Viscoelastic mechanics of tidally induced lake drainage in the Amery grounding zone

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Abstract. Drainage of supraglacial lakes to the ice-sheet bed can occur when a hydrofracture propagates downward, driven by the weight of the water in the lake. For supraglacial lakes in the grounding zones of Antarctic glaciers, the mechanics of drainage is complicated by their proximity to the grounding line. Recently, a series of supraglacial lake-drainage events through hydrofractures was observed in the Amery Ice Shelf grounding zone, East Antarctica. The lake depth at drainage varied considerably between events, raising questions about the mechanisms that induce hydrofracture, even at low lake depths. Here we use a modelling approach to investigate the contribution of tidally driven flexure to hydrofracture propagation. We model the viscoelastic response of a marine ice sheet to tides, the stresses that are induced, and the contribution of tidal stresses to hydrofracture propagation. Our results show that ocean tides and lake-water pressure together control supraglacial lake drainage through hydrofractures in the grounding zone. We give a model-based criterion that predicts supraglacial lake drainage as a function of daily maximum tidal amplitude and pre-drainage lake depth. Our model-based criterion agrees with remotely sensed data, indicating the importance of tidal flexure to processes associated with hydrofracturing such as supraglacial lake drainage and crevassing.

1 Introduction

Atmospheric warming is driving increasing meltwater production on ice-sheet and ice-shelf surfaces (Trusel et al., 2015). In the melt season, the meltwater ponds in topographic lows and forms supraglacial lakes. Lakes drain either slowly, through surface drainage channels (Banwell et al., 2019), or rapidly, through hydrofractures (Das et al., 2008). Lake drainages through hydrofractures can potentially impact ice-sheet mass balance in various ways. For grounded ice sheets, hydrofracture efficiently transports surface meltwater to the subglacial hydrological system. This reduces bed friction and thus modulates ice-flow velocity and flux (Das et al., 2008; Doyle et al., 2013; Tedesco et al., 2013; Stevens et al., 2015; Dunmire et al., 2020). At ice shelves, hydrofractures can initiate or promote rifts. When propagating through ice shelves, rifts can destabilise them by triggering iceberg calving and ice-shelf collapse (Scambos et al., 2000; Glasser and Scambos, 2008; Banwell et al., 2013, 2019; Warner et al., 2021; Lipovsky, 2020), which leads to a loss of buttressing and increased ice-sheet mass loss.

In East Antarctica, satellite imagery suggests that supraglacial lakes often cluster in the grounding zone, particularly at low elevations and bedslopes. Many of these lakes are connected to surface drainage systems or located in regions that are vulnerable to hydrofracturing (Stokes et al., 2019). In the grounding zone, besides lake–water pressure, tensile stress due

to tidal flexure can promote hydrofracturing. Meanwhile, advection of damaged ice produced in the grounding zone could destabilise the downstream ice shelf (Borstad et al., 2012). Thus it is important to understand how hydrofractures are initiated and promoted in the grounding zone.

Recently, Trusel et al. (2022) reported a series of repeated drainage events of a supraglacial lake at the grounding line (GL) of the Amery Ice Shelf, East Antarctica. Interestingly, these drainage events did not occur past a threshold in lake volume, but rather tended to coincide with times of high daily tidal amplitude. These observations raise the question: *how do tides near the GL contribute to lake drainage through hydrofracturing*? Trusel et al. (2022) hypothesised that near the GL, the drainage events are promoted by tensile stress due to tidal flexure. To test this hypothesis and further explore the dependence of lake drainage on ocean tides and lake depth, we present a numerical model that accounts for the tidal contribution to hydrofracturing near the Amery Ice Shelf GL.

As an important component of the marine ice sheet, the GL serves as an internal boundary connecting the grounded ice sheet and floating ice shelf. Variations in its position control the total ice flux from inland to the sea. In Antarctica, the GL response to diurnal ocean tides has been documented by various observations. On the Rutford Ice Stream, West Antarctica, ice flow is modulated by semi-diurnal tides, with tidal effects on the ice-flow rate propagating tens of kilometres upstream (Gudmundsson, 2006; Murray et al., 2007; Minchew et al., 2017). At the Amery Ice Shelf, kilometre-scale tidal GL migration with seawater intrusion has been observed from differential radar interferometry (Chen et al., 2023). The observed grounding zone is much larger than predicted from hydrostatic equilibrium, raising questions about whether the observed tidal flexure within the grounding zone is associated with stresses that contribute to hydrofracturing.

Ice-shelf flexure at the grounding line can be modelled using thin-plate theory with an elastic (Vaughan, 1995; Sayag and Worster, 2011; Wagner et al., 2016; Warburton et al., 2020) or a viscoelastic constitutive relationship (Reeh et al., 2003; Gudmundsson, 2007). In these models, the GL is treated as a peeling front or as the clamped end of the ice shelf. Thin-plate models capture the large-scale flexure of ice shelves, and neglect the membrane stress that is governed by sliding.

Various studies have used a vertically integrated theory with a viscous rheological law to investigate the dependence of steady-state grounding-line position on ice thickness, sliding laws, buttressing effects and bed topography (Schoof, 2007a, b; Katz and Worster, 2010; Schoof, 2012; Tsai et al., 2015; Pegler, 2018; Haseloff and Sergienko, 2022; Sergienko and Haseloff, 2023). Besides depth-integrated models, full-Stokes models have also been used to study the migration of grounding lines on both longer timescales (Nowicki and Wingham, 2008; Durand et al., 2009; Favier et al., 2012; Gudmundsson et al., 2012; Cheng et al., 2020) and tidal timescales (Gudmundsson, 2011; Rosier et al., 2014, 2015; Rosier and Gudmundsson, 2020). In these models, the ice-sheet–bed contact problem has been incorporated as boundary conditions. A more recent development by Stubblefield et al. (2021) and de Diego et al. (2022) has incorporated the contact boundary conditions into variational inequalities. This formulation enabled the representation of the contact condition within a variational framework that was implemented in the Finite Element method. In this study, we adopt the framework by Stubblefield et al. (2021) to study the tidal effects on GL dynamics of the Amery Ice Shelf.

