Supplementary material SM 1

THEORY

Summary of formulae of the gravity aspects

The theory is mostly from Pedersen and Rasmussen (1990), Beiki and Pedersen (2010) and from our own papers/books Kalvoda et al. (2013) or e.g. Klokočník et al. (2017, 2020). These last two are the source for this *Supplement*. Examples follow in the second Supplement. References are in the main text and in (Klokočník et al. 2020).

The *disturbing static global gravitational potential* outside the masses of a celestial body (planets, moons) in the spherical harmonic expansion is given by

$$T(r,\varphi,\lambda) = \frac{GM}{r}\sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{R}{r}\right)^{l}\left(C'_{l,m}\cos m\lambda + S_{l,m}\sin m\lambda\right)\ P_{l,m}(\sin\varphi), \qquad \text{(A1)}$$

where GM is a product of the universal gravitational constant G and the mass M of the planet (also known from satellite analyses as the geocentric gravitational constant in the case of the Earth), r is the radial distance of an external point where T is computed, R is the radius of the planet (which can be approximated by the semi-major axis of a reference ellipsoid), $P_{l,m}(\sin\varphi)$ are the Legendre associated functions, l and m are the degree and order of the harmonic expansion, (φ, λ) are the (planeto)centric latitude and longitude, and $C'_{l,m}$ and $S_{l,m}$ are the *harmonic geopotential coefficients* (also known as *Stokes parameters*); fully normalized, $C'_{l,m} = C_{l,m} - C^{el}_{l,m}$, where $C^{el}_{l,m}$ belongs to the reference ellipsoid. The word "*disturbing*" here means the difference between the total gravitational potential of the actual body and the gravitational potential of a reference body, i.e. the reference ellipsoid, usually taken as a rotational ellipsoid with some flattening on the poles due to the rotation of that body.

A set of numbers $C'_{l,m}$ and $S_{l,m}$, presented to a maximum degree L_{max}, is called the *gravity/gravitational (field) model of the Earth*. ("Gravity" means gravitational effect plus effect of centrifugal force of the studied body).

There is $l \times (l-1)$ terms in such a model, if is complete to the maximum degree and order l, m (or d/o) and if a few first (lowest degree) terms are not omitted (sometimes these terms are set at zero, due to reasons which will not be discussed here).

The gravity/ gravitational models are usually based on a great amount of diverse satellite and terrestrial data collected from around the world over a long time; then such a result is known as a high-resolution "*combined model*" (e.g., GEM 2008, Pavlis et al. 2012; EIGEN 6C4, Förste et al. 2014), for references see Förste et al. (2014) in contrast to "*satellite-only models*".

Let us recall that $C'_{l,m}$ and $S_{l,m}$ are considered to be constants (excluding a few lowest degree zonal harmonics, which have often been published with a secular trend and semi/annual or other time variable components). We speak about *static gravity/gravitational models.*

There are also *variable gravity/gravitational field solutions*, derived from the global satellite data (mainly from the GRACE mission). They are based on short arc solutions (from observations gathered for one month or a shorter interval), so they are available for a much lower L_{max} than the static models (say to d/o = ~100 instead of ~2000), and only for the Earth.

The *gravity/gravitational aspect* is a functional/function of the gravity gravitational) field potential T. It can be its derivative or any other function, often non-linear. We work with the following *gravity aspects* (descriptors): the gravity anomaly (or disturbance) Δg, the Marussi tensor ($\mathbf{\Gamma}$) of the second derivatives of the disturbing potential (T_{ij}), two gravity invariants (I_j), their specific ratio (I), the strike angles (θ) and the virtual deformations (vd) – Klokočník et al. (2017, 2020).

All the gravity aspects together provide thorough information about the density anomaly due to the causative body that is more complete than, for example, information that the traditional and usual gravity anomalies themselves could yield. The set of gravity aspects informs about location, shape, orientation, a tendency to a 2D or 3D pattern, and stress tendencies and may partly simulate "dynamic information" although the input data are always the same – those harmonic geopotential coefficients $C'_{l,m}$ and $S_{l,m}$ of a static gravity field model. The whole theory is arranged in such a way that we cannot use any input other than the harmonic coefficients of a gravity model.

The spherical approximation of the *gravity anomaly* Δg (free air, without any geophysical model) is computed as the first radial derivative of T by

$$\Delta g = -\frac{\partial T}{\partial r} - 2\frac{T}{r}. \tag{A2}$$

Instead of (A2), one can use the *gravity disturbance*, which is as (A2), but without the second term (often small). The gravity anomalies/disturbances are computed from measurements by ground, airplane or marine gravimeters or derived from measurements performed by means of satellite altimetry.

