

Reviewer 1:

The authors processed 4861 MODIS images of sea ice in the Beaufort Sea spanning the months of March through September in the years 2000 through 2022, identifying more than 9 million ice floes. They then constructed and analyzed the sea-ice floe size distribution (FSD). They found that the FSD follows a power law with an exponent that varies seasonally. They also calculated the mean floe size and the orientation of floes.

Previous studies of the FSD have typically been based on the analysis of a handful of images, or at most a few hundred. The amount of imagery analyzed in this work exceeds all previous studies by a factor of 15 or more, and for that reason alone it is worthy of publication. But regarding the seasonal evolution of the FSD, the authors did not make a careful comparison of their results with previous studies, and there are some mathematical issues to resolve.

Response: We are grateful to the reviewer for a thorough review of our manuscript. We have revised the manuscript per the reviewer's constructive suggestions, which we believe has greatly improved the clarity of the presentation. We have addressed the stated concerns and made the necessary changes to the manuscript (shown in blue font). Significant changes were made to Figure 5 (now Figure 6) per the reviewer's comments and added a new appendix to show the analytical derivation of the observed relationships in our dataset. The specific revisions are described in the point responses below:

Lines 89-90. "The dynamic threshold value is the weighted mean for the 400-pixel (100 km) neighborhood of a pixel subtracted by a constant." A few questions:

Thank you for your comment. The text has been revised to clarify:

"The dynamic threshold value is the weighted mean for the 399 pixel x 399 pixel neighborhood (~100 km x ~100 km)."

1. A neighborhood is an area. Is the neighborhood 400 x 400 pixels (100 x 100 km), or is it 20 x 20 pixels (400 total, 10 x 10 km)?

Correct. In this case, it is 399 pixels x 399 pixels (centered around the pixel of interest).

2. Is the neighborhood a square or some other shape?

Yes, square.

3. What does "subtracted by a constant" mean? What constant?

Thank you for catching this detail. We do not subtract by a constant, so this has now been deleted. This was the description of the python function we used. But the default constant is zero.

Lines 161-167. I do not understand the bootstrapping procedure. Samples of (what?) are drawn from (what?).

Thank you for your feedback. We are referring to sample of floes in an image, drawn from all the floes in an image with replacement. Text has been modified for clarity:

Finally, we apply a bootstrap approach to quantify uncertainty in the derived parameters. We randomly select 100 days of segmented MODIS imagery with at least 50,000 km² of identified floes, and examine the uncertainty in the derived FSD for each of those days. We create 1000 bootstrap samples (of floes in the image) with replacement. Each bootstrapped sample is the length of the original dataset (the number of identified floes in the image), and we calculate the power law distribution of the sampled floe areas. We then calculate the standard deviation of the alpha values for each image. The standard deviation of alpha generated from the bootstrapping of an image is on average, 0.024, ranging from 0.007 to 0.08. The standard deviation increases as the alpha value increases, indicating more uncertainty.

Lines 178-182. These sentences discuss the mean and median floe areas as shown in Figure 5(a). It should be noted that for a power-law distribution, the mean and the median are directly proportional to the minimum floe area, A_{min} , which is 5 km² in the present work. The power-law pdf with exponent $-\alpha$ is:

$$f(A) = ((\alpha-1)/A_{min}) * (A/A_{min})^{-(\alpha)}$$

The integral of $f(A)$ from A_{min} to infinity is 1.

The mean floe area is:

$$A_{mean} = A_{min} * (\alpha-1)/(\alpha-2)$$

The median floe area is:

$$A_{med} = A_{min} * (2^{1/(\alpha-1)})$$

The point is this: The mean and median floe areas are completely dependent upon the resolution of the measuring device. If the resolution of the sensor were to allow the detection of floes 10 times smaller, the mean and median floe areas would be 10 times smaller (provided the data continued to follow the same power law). The mean and median floe areas reported here do not reflect any kind of objective reality but rather the limitations of the sensor. They cannot be meaningfully compared to the mean and median floe areas detected by sensors with different resolutions. It's fine to calculate and plot the mean and median floe areas, and to note their variation over time, but their direct proportionality to the resolution (A_{min}) of the detection mechanism should be noted, if in fact the floe areas are power-law distributed.