For a Maxwell viscoelastic ice shelf subject to external tidal forcings, the shelf initially responds elastically, followed by viscous creep. The Maxwell time represents the characteristic timescale over which the ice transitions from behaving elastically

to viscously. Since the semi-diurnal tidal period at the Amery Ice Shelf (approximately 12 hours) is close to the Maxwell time of ice (less than 5 days in our estimation at the GL), we use a viscoelastic constitutive relationship to model the tidal flexure. In particular, we extend the framework for a marine ice sheet with viscous ice flow by Stubblefield et al. (2021) to an upper-convected Maxwell (UCM) model (e.g., Snoeijer et al., 2020) to capture the viscoelastic stress and GL migration induced by the large deformation associated with tides. The upper convected time derivative provides an objective measure of the rate of stress change within a fluid material parcel, meaning that the stress response is independent of the observer's frame of reference.

We use this framework to predict the tensile stress at the GL during daily maximum tidal amplitudes. Then, using Linear Elastic Fracture Mechanics (LEFM) analysis, we estimate the contributions from tidal stress and lake-water supply to quasi-static hydrofracturing. This enables a model-based criterion in terms of tidal amplitude and lake-water depth for tidally-induced supraglacial lake drainage, which we compare to the lake-drainage time series presented by Trusel et al. (2022). The results support the hypothesis that at the Amery Ice Shelf GL, supraglacial lake drainage is controlled by both lake depth and tidal amplitude. Moreover, this new model-based criterion enhances our understanding of the role ocean tides play in driving lake drainage through tidal flexure.

The paper is organised as follows. In Sect. 2, we show an analysis of the ice-flow patterns near the lake, the viscoelastic marine ice-sheet model, and the corresponding numerical implementation. In Sect. 3, we demonstrate the viscoelastic tidal response of a marine ice sheet, then use LEFM analysis to construct a drainage criterion and compare it with observations. In Sect. 4, we present an analysis of the model's sensitivity to ice Maxwell time and bedslope angle, and summarise model limitations.

2 Method

In this section, we first use remotely-sensed ice-surface velocity fields to estimate the local strain-rate and stress fields near the lake studied by Trusel et al. (2022). Our estimate indicates the lake region is extension-dominated with negligible shear stress. The background stress is insufficient to produce hydrofracturing, indicating the importance of tidal stress. Therefore, we adopt the viscous, marine ice-sheet model by Stubblefield et al. (2021) and incorporate a viscoelastic rheological formulation. We introduce the model set-up and describe how we predict lake drainage events. Then we explain how the observational data is processed to enable comparison with our models.

2.1 Data extraction and model selection

Fig. 1(a) shows the ice-surface velocity field v from the MEaSUREs InSAR-Based Antarctica Ice Velocity Map (Rignot et al., 2011b, 2017; Mouginot et al., 2012, 2017) near the lake, from which we calculate ice-flow streamlines and the strain rate $\dot{\varepsilon}$ (Fig. 1b). The lake is located on the grounding line extracted from MEaSUREs Antarctic Grounding Line from Differential Satellite Radar Interferometry, Version 2 (Rignot et al., 2016b, 2011a, 2014; Li et al., 2015). Following Wearing (2017), we denote the direction of velocity $\hat{v}$, and the transverse direction $\hat{t}$. The along-flow strain rate can be defined as

$$\dot{\varepsilon}_p = \hat{v} \cdot \dot{\varepsilon} \cdot \hat{v}, \tag{1}$$

and the transverse strain rate as

$$\dot{\varepsilon}_t = \hat{\boldsymbol{v}} \cdot \dot{\boldsymbol{\varepsilon}} \cdot \hat{\boldsymbol{t}}. \tag{2}$$

95   Fig. 1(c,d) shows $\dot{\varepsilon}_p$ and $\dot{\varepsilon}_t$ near the lake. We are particularly interested in $\dot{\varepsilon}_p$ and $\dot{\varepsilon}_t$ along the streamline that crosses the lake. Assuming that the z-components of ice deviatoric stress $\boldsymbol{\tau}$ are zero on the surface ($\tau_{xz} = \tau_{yz} = \tau_{zz} = 0$), we calculate the viscosity η using Glen's flow law (Glen, 1955). The deviatoric stress along (τ_p) and transverse (τ_t) to the streamline are $\tau_p = 2\eta\dot{\varepsilon}_p$, $\tau_t = 2\eta\dot{\varepsilon}_t$, as shown in Fig. 1(f). At the lake, there is negligible background shear stress and an extensional stress with a magnitude of about 90 kPa in the flow direction, indicating that the hydrofracture is associated with along-flow extension.

100 Therefore, for simplicity, we adopt a 2-D, flow-line model that accounts for tidally induced extension along the flow direction.

     We use BedMachine Antarctica v2 to obtain the basal topography and ice geometry along the flow line (Fig. 1e) (Morlighem et al., 2017, 2020). The surface gradient is relatively uniform upstream of the GL. The subglacial cavity downstream of the grounding zone is more than 20 m in height, which is large enough to allow free tidal oscillation without the formation of pinning points. Thus, we assume that the ice shelf downstream of the GL does not contact the bedrock, and the water pressure

105 on the ice–ocean interface is hydrostatic. In the computation, we use a linear bedrock topography (Fig. 1e) with a slope angle chosen to approximately match observations. This simplification enables us to further explore the sensitivity of GL dynamics to bedslope angle. In Appendix B we present a set of model results with real bed topography for comparison. While the magnitude of tidal stress is modified quantitatively in the real-bed-topography case, tidal stress remains the main contribution to the extension on the GL, compared with the background extensional and shear stress.