The gravity gradient tensor $\mathbf{\Gamma}$ (the *Marussi tensor* or simply the *gravity tensor*) is a tensor of the second derivatives of the disturbing potential T of the gravity field model. The Marussi tensor was considered the centerpiece of traditional differential geodesy; up to the second order this tensor systematically synthesizes all the dynamical and geometric properties of the Earth's gravity field (see Klokočník et al., 2020 for references).

The tensor $\mathbf{\Gamma}$ is given in the local north-oriented reference frame (x, y, z), where z has the geocentric radial direction, x points to the north and y is directed to the west (Pedersen and Rasmussen, 1990):

$$\mathbf{\Gamma} = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 V}{\partial x^2} & \frac{\partial^2 V}{\partial x \partial y} & \frac{\partial^2 V}{\partial x \partial z} \\ \frac{\partial^2 V}{\partial y \partial x} & \frac{\partial^2 V}{\partial y^2} & \frac{\partial^2 V}{\partial y \partial z} \\ \frac{\partial^2 V}{\partial z \partial x} & \frac{\partial^2 V}{\partial z \partial y} & \frac{\partial^2 V}{\partial z^2} \end{bmatrix} \tag{A3}$$

Outside of the body masses $\mathbf{\Gamma}$ satisfies Laplace's differential equation, i.e. the trace of the Marussi tensor (A3) is zero. The tensor $\mathbf{\Gamma}$ is symmetric ($T_{yx}=T_{xy}$, $T_{zx}=T_{xz}$, $T_{zy}=T_{yz}$) and harmonic ($T_{xx}+T_{yy}+T_{zz}=0$); it contains nine components, but just five linearly independent components.

Gravity gradiometry is the measurements of T_{ij}. The gravimeters measure the first derivative of T, i.e. the accelerations Δg, the gradiometers measure the second derivatives of T. From the actual formulae for their computation (elsewhere) one can see that the gravity gradients are more sensitive to the close-by mass distribution (density anomalies) than the gravity accelerations.

Terrestrial gradiometers (torse balances) to measure T_{ij} are known from geophysics (see any textbook), but they were not too successful in practical use (too noisy). Now, owing to technical progress, they can successfully be used for measurements on board of airplanes and are used for prospection at local scales.

The *GOCE* mission (Gravity and [steady state] Ocean Circulation Explorer, ESA) was the first (2009) and until now (2022) still the last gradiometric instrument working successfully in space on low orbit (measuring six of T_{ij} components million times during its ~5-year lifetime at a carefully selected orbit fulfilling special criteria on orbital resonances), based on micro-accelerometers (a pair of them in each spatial direction x,y,z).

The Marussi tensor has already been used locally (this means in areas of a few per few kilometers) for petroleum, metal, diamond, groundwater, etc., explorations (for more information and for further references see Klokočník et al. 2020a).

The Marussi tensor is a rich source of information about density anomalies providing useful details about the target objects shallowly located beneath the Earth's surface. This extra information can be used by tensor imaging techniques to enhance the source anomalies; it has been tested for local features (economic minerals, oil and gas deposits, fault location, etc.). The tensor components are used at local scales to identify and map the geological contact information, either the edges of the source targets or the structural/stratigraphic contact information. The horizontal components identify the shape and the geological setting of a responsible body. The quantity T_{zz} is best suited for target body detection; T_{zz} helps to define the isopath/density relationships of body mass with relation to its geological setting, see e.g. Saad (2006).

Under arbitrary coordinate transformation, any gravity field and any $\mathbf{\Gamma}$ have just three global *gravity invariants* which remain constant. Here they are labelled $\boldsymbol{I_0}$, $\boldsymbol{I_1}$, and $\boldsymbol{I_2}$:

$$\boldsymbol{I_0} = trace\ (\mathbf{\Gamma})$$

and this one is zero outside the masses of the studied body (known also as the Laplace equation). The remaining two invariants read in general:

$$\boldsymbol{I_1} = \tfrac{1}{2}\ [\mathrm{trace}\ (\mathbf{\Gamma})^2 - \mathrm{trace}\ (\mathbf{\Gamma}^2)],$$

$$\boldsymbol{I_2} = det\ (\mathbf{\Gamma}).$$

This can be transformed (using the components of **Γ** in Eq. 3) to:

$$\boldsymbol{I_0} = T_{xx} + T_{yy} + T_{zz}$$

$$\boldsymbol{I_1} = (T_{xx}T_{yy}+T_{yy}T_{zz}+T_{xx}T_{zz}) - (T_{xy}^2+T_{yz}^2+T_{xz}^2) = \sum_{\{i,j\}\in\{x,y,z\}}\left(T_{ii}T_{jj} - T_{ij}^2\right) = -(T_{xx}^2+T_{yy}^2+T_{xx}T_{yy}+T_{xy}^2+T_{yz}^2+T_{xz}^2), \quad \text{(A4)}$$

$$\boldsymbol{I_2} = det\,(\boldsymbol{\Gamma}) = T_{xx}(T_{yy}T_{zz}-T_{yz}^2) + T_{xy}(T_{yz}T_{xz}-T_{xy}T_{zz}) + T_{xz}(T_{xy}T_{yz}-T_{xz}T_{yy}) \quad \text{(A5)}$$

The invariants are mathematically independent of the coordinate system chosen, so invariant ("resistant") with respect to any rotation. The invariant $\boldsymbol{I_0}$ is useful for numerical checks of the actually measured T_{ii}. The invariant $\boldsymbol{I_1}$ is the sum of the six products of two tensor coefficient matrix elements, a nonlinear functional model with regard to the geopotential harmonics. The invariant $\boldsymbol{I_2}$ is the determinant "det" of **Γ**.

The invariants can be looked upon as non-linear filters enhancing sources with big volumes (Pedersen and Rasmussen, 1990). They discriminate major density anomalies into separate units. It is useful and helpful that the resultant computed anomaly response retains the same shape and orientation, i.e. it is independent of the observer's choice of axes; this is significant for interpretation when mapping geological structures.

Pedersen and Rasmussen (1990) showed that the ratio I of the invariants $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$, defined as

$$0 \le I = -\frac{(\boldsymbol{I_2}/2)^2}{(\boldsymbol{I_1}/3)^3} \le 1\,, \quad \text{(A6)}$$

always lies between zero and unity for any potential field. If the causative body is strictly 2D (flat), then I=0. Thus, the ratio can be an indicator of two-dimensionality, sometimes called the "2D factor". If I=0, then we have the necessary but not sufficient condition for two-dimensionality. If the causative body – as seen from the observation point – looks more "3D-like" (for example, a volcano), then I grow and eventually approach 1.

The gradient tensor **Γ** contains information about subsurface strike (stress) directions. Pedersen and Rasmussen (1990) defined the *strike angle* θ (strike lineaments, strike direction) as follows:

$$\tan 2\theta_s = 2\frac{T_{xy}(T_{xx}+T_{yy})+T_{xz}T_{yz}}{T_{xx}^2-T_{yy}^2+T_{xz}^2-T_{yz}^2} = 2\frac{-T_{xy}T_{zz}+T_{xz}T_{yz}}{T_{xz}^2-T_{yz}^2+T_{zz}(T_{xx}-T_{yy})} \qquad \text{(A7)}$$

where θ is estimated within a multiple of $\pi/2$; and only one value represents the main direction of **Γ.** Provided that the ratio I in (A6) is small, the strike angle may indicate a dominant 2D structure. If one were able to rotate with the structure in such a way that the elements of the first row and first column of **Γ** were identically equal to zero, then one would reach a "correct" direction of "stress fields" described by θ (Beiki and Pedersen, 2010).

Mathematically, θ is the main direction of **Γ**. Geophysically, it is an important direction for the ground structures; it may indicate areas with a lower porosity or "stress directions".

The strike angles usually show chaotic directions. Sometimes, they are oriented dominantly in one prevailing direction (linearly or creating a halo around the object), they are aligned, combed. The combed values, mostly for small I ($I<0.3$), may signalize possible oil or gas fields, ground water, paleolakes or impact craters (e.g., Klokočník et al. 2020, and further references there).

The situation remains, however, not unambiguous when solely using the gravity data. The reason is that not only can oil and gas fields be detected by the combed θ but also groundwater reservoirs, water-filled depressions, paleolakes or stress fields after impact at and near the impact craters. The combed θ probably relates to changes of porosity and stresses, for example due to impact pressure deformations. It is evident that we need additional information to the gravity aspects, geological or geophysical information, namely magnetic anomalies, archaeological data, detailed surface or subglacial topography, etc.