Thank you for the comment and the detailed derivation of these equations. The mean and median values, while dependent on the A_{min} value (as pointed out), are also a function of alpha, which we show has a seasonality. Therefore, showing the evolution of the mean and median floe size is

an interesting result. Throughout our analysis, we have held the x_{min} and x_{max} values constant and changes in the mean and median are a result of changes in α . When comparing different sensors, we held x_{min} and x_{max} fixed to ensure comparability of the calculated FSDs. We have added a discussion on the relationship between these parameters (section 4.1 second paragraph) and have described the choice of x_{min} and x_{max} values in more detail (section 3.5). Thank you for pointing this out, we believe the discussion has improved with this suggestion.

The previous comment raises a further difficulty. For a power-law distribution with exponent $-\alpha$, the mean does not exist when $\alpha \leq 2$. In other words, the integral of $A \cdot f(A) dA$ from A_{min} to infinity is infinite when $\alpha \leq 2$. Now look at Figure 5(b). It shows that α is ≤ 2 quite often, and yet Figure 5(a) shows a finite mean floe area. It is not mathematically possible for a power-law distribution with the reported values of α to have a finite mean floe area. Fortunately, this is easily fixed by postulating that a maximum floe area (A_{max}) exists such that the distribution of floe areas follows a power law only for the finite range $A_{min} \leq A \leq A_{max}$. If this approach is taken, the authors should estimate A_{max} . If this approach is not taken, the authors should explain how to reconcile the mathematical contradiction.

Thank you for raising this point. We have been using a constant A_{max} value throughout our analysis. This value is now in the text and the choice of this value is now described in the manuscript (section 3.5).

Line 188. "The power law α is inversely correlated to the mean floe area". This finding is based on the data. It could also be checked analytically for a power-law distribution. As noted above, if $\alpha > 2$ then $A_{mean} = A_{min} \cdot (\alpha - 1) / (\alpha - 2)$. As α becomes larger, A_{mean} does indeed become smaller, although not linearly. But most of the values of α in this paper are ≤ 2 . If the power law is defined on the finite interval $[A_{min}, A_{max}]$ then A_{mean} is a more complicated function of A_{min} , A_{max} , and α , but A_{mean} still decreases as α increases, although not linearly. Whether the authors wish to include such an analytical comparison to the data is up to them, I am only pointing out the possibility to do so.

Thank you for detailing the mathematical functions behind this relationship. The authors agree that this derivation would complement the information presented in the manuscript. We have added an Appendix to derive the analytical mean and median which are now shown in Figure 6a and b.

Lines 210-218. This paragraph compares the power-law exponents of the present study to those found by Stern et al (2018a), but the comparison is not done correctly. The present study uses floe area as the measure of floe size, while Stern et al (2018a) used floe diameter. Assuming that floe area (A) is proportional to the square of floe diameter (x^2), a power-law distribution in A with exponent $-\alpha$ is equivalent to a power-law distribution in x with exponent $-2\alpha + 1$. With this conversion, the authors should make a quantitative month-by-month comparison of their results with those of Perovich and Jones (2014), Hwang et al (2017), and Stern et al (2018a), all of which looked at the seasonal evolution of the FSD in the Beaufort Sea.

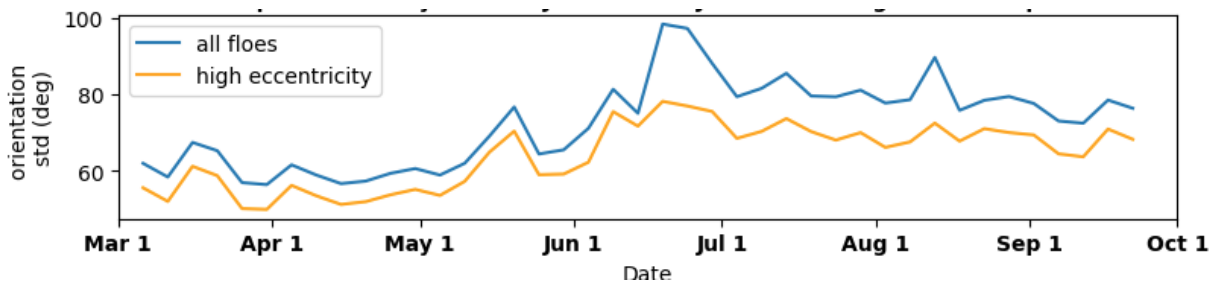
We agree. We have included a description of this conversion to the manuscript and added a quantitative comparison with the studies mentioned here (Section 5.2).