110 ## 2.2   Lake basin bathymetry

Trusel et al. (2022) report the characteristics of the supraglacial lake located at 70.59 °S, 72.53 °E. As an input to a model of hydrofracture beneath this lake, we estimate the hydrostatic pressure at the bottom of the lake basin. To this end, we obtain the elevation of the basin from a 2-m resolution WorldView-1 DEM captured when the lake was dry. The elevation of the flat basin, excluding any craters or hydrofractures, is considered the lowest point of the lake. The basin is approximately 10 m deeper

115 than the surrounding terrain and spans about 1 km in the direction perpendicular to the grounding line. The lake-water depth prior to each drainage event is calculated by subtracting the lake-basin elevation from the median shoreline elevation, which we obtain using a shoreline-extraction technique as outlined in Moussavi et al. (2016) and Trusel et al. (2022).

2.3   Model domain

Fig. 2 shows a schematic of the computational domain. We consider a segment of marine ice sheet with length $2L$ and thickness

120 $H(x,t)$ in a Cartesian coordinate system with position vector $\boldsymbol{x} = (x, z)$, where z increases upward. The inflow and outflow boundaries are denoted Γ_{in} and Γ_{out}. The top surface is denoted Γ_h. The bottom is divided into two parts, according to whether the ice is in contact with the bedrock or the ocean. The ice–bedrock interface Γ_s is where ice is in contact with the bedrock at a height $b(x)$. As a simplification, we assume that the bedrock has a uniform slope θ. The ice–ocean interface Γ_w is where ice is detached from the bedrock. The two boundaries Γ_s and Γ_w meet at the GL whose horizontal position, denoted $x_g(t)$, migrates
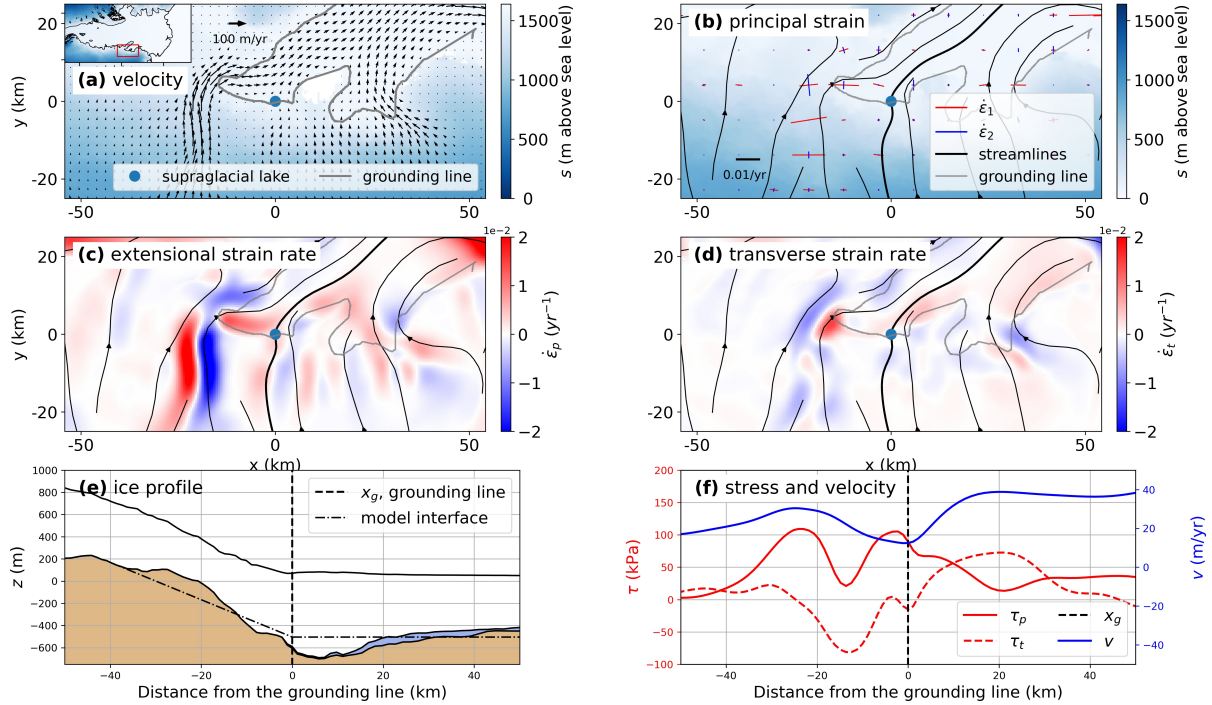
Figure 1. Ice surface velocity, strain rate and stress in the region of the supraglacial lake. **(a)** Velocity field near the grounding line (grey), where the supraglacial lake is denoted with the blue dot. Here the largest arrow in the figure represents a maximum velocity of 20 m yr^{-1}. The colour represents the ice-sheet surface elevation above sea level. The map inset on the top left corner shows the full Amery Ice Shelf topography, with the plotted region outlined with a red box; **(b)** principal strain rate and streamlines. The streamline that crosses the lake is marked with the bold line; **(c)** along-flow strain rate with positive values representing extension; **(d)** transverse strain rate; **(e)** local ice-sheet geometry and bed topography; **(f)** For the streamline that crosses the lake, along-flow deviatoric extension τ_p (solid red line) and shear stress τ_t (dashed red line), and speed v (blue). Note that $x = x_g = 0$ is the position of the supraglacial lake as well as the grounding line.

125 with time t. The origin of the coordinate system is set at the middle of the domain on the ice–bedrock interface (at the position of the GL shown in the schematic).