Now let us define the *"virtual deformation"* (*vd*), introduced for the first time by Jan Kostelecký in Kalvoda et al. (2013). It is analogous to the tidal deformation known from geodesy and geophysics; one can imagine the directions of such a deformation due to "erosion" brought about solely by gravity.

If there were a tidal potential represented as in our case by T (A1), then horizontal shifts (deformations) would exist due to this and they could be expressed in the north-south direction (latitude direction) as

$$u_\Phi = l_S \, \frac{1}{g} \frac{\partial T}{\partial \varphi} \qquad \text{(A8)}$$

and in the east-west direction (longitudinal direction) as

$$u_\Lambda = l_S \frac{1}{g\cos\varphi} \frac{\partial T}{\partial \lambda} \tag{A9}$$

where g is the gravity acceleration 9.81 m·s^{-2}, l_S is the elastic coefficient (called the Shida number) expressing the elastic properties of the Earth as a planet (generally l_S = 0.08), φ and λ are the geocentric latitude and longitude of the point P where we measure T; and the potential T is expressed in [m^2 s^{-2}]. In our case, T is represented by Eqs. (A1), (A8) and (A9). The practical problem is that the actual values of the Shida parameters l_S for the Earth's surface (for the specific locations) are not known; thus, we will know (A8) and (A9) and subsequent quantities only as relative values.

The formalism of continuum mechanics was applied to derive the main directions of the deformations, meaning to transform the horizontal shifts to a small deformation. The tensor of a small deformation $\boldsymbol{E}$ is defined as a gradient of the horizontal shifts (A8) and (A9):

$$\boldsymbol{E} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} \end{pmatrix}. \tag{A10}$$

The tensor $\boldsymbol{E}$ has two parts: $\mathbf{e}$ is the symmetrical, and $\boldsymbol{\Omega}$ is the anti-symmetrical part:

$$\boldsymbol{E} = \mathbf{e} + \boldsymbol{\Omega} = (\mathrm{e}_{ij}) + (\Omega_{ij}) \tag{A11}$$

The symmetrical tensor $\mathbf{e}$ reads:

$$\mathbf{e} = \begin{pmatrix} \mathrm{e}_{11} & \mathrm{e}_{12} \\ \mathrm{e}_{21} & \mathrm{e}_{22} \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & (\epsilon_{12} + \epsilon_{21})/2 \\ (\epsilon_{12} + \epsilon_{21})/2 & \epsilon_{22} \end{pmatrix}, \tag{A12}$$

the parameters of deformation are:

$\Delta = \mathrm{e}_{11} + \mathrm{e}_{22}$	total dilatation	(A13)
$\gamma_1 = \mathrm{e}_{11} - \mathrm{e}_{22}$	pure cut	

$\gamma_2 = 2e_{12}$	technical cut
$\gamma = (\gamma_1^2 + \gamma_2^2)^{1/2}$	total cut
$\boldsymbol{a} = ½ (\Delta + \gamma)$	major semi-axis of the ellipse of deformation
$\boldsymbol{b} = ½ (\Delta - \gamma)$	minor semi-axis of the ellipse of deformation
$\boldsymbol{\alpha} = ½ \,\mathrm{atan}(\gamma_2 / \gamma_1)$	direction of the main axis of deformation.

Note that different specialists make use of different terminology for the same or similar quantities quoted in (A13).

The derivatives of the disturbing geopotential obtained originally as the directional values (A8), (A9) were transformed to small positional deformations, or shifts. But basically, the information content of the Marussi tensor and of *vd* is the same, only exposed in different ways.

To illustrate *vd*, the semi-axes $\boldsymbol{a}$, $\boldsymbol{b}$ of the deformation ellipse are computed. As we already know, the local values of l_S are not known, and, in turn, only the main directions of *vd* (and not their amplitudes) can be computed.

It is very interesting and may sounds unusual that *vd* provide dynamical information, even though they are, as well as all the gravity aspects mentioned here, computed from static gravity models (represented by a set of $C'_{l,m}$ and $S_{l,m}$).

As already mentioned, the *vd* is analogous to the tidal deformation and characterizes the "tensions" (directional compression and dilatation) generated by the causative body (Kalvoda et al. 2013). We can understand the *vd* as a principal axis transformation from the horizontal gradients of the deflections of the vertical (A8) and (A9). Since the potential is forward modelled from the topography (at least in the case of the RET 14 model, see below), it is also related to curvature of topography.

