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Review of *An intercomparison of four gridded precipitation products over Europe using the three-cornered-hat method* by L. Lledó, T. Haiden, and M. Chevallier for EGUsphere.

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This is an excellent paper that applies the three-cornered hat (3CH) and four-cornered hat (4CH) methods to estimate the uncertainties (random error variances) of four precipitation datasets. It is acceptable for publication after the authors consider some relatively minor changes that would improve the clarity of the paper.

A novel aspect of the paper is to use the 4CH method to compute two of the error covariance terms that must be neglected in 3CH error variance estimates. [Sjoberg et al. \(2021\)](#) show that N datasets lead to $(N - 1)(N - 2)/2$ unique error variance estimates for each dataset when using the 3CH method. Furthermore, under the assumption of zero error covariances among all the datasets, a single error estimate for each dataset can be computed using all N datasets simultaneously (N-cornered hat method, a generalization of the 3CH method), which is identical to the mean of all the individual estimates. This paper goes further and shows that the N -cornered hat method is potentially more powerful than multiple applications of the 3CH method to various combinations of N datasets. It allows the estimation of some of the error covariance terms, which can be chosen arbitrarily, but based on insights of which pairs of datasets are likely to be most independent (have smallest error covariances). The authors illustrate this with four different precipitation datasets.

It is important when describing the 3CH and 4CH to be very clear in describing the methods, notation, and equations, because the equations can quickly become confusing, especially to those unfamiliar with these relatively new methods. My major suggestion is to improve the description of the 3CH and 4CH method in Section 3.3. I found the discussion a bit unclear, in part due to the notation in Equations (1)-(3):

1. There are currently two ways of writing the variance of the difference between two datasets. For example, in (1) $\text{var}(A-B)$ is used, while in (2) and (3) V_{A-B} is used. Please use one or the other for consistency.
2. Also, V_{A-t} is not defined, although one can assume it is the error variance of A . I suggest writing it as something like $\text{var}(A_{\text{err}})$ or $\text{var}(A\epsilon)$, where A_{err} or ϵ denotes random errors.
3. More seriously, the covariance terms in (2) and (3) are not defined and could be misinterpreted. For example, c_{AtBt} should be the covariance of the errors of A and B , but it could be misinterpreted as the covariance between the true values of A and B . Write something like $\text{cov}(A\epsilon B\epsilon)$ or $\text{cov}(A_{\text{err}} B_{\text{err}})$.
4. Finally, in Eqs. (2) and (3), each equation actually represents two equations. For example, the first equation represents the following two equations:

$$V_{A-B} = V_{A_{\text{err}}} + V_{B_{\text{err}}} - 2\text{cov}(A_{\text{err}} B_{\text{err}})$$

$$V_{A-B} \equiv V_{A_{\text{err}}} + V_{B_{\text{err}}}$$

The second equation is only an approximation, and equality holds only when the error covariance term is zero.

In addition to the notation issues, I suggest a rewriting of lines 142-150 (I did not change the notation in this suggested revision):

To that end, if we have a third independent observation system C, we can repeat the above steps with differences between A and C, and B and C to obtain a system of three equations and six unknowns (the error variances of the three datasets and three error covariance terms):

$$\begin{aligned} v_{A-B} &= v_{A-t} + v_{B-t} - 2c_{AtBt} \\ v_{A-C} &= v_{A-t} + v_{C-t} - 2c_{AtCt} \\ v_{B-C} &= v_{B-t} + v_{C-t} - 2c_{BtCt} \end{aligned} \quad (2)$$

If the three error covariance terms are small and can be neglected, the equations for the estimates of the true error variances can be solved from the remaining terms, which can be computed from the three collocated datasets.

In our case we have identified four observational systems, therefore we can constrain the computation a bit more. The four-cornered hat (4CH) method extends the system of equations with three more equations, (six in total, one for each pair of products), with 10 unknowns (the error variances of the four datasets and six error covariance terms (Eq. (3):

$$\begin{aligned} v_{A-B} &= v_{A-t} + v_{B-t} - 2c_{AtBt} \\ v_{A-C} &= v_{A-t} + v_{C-t} - 2c_{AtCt} \\ v_{A-D} &= v_{A-t} + v_{D-t} - 2c_{AtDt} \\ v_{B-C} &= v_{B-t} + v_{C-t} - 2c_{BtCt} \\ v_{B-D} &= v_{B-t} + v_{D-t} - 2c_{BtDt} \\ v_{C-D} &= v_{C-t} + v_{D-t} - 2c_{CtDt} \end{aligned} \quad (3)$$

If any four of the error covariance terms are assumed zero, we can solve for the error variance of the four datasets and the remaining two error covariance terms. Assessing which two covariance terms should be computed is a science-informed but subjective matter. The independence.....

Minor comments:

1. The paper uses “data” as a singular noun. I realize many people use it this way, but technically “data” refers to more than one datum, and hence is plural. Consider changing all the “data is” to “data are”.
2. Fig. 2—What is the circular feature over central Spain in three of the panels? And one panel in Fig. 4. A comment about this in the caption would be useful.
3. Line 175—It appears that the largest values of the differences between IMERG and ERA5 are along the coasts of western Europe and northern Italy (2-5 mm/day), not over the oceans where the largest magnitudes are 0.5-1.0 mm/day.
4. Both E-OBS and EOBS are used for the rain gauge dataset; please use one of these throughout. EOBS is used in many of the figures and in Line 180. E-OBS is used in many other places.
5. A. Line 183-It might be better to change the wording from “worst” and “best” to the less pejorative “highest” and “lowest” e.g. ranking the products from highest to lowest.
B. Line 185-Same as A-highest instead of worst.

- C. Line 191-Change “better” to “lower”
- D. Line 245-Change “best to worst” to “lowest to highest.”
- 6. Line 195—Sjoberg et al. (2021) discuss in some detail how negative error variances can be obtained and could be included here as a reference. Line 249 as well.
- 7. Are the units of Figs. 2, 4 and 5 mm^2 or $(\text{mm}/\text{day})^2$?
- 8. Line 230-You could compute the error covariance of ERA5 and Opera in this case using the 4CH method, but it is not necessary.
- 9. Fig. A1 is outside Appendix A.
- 10. The error covariances shown in Fig. A2 are interesting, but it would also be interesting, and easier to interpret, if the corresponding error correlation map were shown.