Following Stubblefield et al. (2021), we first construct the mesh with piecewise linear bottom profile $s(x)$ and surface profile $h(x)$

$$s(x) = \max(b(x), 0),$$
$$h(x) = s(x) + H,$$

(3)

130 which are shown in Fig. 2. We evolve this initial profile with no tides until the ice-flow geometry (i.e., $s(x)$, $h(x)$ and GL x_g) reaches a steady state. In practice, the ice flow reaches the steady state when the grounding line attains its steady position, defined as when its migration can not be resolved with the 25-m grid size. This provides a steady profile of the marine ice sheet for use as an initial condition for simulations with tides.
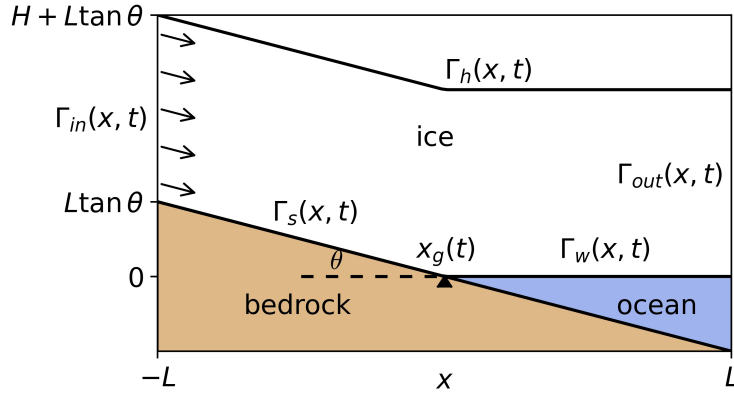
Figure 2. Schematic showing the model domain of a marine ice sheet system.

2.4 Governing equations

135 The governing equations for momentum and mass conservation are

$$\nabla \cdot \boldsymbol{\sigma} + \rho_i \boldsymbol{g} = \boldsymbol{0}, \tag{4}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{5}$$

where $\boldsymbol{\sigma}$ is the total Cauchy stress tensor, ρ_i is ice density, $\boldsymbol{g}$ is gravity, and $\boldsymbol{u}$ is the ice velocity field. The stress $\boldsymbol{\sigma}$ can be decomposed as an isotropic part and deviatoric part, $\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau}$, where p and $\boldsymbol{\tau}$ represent the pressure and deviatoric stress,

140 respectively. Here $\boldsymbol{I}$ is the unit tensor.

To model viscoelasticity, we adopt the upper-convected Maxwell formulation for the deviatoric stress $\boldsymbol{\tau}$. The constitutive relationship is

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta \dot{\boldsymbol{\varepsilon}}, \tag{6}$$

where the Maxwell time, $\lambda = \eta/\mu$, is the ratio of ice viscosity η to shear modulus μ. The strain rate is denoted $\dot{\boldsymbol{\varepsilon}}$. The upper-

145 convected time derivative (Oldroyd rate) $\overset{\nabla}{\boldsymbol{\tau}}$ measures the temporal variation of $\boldsymbol{\tau}$ including the effect of rigid body rotation,

$$\overset{\nabla}{\boldsymbol{\tau}} = \partial_t \boldsymbol{\tau} + \boldsymbol{u} \cdot \nabla \boldsymbol{\tau} - (\nabla \boldsymbol{u})^T \cdot \boldsymbol{\tau} - \boldsymbol{\tau} \cdot \nabla \boldsymbol{u}, \tag{7}$$

where $(\cdot)^T$ represents tensor transpose.

We assume a constant shear modulus μ and non-Newtonian viscosity η that is governed by Glen's flow law with regularisa-

150 tion

$$\eta = \frac{1}{2}B\left(|\dot{\boldsymbol{\varepsilon}}|^2 + \delta_\nu\right)^{-(n-1)/2n}, \tag{8}$$

where $B = 2^{(n-1)/2n} A_0^{-1/n}$ is determined by the two flow law parameters A_0 and n, and $|\dot{\varepsilon}| = \sqrt{\dot{\varepsilon} : \dot{\varepsilon}}$ is the Frobenius norm of the strain rate. The regularisation with the numerical parameter δ_ν is used to prevent infinite viscosity at vanishing strain rate (Jouvet and Rappaz, 2011; Helanow and Ahlkrona, 2018; Stubblefield et al., 2021). The value of δ_ν sets an upper limit on the viscosity and, therefore, also on the Maxwell time. In our reference parameter set, used below, $\eta \leq 1.3 \times 10^{14}$ Pa s and $\lambda \leq 120$ hr. For $\delta_\nu = 0$, Eq. 8 reduces to the classical form of Glen's flow law. In Sect. 4, we test the sensitivity of the model to λ, highlighting the importance of a good estimate of λ for modelling GL dynamics.

2.5 Boundary conditions

Neglecting atmospheric pressure and other surface loading, the top boundary is assumed to be stress-free. Its elevation h is governed by the kinematic condition

$$\frac{\partial h}{\partial t}(x,t) = \left[\left(\frac{\partial h}{\partial x}\right)^2 + 1\right]^{1/2} \boldsymbol{u} \cdot \boldsymbol{n} \qquad \text{on } \Gamma_h, \tag{9}$$

where $\boldsymbol{n}$ is the outward-pointing unit normal vector and Γ_h is the top boundary.

On the inflow boundary Γ_{in}, we impose a uniform horizontal inflow rate u_0 and zero shear stress

$$\begin{cases} \boldsymbol{u} \cdot \boldsymbol{n} = u_0, \\ \boldsymbol{t} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} = 0, \end{cases} \qquad \text{on } \Gamma_{in}, \tag{10}$$

where $\boldsymbol{t}$ is the tangential unit vector. The inflow velocity u_0 is set to be the satellite-derived surface velocity, 18 m y^{-1} (Rignot et al., 2016a). On the outflow boundary, we impose the ice-overburden pressure

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = -\rho_i g (h - z) \boldsymbol{n}, \qquad \text{on } \Gamma_{out}, \tag{11}$$

which means that at the downstream boundary Γ_{out}, the ice shelf floats at hydrostatic equilibrium, without bending stress.

