Ensemble Kalman filter in geosciences meets model predictive control – Review report

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In this manuscript, the author propose to use a well-known data assimilation method, namely the ensemble transform Kalman smoother, for a model control problem. The method is illustrated using the three-variable Lorenz 1963 system.

The manuscript is well written and easily understandable. It presents a new method for model control. However, after reading the manuscript, I have a feeling of incompleteness both in the justification of several methodological choices and in the discussion of the results. I will try to explain this in a constructive way.

1 Major comments

1.1 Presentation of the EnKF

In this manuscript, the EnKF is presented as a variational method (L 108-109 'EnKF aims to minimize the following cost function'). While it is true that under linear and Gaussian assumptions, the EnKF analysis provides the solution to a variational problem, the EnKF is *not* a variational assimilation method and the description provided in section 2 is partial and misleading. In particular there is no information about the ensemble update.

1.2 Methodological choices

Throughout the manuscript, strange methodological choices are made. These choices need to be justified. Retrospectively, my impression is that these choices are necessary because otherwise the control problem would not be solvable with an ETKS, but in the end, without justification it makes me believe that the ETKS is not an appropriate choice for the control problem.

Choice of controlling only the end of the trajectory As far as I understand, from a control perspective it would be more efficient to have the entire trajectory in the control horizon. For example, I think that the reason why in the numerical experiments the control fails for large values of $\boldsymbol{C}^r$ is precisely that the control is only applied at the end of the control window.

Choice of the ETKF/ETKS With a time interval between observations of 0.08 model time units (MTU), the dynamics of the system is mildly nonlinear, and therefore using the ETKF for the assimilation step totally makes sense. On the other hand, the control horizon is $300 \times 0.01 = 3$ MTU, which means that the dynamics will be strongly nonlinear throughout the control window. In a strongly nonlinear regime, the choice of a Kalman smoother with its inherent linear and Gaussian assumptions is questionable. Furthermore, from a control perspective the goal is to control the *dynamics* of the system, which is why a nonlinear optimisation method would make much more sense than the ETKS here.

Choice of the control operator You chose to use a logistic function as control operator. This is a nonlinear operator and hence makes the ETKS step more difficult. Beyond this considerations, using 1 as pseudo observation is roughly equivalent (since the logistic function is invertible) to having $X = \infty$ as observation which seems extreme. From a control perspective, it would make much more sense to have in the cost function a term that would penalise negative values of X (since the objective is to avoid these negative values).

Choice of the error covariance matrix for the perturbation increment For simplicity, $\boldsymbol{C}^u$ is set to $\boldsymbol{P}^a$, but this choice does not make sense at all. Indeed $\boldsymbol{P}^a$ measures the uncertainty in the analysis (which varies over time depending on the current observation errors and the position in the attractor in particular), while $\boldsymbol{C}^u$ measures the cost of applying a given perturbation increment (economic cost). There is absolutely no relationship between both quantities. Without further knowledge on the economic constraints, I would suggest to use a diagonal $\boldsymbol{C}^u$.

1.3 Discussion of the results

Overall, I have the impression that the manuscript lacks a proper discussion of the results. In particular, the following points are left unanswered.

Objective of the experiment In the end, what is the real objective in the experiment? Is it to ensure $X > 0$ at all economic cost? Is it to minimise the economic cost to ensure $X > 0$ all the time or at a fixed percentage of the time? Is it something else? In connection with this question, I would have like to see, for each experiment, the averaged economic cost (as a complement to figure 3b). Furthermore, I think that a baseline method must be included in the experiments must be compared to the proposed method. The last paragraph of the manuscript is not sufficient in that sense.

Is the experiment successful? For small values of $\boldsymbol{C}^r$, it seems that the method is indeed able to enforce $X > 0$. However, there is an important side-effect: X is now restricted to values between 6.5 and 10.5. This must be discussed. In particular, to what extent does this side-effect provoke damage and can it be avoided? In the end, this raises the following question: can we really consider that the experiment is a success?