Regarding floe orientation (section 4.3), a couple of comments:

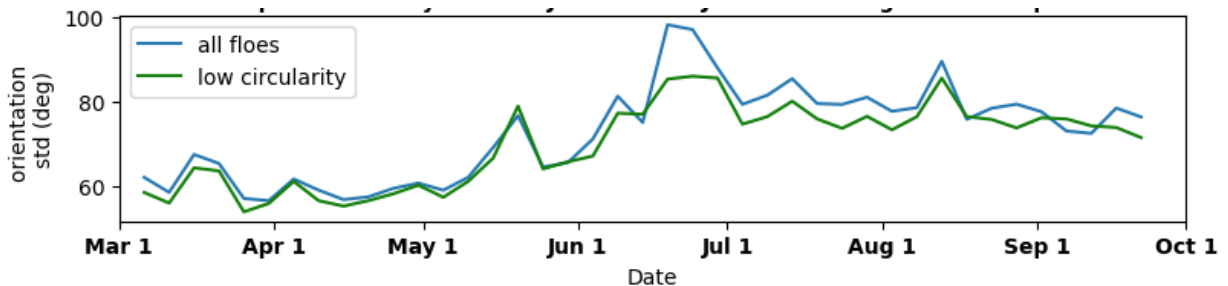
(1) It's possible that nearly circular floes could have a more variable orientation than elongated floes, because small changes in their perimeter could trigger large changes in their orientation. Therefore, as floe orientation becomes more variable (as in Figure 5(c) in June), how do we know whether it's due to floes becoming more randomly oriented or floes becoming more circular?

This is a good point. We looked at a subset of larger floes (>75th percentile), a subset of less circular floes (<25th percentile) and a subset of higher eccentricity (major axis/minor axis) floes (>75th percentile). We observed similar patterns in the evolution of the standard deviation of orientation to ensure that the pattern we were seeing is not just because floes are becoming smaller, more circular, and the eccentricity decreasing. We have added a sentence to the paper to address this comment.

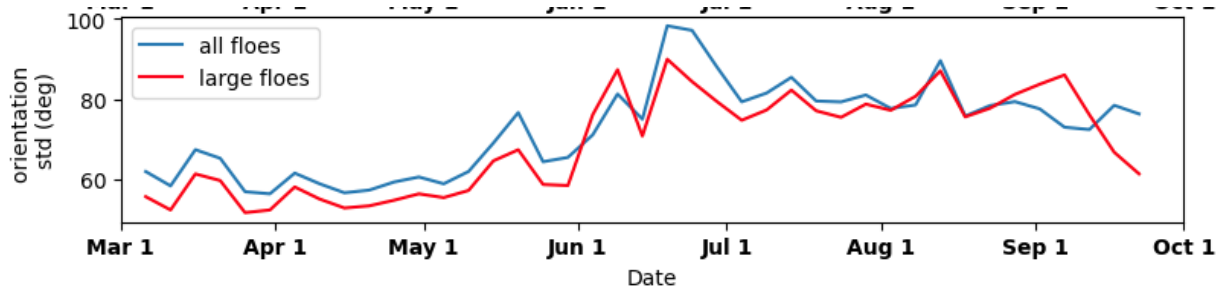
Here is an example of the evolution of all floes (blue) and the subset of just higher eccentricity (orange) - the trend is less extreme, but still present.



all floes (blue), low circularity (green):



all floes (blues) large floes (red):



(2) Lines 227-231 discusses the orientation of large rectilinear floes, concluding that their orientation is less variable in the early season compared to the full data set, but increasingly variable into the summer. It's not clear what the point of this comparison is. The reader is left hanging, with no figures or numbers or explanation of the significance of the result.