Similar to Eq. 9, the bottom profile $s(x,t)$ is governed by the kinematic equation

$$\frac{\partial s}{\partial t}(x,t) = -\left[\left(\frac{\partial s}{\partial x}\right)^2 + 1\right]^{1/2} \boldsymbol{u} \cdot \boldsymbol{n}, \qquad \text{on } \Gamma_w, \tag{12}$$

where Γ_w is the ice–ocean interface. The stress on the bottom boundary depends on the local contact condition. To introduce the boundary conditions related to the contact problem, we consider hydrostatic water pressure p_w on the ice–ocean interface, defined as

$$p_w = \rho_w g [h_w(t) - s^*], \qquad \text{on } \Gamma_w, \tag{13}$$

where ρ_w is water density, g is gravitational acceleration, h_w is the sea level and s^* is the approximated bottom boundary that will be introduced in subsection 2.6. The atmospheric pressure is neglected at the ice–ocean interface, as it is generally small compared to the hydrostatic pressure.

The sea level h_w is a superposition of a steady state h_0 and a sinusoidal function of time, representing ocean tides with amplitude A and frequency f,

$$h_w(t) = h_0 + A \sin(2\pi f t). \tag{14}$$

On the ice–ocean interface, the hydrostatic pressure p_w is imposed as the traction

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = -p_w \boldsymbol{n}, \qquad \text{on } \Gamma_w. \tag{15}$$

On the ice–bedrock interface, ice can be either attached or detached from the bed. In the normal direction, the contact condition is established by the following boundary conditions

$$\begin{cases} \sigma_n \geqslant p_w, \\ u_n \leq 0, \qquad\qquad \text{on } \Gamma_s, \\ (\sigma_n - p_w)\, u_n = 0, \end{cases} \tag{16}$$

where σ_n is the normal component of traction. Here the water pressure p_w follows Eq. 13. When $u_n = 0$, ice is attached to the bed, $\sigma_n \geq p_w$. When $u_n < 0$, ice is detached from the bed, thus $\sigma_n = p_w$. The impenetrability condition is implemented using a penalty term shown in subsection 2.6, originally proposed by Stubblefield et al. (2021).

In the tangential direction, ice sliding is resisted by friction that is governed by a Weertman-type sliding law (Weertman, 1957)

$$\boldsymbol{t} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} = -C \left[(\boldsymbol{u} \cdot \boldsymbol{t})^2 + \delta_s \right]^{-\frac{n-1}{2n}} \boldsymbol{u} \cdot \boldsymbol{t} \qquad \text{on } \Gamma_s, \tag{17}$$

where C is the friction coefficient, δ_s is a numerical factor preventing singularity, and n is the exponent in Glen's flow law Eq. 8. In the computation, we choose C such that the surface velocity at the GL matches the inflow speed u_0. This choice gives a relatively low surface velocity gradient, which agrees with satellite observations at the lake region (Rignot et al., 2016a).

2.6 Numerical Implementation

When implementing the hydrostatic water pressure on the ice–ocean interface, for numerical stability, rather than the bottom elevation from the previous time step, s^* is an approximation to the current step elevation (Durand et al., 2009; Stubblefield et al., 2021), defined as

$$s_*(x,t) = s(x, t - \Delta t) - u_n(x, s, t)\Delta t, \tag{18}$$

where Δt is the numerical time step, $s(x, t - \Delta t)$ is the bottom profile at the previous time step, and u_n is the normal velocity on the bottom boundary ($u_n > 0$ accounts for downward motion).

In the variational formulation, on the ice–bed interface, the contact condition Eq. 16 is accounted for by imposing the hydrostatic pressure Eq. 13 to basal ice, along with a line integral as a penalty term in the variational formulation

$$\frac{1}{\varepsilon} \int_{\Gamma_s} \frac{1}{2} \left(\boldsymbol{u} \cdot \boldsymbol{n} + |\boldsymbol{u} \cdot \boldsymbol{n}| \right) \boldsymbol{v} \cdot \boldsymbol{n} \, \mathrm{d}s, \tag{19}$$

Table 1. Parameters used in numerical model and their reference values.

Physical property	Notation	Value
Density of water	ρ_w	1027 kg m^{-3}
Density of ice	ρ_i	917 kg m^{-3}
Length of the domain	L	20 km
Ice thickness	H	500 m
Bedslope angle	θ	0.02
Glen's Law exponent	n	3
Viscosity coefficient	A_0	3.1689×10^{-24} Pa^{-n} s^{-1}
Characteristic (inflow) velocity	u_0	9 m y^{-1}
Friction coefficient	C	1.0×10^7 Pa$^{1/n}$ m^{-1}
Shear modulus	μ	0.30×10^9 Pa
Viscosity regularisation parameter	δ_ν	10^{-18} s^{-2}
Friction regularisation parameter	δ_s	10^{-15} m^{2}s^{-2}
Penalty parameter	ε	10^{-13}
Tidal amplitude	A	1 m
Ice fracture toughness	K_C	10^5 Pa m$^{1/2}$

205   where ε is the penalty parameter and $\boldsymbol{v}$ is the test function corresponding to the velocity field $\boldsymbol{u}$. The penalty term Eq. 19 becomes non-zero only when $u_n > 0$ and thus penalises penetration. For the viscous contact problem, when $\varepsilon \to 0$ the solution to the variational formulation weakly converges to the solution governed by the contact condition Eq. 16 (Kikuchi and Oden, 1988). For the viscoelastic case, although the variational formulation cannot be directly cast as a minimization problem for $\varepsilon \to 0$, this limit is still a good approximation to the solution governed by the contact condition.