Notes to the combed strike angles

The *combed strike angles* are strike angles θ oriented roughly in one and the same direction. Here we define the combed coefficient *Comb* for θ as a measure or degree of θ being combed; it is a relative value in the interval $\langle 0,1 \rangle$, where 0 means to be "not combed" (the vectors of θ are in diverse directions) and *1* means to be "combed" (perfectly aligned, the vectors of θ are oriented into one prevailing direction). They are different ways how to arrange such a tool.

The following are the input data to the statistics:

$$\theta_i \in \langle -90°, 90° \rangle, i = 1, \ldots, n$$

for *n* pixels in the studied area or zone. We compute the main direction of the combed θ as the mean value of θ_i; let us denote it as θ_{Comb}:

$$\theta_{Comb} = \frac{\sum_{i=1}^{n} \theta_i}{n}$$

by choosing the angles θ_i either in the interval $\langle -90°, 0° \rangle$ or in the interval $\langle 0°, 90° \rangle$. We use the following important condition:

$$\forall (|\theta_i - \theta_{Comb}| > 90°): \theta_i = 180° - |\theta_i|$$

which means that even two angles θ_i in opposite directions are counted as one direction. For example: for $\theta_{Comb} = 80°$ and $\theta_i = -80°$, a deviation from the main *Comb* direction is 20°.

Let us define a root mean square value of scatter (variance) of θ_i for *n* pixels as:

$$rmsv = \sqrt{\frac{\sum_{i=1}^{n} (\theta_i - \theta_{Comb})^2}{n}}$$

Then the required looked-for value of the main *Comb* direction can be defined as

$$Comb = 1 - \frac{rmsv}{90°}$$

As a measure of the degree of θ "being combed", we make use of the relative values of θ_i:

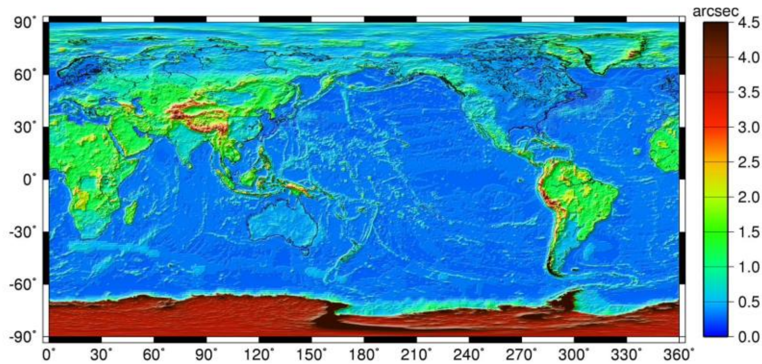
$$\theta_i^{relat} = 1 - \frac{abs(\theta_i - \theta_{Comb})}{90°} \qquad \text{(A14)}$$

The *Comb* value is shows local direction of the tested set of θ_i in the given region; the departures of the individual θ_i from *Comb* are plotted in a preselected optimum size of *n* rectangular pixels in a relative scale; these are the values of (A14); if some of θ_i fit in the main direction of *Comb*, then the pixel has the value 1; if not, then the values (A14) are in the interval $\langle 0,1 \rangle$. This serves as a simple statistical evaluation to compare areas with combed strike angles to those with "non-combed" strike angles.

If *Comb* is smaller than 0.55, we say that θ_i of the given region are "not combed"; if *Comb* >0.65, we say θ_i are "combed". There is a "grey zone" between the two, i.e. *Comb*= 0.55–0.65.

Note on DATA in EIGEN 6C4

The most important is the gravity field model used. We make use of a high resolution combined *E*uropean *I*mproved *G*ravity model of the *E*arth by *N*ew techniques (*EIGEN 6C4*, Förste et al. 2014), expanded to degree and order (d/o) 2190 in spherical harmonics; this corresponds to the ground resolution 5x5 arcmin or ~9 km on surface. Precision of EIGEN 6C4, expressed in terms of Δg, is 10 mGal, but in many civilized land areas and over the oceans and open seas is much better. The authors of EIGEN 6C4 have not access to most of the recent high resolution terrestrial gravity data on the continents, thus they took a synthesized gravity anomaly grid based on EGM2008 (Pavlis et al. 2012). That means that the errors for high d/o terms in EIGEN 6C4 are dominated by the relevant errors in EGM2008. To estimate the precision for the given area of interest, not only a general figure 10 mGal, one needs to inspect gravity anomaly commission error maps of EGM2008 (Pavlis, reference above, the map below). For the northern Yucatan peninsula, we get 4-8 mGal, for Popigai in Siberia a bit worse.



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