2 General minor comments

L 164 '1600 data assimilation cycles'. Why so few cycles? For such a small system, it is easy to make experiments with hundreds of thousands of cycles (e.g. Bocquet 2011).

L 253-254 'as humans cannot (and should not) undertake large-scale alterations of the Earth system' The first part of this sentence is questionable: by changing the climate of the Earth, humans do alter the Earth system at large-scale. Perhaps you should use 'rapid, large-scale alterations' instead. The second part of this sentence 'and should not' is subjective and hence out of subject in a scientific article.

Mathematical notation Please use the journal conventions for the mathematical notation. They can be found here: `https://www.nonlinear-processes-in-geophysics.net/submission.html#manuscriptcomposition`.

3 Technical comments and suggestions

4D-Var '4-D variational' The form of this acronym is highly unusual. The full name is usually 'four-dimensional variational method' and the associated acronym is '4D-Var'.

L 46-47 'the duration of an data assimilation window' this is usually called 'the data assimilation window length'.

L 52-53 'to predict the future behavior of the controlled system' Is 'predict' to right term here? Shouldn't it be 'control'?

L 58 'where $\boldsymbol{u}_t$ is control inputs at time t' Please reformulate.

L 108 'EnKF aims to minimize' → 'the EnKF aims to minimize'.

L 113 I would recommend here to also cite Hunt et al. (2007), who provided a highly simplified and efficient implementation of the ETKF (alongside a localisation technique) compared to the original one.

L 114-115 'assuming that H is linear and the ensemble members are Gaussian-distributed' In a sense, this is redundant since the EnKF in general, and the ETKF in particular, make these assumptions (even though they can be relaxed to some extent).

L 130 Please provide a citation for the ETKS.

L 136-150 This could be included in a dedicated algorithm environment.

L 137 ‘Perform ETKF’ → ‘Perform an ETKF analysis step’

L 161 ‘The data assimilation window was set to 8 timesteps’. Since you are using a filter and not a smoother, it seems weird to use the term ‘data assimilation window’. I would rather speak of ‘time interval between observations’.

L 161-162 ‘Observation error was set to $\sqrt{2}$, and it was assumed that all variables X, Y, and Z were observed.’. Is this the variance or the standard deviation? In addition, are the observation errors for all three variables independent to each other? Please be accurate.

L 163 ‘Observation was generated from the nature run by adding Gaussian noises.’ For consistency, I think you should use plural everywhere here (‘observations were’ and then ‘noises’) and not singular.

L 179-180 Please use scientific notation for the values of $\boldsymbol{C}^r$ (e.g. 10^{-1}, 10^{-5}, etc.).

Figure 1 3D plots are very hard to read in general and I would advise to avoid them. In the present case, the attractor could be visualised used 2D projections.

Figure 2 The figure would be easier to read with a grid. Is panel (a) representing the true X or the analysis? In the first case, I would suggest to plot all the 0.01 time steps to avoid ugly spikes For D, you could potentially use only markers (again to avoid ugly spikes). For both panels, I would recommend to remove the white space at the left and right of the panels because it gives a false impression that the experiment starts at cycle 620 and ends at cycle 700.

Figure 3 I would recommend to show violin plots instead of box plots and to use an horizontal grid. Also, it would be good to use a scientific notation for $\boldsymbol{C}^r$.

References

Bocquet, Marc (20th Oct. 2011). ‘Ensemble Kalman filtering without the intrinsic need for inflation’. In: *Nonlinear Processes in Geophysics* 18.5, pp. 735–750. DOI: 10.5194/npg-18-735-2011.

Hunt, Brian R., Eric J. Kostelich and Istvan Szunyogh (June 2007). ‘Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter’. In: *Physica D: Nonlinear Phenomena* 230.1-2, pp. 112–126. DOI: 10.1016/j.physd.2006.11.008.