210   The variational formulation is implemented using the finite-element library FEniCS (Logg and Wells, 2010; Logg et al., 2012; Langtangen and Logg, 2017). A mixed finite element is used to solve for a combined field $(\boldsymbol{u},\ p,\ \boldsymbol{\tau})$. We use triangular elements in which the pressure varies linearly and the velocity and deviatoric stress vary quadratically. In Appendix A, we report convergence tests showing the results are mesh-independent and, in the limit of no elastic deformation ($\mu \to \infty$), converge to the viscous solutions by Stubblefield et al. (2021). For further details about the variational formulation and its numerical

215   implementation, the reader is referred to Stubblefield et al. (2021).

3   Results

We first present a reference case representing the tidal response of the marine ice sheet near the lake studied by Trusel et al. (2022). The lake depth (≤ 10 m) is much smaller than the ice thickness (≈ 500 m) at the location, and hence its effects on ice
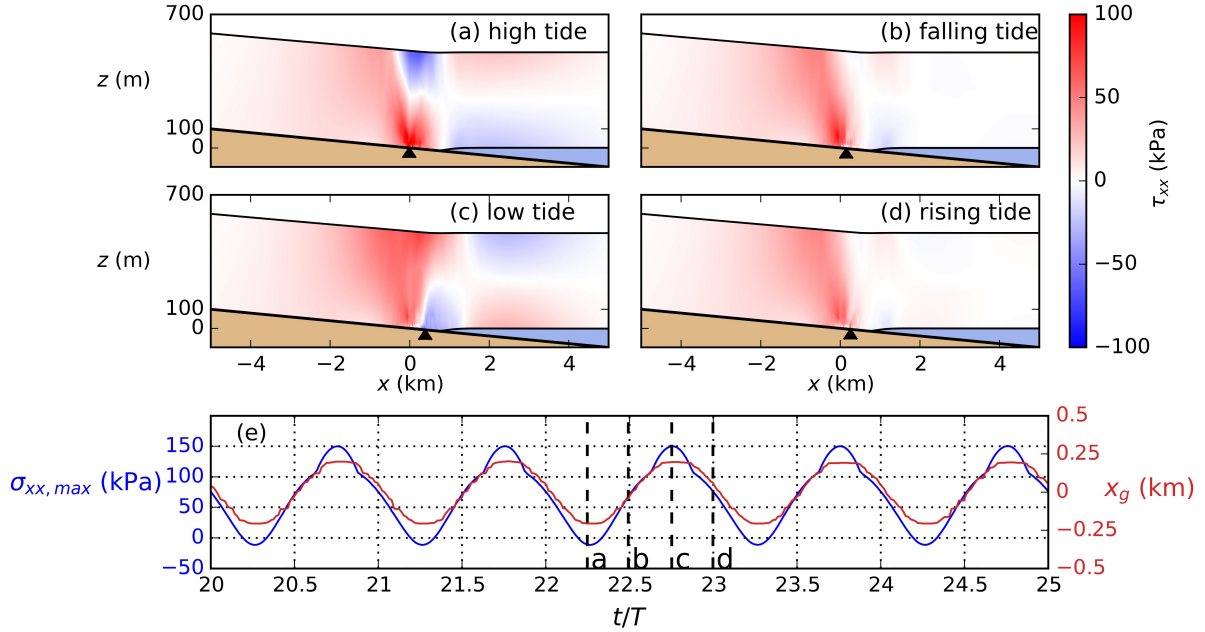
Figure 3. Tidal response of a marine ice sheet at different tidal phases. **(a)–(d)** Deviatoric tensile stress τ_{xx} in one tidal period, with red indicating tensile stress and blue indicating compressive stress. The black triangle marks the position of the grounding line. **(e)** The maximum tensile stress $\sigma_{xx,max}$ (blue) on the top boundary within the lake region ($\bar{x}_g - 0.5$ km $\leq x \leq \bar{x}_g + 0.5$ km) and the GL position x_g (red) versus time (scaled by the tidal period T) with positive values representing downstream migration. Vertical dashed lines show the time of panels (a)-(d).

dynamics are assumed to be negligible. We consider a 20-km long section of a 500-m thick ice sheet, sliding on bedrock with
220   constant bedslope angle $\theta = 0.02$. This angle is characteristic of bedslope upstream of the lake, as shown by the dash-dotted line in Fig. 1(e). The ice thickness and bedslope angle, representing the local topography, are adopted from Morlighem et al. (2017). Initially, the grounded ice sheet and floating ice shelf each cover 10 km of the domain. In the model of elasticity, we use Young's modulus $E = 0.88$ GPa and Poisson's ratio $\nu = 0.41$, as suggested by Vaughan (1995) for tidal flexure problems. A list of of parameters and their references values is provided in Table 1. The tidal amplitude $A = 1$ m is a characteristic local
225   tidal amplitude calculated from the CATS2008 (Circum-Antarctic Tidal Simulation) model (Howard et al., 2019).

3.1   Tidally-induced grounding line migration and stress

As shown in Eq. 3, we first construct the mesh with piecewise linear bottom profile and evolve this initial profile with no tides for 40 yr, by which time the ice-flow geometry (i.e., $s(x)$, $h(x)$ and GL x_g) reaches a steady state. The steady profile is then used as an initial condition for simulations with tides.

230   We find tidally modulated GL migration and corresponding change in stress. The GL position x_g is shown in Fig. 3e. Whereas Stubblefield et al. (2021) find double GLs at low tides with a relatively small bedslope angle $\theta = 2.5 \times 10^{-4}$ (Stubblefield et al., 2021), in our model we find only a single GL migrating in phase with the tides. This migration results in a 600-m wide grounding zone, which is larger than estimated from hydrostatic equilibrium assuming a uniform ice thickness ($2A/\theta = 100$ m). Chen et al. (2023) studied the GLs of the feeding glaciers of the Amery Ice Shelf. Although the location is dif-

235 ferent from the lake position we discuss, they reported that near the GL with prograde bedslopes, the observed grounding-zone width is one order of magnitude larger than predicted from hydrostatic equilibrium.