Thank you for your feedback. Floe orientation is related to the generation of fractures on the ice as a result of the stress being applied (e.g., from atmospheric features, like the Beaufort High and anticyclonic winds) and the effects of coastal geometry. By looking at the standard deviation of floe orientation, we can infer at what point the ice movement transitions to free rotation- and if floe rotation can be linked to ocean vorticity underneath the ice. We have added text discussing floe orientation in more depth:

“The orientation of the floes is an indication of the stresses that have caused ice breakup and can provide insight into the structural properties of the ice pack to inform how it may respond to future stresses. When ice floes are able to rotate without interaction with other floes, their rotation rates can be related to ocean vorticity (Manucharyan et al., 2022).”

Minor Comments

First line of the Introduction. Consider changing "from the atmosphere to the ocean" to "between the atmosphere and the ocean" because the fluxes go both ways -- atm to ocean and ocean to atm.

Thank you, changed.

Lines 52-53. "The older and thicker multiyear ice is now melting out" -- but the loss of older and thicker multiyear ice has been going on for a long time. It was first noted in 2007:

Maslanik, J. A., C. Fowler, J. Stroeve, S. Drobot, J. Zwally, D. Yi, and W. Emery (2007), A younger, thinner Arctic ice cover: Increased potential for rapid, extensive sea-ice loss, *Geophys. Res. Lett.*, 34, L24501, doi:10.1029/2007GL032043.

This phrase “now melting out” refers not to recent years, but to recent decades- as described in the study referenced at the end of this sentence that is specific to the Beaufort Sea. We removed this confusing phrase and added the Maslanik reference to the end of the sentence. Thank you for mentioning this study.

Lines 95-102. This same segmentation procedure was also used by Stern et al (2018a) -- see their section 3.2.

We added text to describe that this algorithm is based on Denton and Timmermans (2022) and Stern et al, (2018).

Lines 95-96. "where where"

Corrected

Lines 110-111. "from from" should be "range from"

Corrected

Line 119. "This quality assurance steps" -- singular or plural?

Singular, changed to "step"

Line 130. "The standard error of the fitted distribution" -- is this supposed to say the standard error of alpha?

Yes, thank you for catching that, corrected.

Line 150. "a correlation of 0.99" -- is this supposed to be a squared correlation? Figure 4 says " $R^2 = 0.99$ "

Yes, corrected to "squared correlation"

Line 166. "The the"

Corrected.

Line 167. "values increases" -- singular or plural?

Changed to value increases

Figure 3 caption: Line 3. "imaeg" Line 4. Either write "captured" or delete the word "capture"

Corrected

Figure 5(b). The caption refers to "the slope anomaly" but the figure shows the actual slope, not the anomaly. Figure 5(c). The caption refers to "low circularity floes" in green but I don't see a green curve. Figure 5(d). The caption refers to "land (magenta)" but I don't see any magenta color. Figure 5(d). Is it really necessary to show all that gray area (clouds)? That makes it harder to see the green and blue areas, which are really the quantities of interest. Line 204. "Fig. 5b, shaded" -- I don't see any shaded part of Figure 5b.

Corrected. Cloud bars were removed, no shaded regions.

Lines 203-204. "the standard deviation calculated as in Equation 3." Is this supposed to say "standard error" or "standard deviation"? Does Equation 3 give the standard error or the standard deviation? See line 130.

Standard error. This has been revised.

Line 211. 59400 should be 594

Corrected.

Line 225. "that are can be"

Corrected.

Line 239. "increasing power law slopes" -- does this mean steeper or shallower? When alpha increases, the slope decreases (becomes more negative). When alpha decreases, the slope increases (becomes less negative). That's why it might be clearer to say steeper or shallower.

I see why this is confusing. We do not include the negative in the alpha- so increasing alpha means more negative and steepening slope. But this is not an increasing slope. Changed to increasing alpha (steepening power law slope)

Lines 241-242. This is the first mention of "discrete element models" in the paper. I would suggest that on line 37 the words "discrete element" be added just before the word "models".

Corrected.

Line 245. "new segmentation algorithm presented here" -- the segmentation algorithm is not new. See the comment above for lines 95-102.

Thank you for bringing this up. We changed this to "modified" and added a longer description in the introductory part of the methodology (Section 3) to describe how this has been adapted from previous work.

There are other minor typos not noted here.

The manuscript has been proofread to identify typos (see new version of the manuscript).