As in Fig. 5, scaling results for individual CMIP5 models are added as a scatter plot.

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4 Discussion and Conclusion

As presented in Fig. 5, the recently updated estimate of *ECS* of 4.8°C ± 1.2°C by Hansen et al. (2023) on the longer term may lead to values in a range well beyond the AR6 *ECS likely* range of 2.5–4°C by the 'scaling up' process from *ECS* to *ESS*, due to the 'slow' feedbacks by ice sheets and trace gases in warming world. This is supported by an assessment of the (red) gain curve, showing the nonlinear increasing role of the cryosphere feedback for an increasing *ECS*, compared to a linear scale factor. This scaling relation is of the type '$1/(1–f.g_0)$', in close analogy with the '$1/(1–f)$' behavior of the gain curve as identified by Roe et al. (2007) in their paper on fundamental feedback behavior of dynamical systems. As derived in the two-stage combined feedback analysis in Section 2.3, in this expression the (atmosphere-ocean) fast-feedback factor f serves as an additional amplification of the glacial feedback gain g_0, showing the complex cascading nature of this basic two-stage example. As also pointed out by Roe et al. (2009) in their feedback study of climate sensitivity, simply because of this nonlinear nature of the gain curve itself, especially for an increasing *ECS* this will have fundamental implications for the combined assessment of *ESS* uncertainty ranges on the basis of GCM model studies, such as in the CMIP5 archive. In Fig. 5 this is illustrated by the high *ESS* uncertainty range (vertical bar on the right axis) as a scaling result of the updated Hansen *ECS* range. Also *ESS* scaling results for the individual CMIP5 models confirm a relative high sensitivity to an increasing *ECS*, demonstrated by the substantially increasing intermodel scatter as a function of *ECS*.

With respect to the Paris Agreement, to limit the 'allowable warming' in the pipeline for this century to some preset value, e.g. 2°C, the results of Fig. 6 are of interest. The new Hansen results suggest a potential large influence on the (committed) temperature response for the different timescales at or beyond this 'Paris time horizon':

- On a century timescale, on top of the 1.2°C warming currently already realized, according to the recent Hansen *ECS* update, an additional warming commitment of about 2°C may be expected, already giving rise to an overshoot of 1.2°C with respect to the 2°C 'Paris setpoint'.
- Beyond the century (to millennial) timescales the 'slow' ice-sheet albedo and trace-gas feedbacks come into play. Current century Paris targets and *ECS* ranges are more or less setting the stage for the magnitude of the long-term temperature response (*ESS*) caused by these feedbacks, further increasing the additional warming commitment according to the Hansen analysis by about 3°C (right axis).

In total, the above analysis leads to a 4°C future warming in the pipeline on top of the 2°C 'Paris' setpoint, resulting in an Equilibrium global warming of about 6°C. This value sits more or less in between today's actual warming of 1.2°C and the expected 10°C warming for today's GHGs concentrations in the atmosphere reported in the recent Hansen study. However, as demonstrated in this paper, this estimate is accompanied by considerable fundamental uncertainties in projected changes, caused by cascading feedbacks from the at present more or less still stable Holocene glacial boundary conditions in a warming world.

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Appendix A: CMIP5 Models

Table A1. Radiative damping coefficients λ and Equilibrium Climate Sensitivity *ECS* for the different Earth system models as provided by the CMIP5 archive (Taylor et al., 2012).

Model	λ (W m^{-2} K^{-1})	*ECS* (K)
ACCESS1-0	0.8	3.8
CanESM2	1.0	3.7
CCSM4	1.2	2.9
CNRM-CM5	1.1	3.3
CSIRO-MK3-6-0	0.6	4.1
GFDL-ESM2M	1.4	2.4
HadGEM2-ES	0.6	4.6
inmcm4	1.4	2.1
IPSL-CM5B-LR	1.0	2.6
MIROC-ESM	0.9	4.7
MPI-ESM-LR	1.1	3.6
MRI-CGCM3	1.2	2.6
NorESM1-M	1.1	2.8
bcc-csm1-1	1.1	2.8
GISS-E2-R	1.8	2.1
BNU-ESM	1.0	4.1
GFDL-ESM2G	1.3	2.4
GFDL-CM3	0.8	4.0
IPSL-CM5A-LR	0.8	4.1
MIROC5	1.5	2.7
bcc-csm1-1-m	1.2	2.9
GISS-E2-H	1.7	2.3

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Appendix B: List of variables and parameters

Symbol	Description	Units
ECS	Equilibrium Climate Sensitivity	°C
ESS	Earth System Sensitivity	°C
S	Climate sensitivity	K W^{-1} m^2
F	Specified unit radiative forcing	W m^{-2}
ΔR	Radiative forcing	W m^{-2}
ΔR_{2x}	Radiative forcing for a doubling of atmospheric CO_2	W m^{-2}
ΔT_{2x}	Equilibrium mean surface temperature change for a doubling of CO_2 forcing	°C
ΔT_{eq}	Equilibrium global mean surface temperature change	°C
ΔT_s	Global mean surface temperature change	°C
ΔT_0	Global mean surface temperature change in the absence of climate feedbacks	°C
ΔT_f	Global mean surface temperature change including fast (hydrological) feedbacks	°C
f	Climate feedback factor	unitless
f_c	Combined feedback factor for multiple feedback gains	unitless
g	Climate feedback gain	unitless
g_0	Glacial feedback gain	unitless
g_c	Combined gain for multiple feedbacks	unitless
λ	Radiative damping coefficient	W m^{-2} K^{-1}
λ_0	Radiative damping coefficient in the absence of climate feedbacks	W m^{-2} K^{-1}
$H_r(s)$	Radiation transfer function	
C	Earth system's (mainly ocean) heat capacity	J K^{-1} m^{-2}
τ	Time constant of the first-order radiation transfer function	s

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