     To demonstrate tidal flexure, we plot the deviatoric stress component τ_{xx} at four tidal phases (Fig. 3a–d). The ice undergoes upward and downward flexure at high and low tides, respectively. At high tide (Fig. 3a), the stress is concentrated close to the GL, with compression near the top and tension near the bottom, closely downstream of the GL. This resembles the stress

240 pattern of a thin-plate (Timoshenko, 1955), indicating a region where the ice vertical velocity transitions from the ice-sheet flow to the ice-shelf oscillation with tides. Further downstream at the floating shelf, the stress is predominantly cryostatic without bending. At low tide (Fig. 3c), the tensile stress dominates the ice-sheet top surface near the GL. The region experiencing tensile stress is larger and located further upstream. At rising tides (Fig. 3b) and falling tides (Fig. 3d), τ_{xx} is tensile at the GL, but the magnitude is smaller than τ_{xx} at low tide.

245   The full horizontal tensile stress σ_{xx} is considered for hydrofracturing at the lake. Assuming that the lake covers the ice-sheet surface within $|x - \overline{x}_g| \leq 0.5$ km, where $\overline{x}_g$ is the time-averaged GL position in a tidal period, we calculate the maximum σ_{xx} on the ice-sheet surface within the lake region for any given time, which is denoted $\sigma_{xx,max}$. The temporal variation of $\sigma_{xx,max}$ is shown in Fig. 3e. In each tidal period, $\sigma_{xx,max}$ reaches its peak at low tide, corresponding to the downward flexural stress in Fig. 3c.

250   The reference case gives the tidal stress at tidal amplitude $A = 1$ m. We further consider cases with a series of tidal amplitude from 0 to 1 m, and thus obtain a stress–amplitude relationship for sinusoidal semi-diurnal tides, which is referred to as the "σ–A relationship" from this point forward. Specifically, it's the relationship between $\sigma_{xx,max}$, the maximum tensile stress in the lake region within one tidal period, and the tidal amplitude A, assuming that the sea level variation due to tides follows a monochromatic sinusoid over time. However, with solar tides, tidal amplitude is modulated in a two-week cycle. Given the

255 viscoelastic rheology with its history-dependence, such an amplitude modulation might complicate the σ–A relationship from monochromatic tides.

     To explore the σ–A relationship with solar tides, we replace the sinusoidal tides in Eq. 14 with a modulated sine wave over a 14-day period, with sea-level variation shown in Fig. 4. Applying this forcing to the reference case, the GL migrates in phase with tides (Fig. 4a). In each tidal period the low-tide $\sigma_{xx,max}$ tracks the σ–A relationship for sinusoidal tides, with slight

260 discrepancies observed at small tidal amplitudes (Fig. 4b), indicating that solar tidal amplitude modulation does not change the σ–A relationship. Therefore, daily maximum tidal amplitude proves to be a good metric to estimate the daily maximum tidal stress that contributes to hydrofracturing.
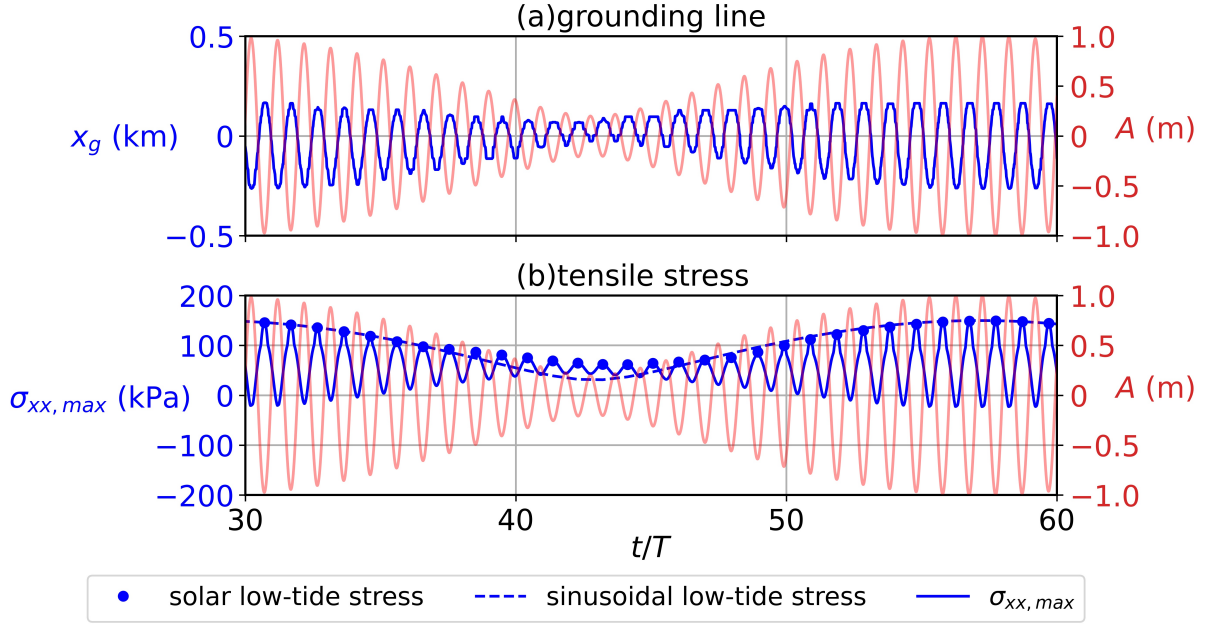
Figure 4. (a) Modulated tidal amplitude (red) and corresponding GL migration (blue). The horizontal axis is time scaled by the tidal period. **(b)** (blue) Maximum deviatoric tensile stress $\sigma_{xx,max}$ on the ice-sheet surface within the lake region with modulated tidal amplitude. The dots denote the low-tide stress in one tidal period. The dashed blue line is the estimated low-tide stress calculated from the modulated tidal amplitude using the σ–A relationship from sinusoidal semi-diurnal tides in subsection 3.1.

3.2 Linear Elastic Fracture Mechanics model of the hydrofracture

Since hydrofracturing typically occurs on a short timescale over which ice behaves elastically, we consider the fracture propagation in the LEFM framework. The hydrofracture is assumed to be a quasi-static elastic fracture occurring at the location with $\sigma_{xx,max}$ at low tides. The stress that drives its propagation is the sum of the water pressure and tidal stress. The water pressure in the fracture p_w is assumed to be hydrostatic; the tidal stress $\sigma_{xx,max}$ is calculated by the viscoelastic model mentioned above. We use the weight-function method to calculate the stress intensity factor K_I (Tada et al., 2000). Since at low tides the GL goes downstream and leaves the ice beneath the lake attached to the bedrock, we use a weight function that is designed for ice grounded on rigid bedrock, as suggested by Jimenez and Duddu (2018). Details about the calculation of K_I are provided in Appendix C.

Fig. 5a shows a schematic of the fracture model. The lake basin has a depth $d_b = 10$ m (obtained from the DEM) and is filled with water to a depth d_w. The horizontal stress $\sigma_{xx}(z)$ represents the low-tide tidal stress, and is obtained from the numerical results with a given tidal amplitude A. For a vertical fracture, we can calculate the stress intensity factor K_I as a function of its length d_l. If K_I exceeds the ice fracture toughness K_C, the fracture can propagate, until $K_I = K_C$. We assume that lake drainage occurs when a vertical hydrofracture reaches the ice-sheet bottom.
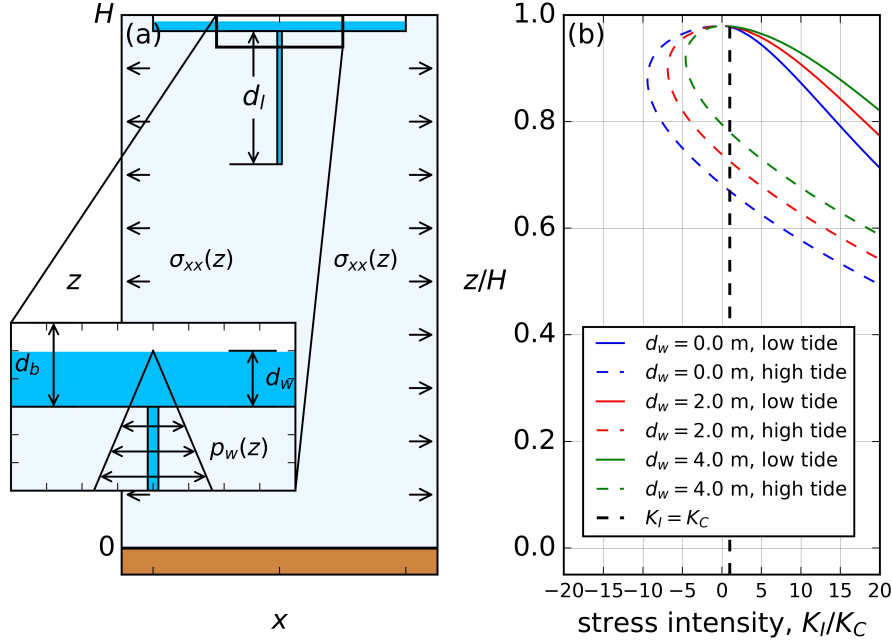
Figure 5. (a) The LEFM model of the hydrofracture. The lake basin with depth d_b, is filled with water to a depth d_w. Here d_w serves as a measurement of the water pressure p_w, as shown in the zoom-in window. Promoted by the tidal stress $\sigma_{xx}(z)$ and lake-water pressure p_w, a vertical fracture with length d_l is initiated from the lake bottom. **(b)** A reference case showing K_I/K_C varying with depth (scaled by the ice thickness) and tidal phases, with $A = 1.0$ m, $d_b = 10$ m, and $d_{l,init} = 0.1$ m. The solid lines represent K_I/K_C at low tides. The dashed lines represent K_I/K_C at high tides when upward flexure causes compression, and thus a negative tidal contribution to hydrofracture.

Note that for the initial fracture, K_I is sensitive to its length $d_{l,init}$. However, there is little observational constraint on the lengths of pre-existing fractures at the lake basin. While the choice of $d_{l,init}$ requires further study, the relative importance of the lake pressure versus the tidal amplitude is independent of $d_{l,init}$, which is shown in the model-based criterion below. Here we will choose $d_{l,init}$ such that the model criterion best fits the drainage data. For an initial fracture with $d_{l,init} = 0.1$ m to propagate, the required critical tensile stress is approximately $\sigma_{xx} = 150$ kPa. For $d_{l,init} = 0.2$ m, the critical stress is about 100 kPa, which cannot be achieved by background extension alone.

In Fig. 5b, using the stress distribution σ_{xx} from the numerical results with $A = 1$ m, we plot a reference case showing K_I/K_C versus the fracture length d_l, under different combinations of lake depth d_w ($d_w = 0, 2, 4$ m) and tidal phases (low tide and high tide). The vertical dashed line represents the ice fracture toughness K_C. At low tide, downward flexure gives positive K_I near the upper surface. Conversely, at high tides, compression gives negative K_I there. Fig. 5b can be used to predict lake drainage: at low tides, for a pre-existing fracture with length $d_{l,init}$, if $K_I > K_C$ holds for any depth that the fracture can reach, then A and d_w are predicted to induce lake drainage. Repeating this treatment for different combinations of A and d_w yields the model-based drainage criterion, which is shown in subsection 3.3.
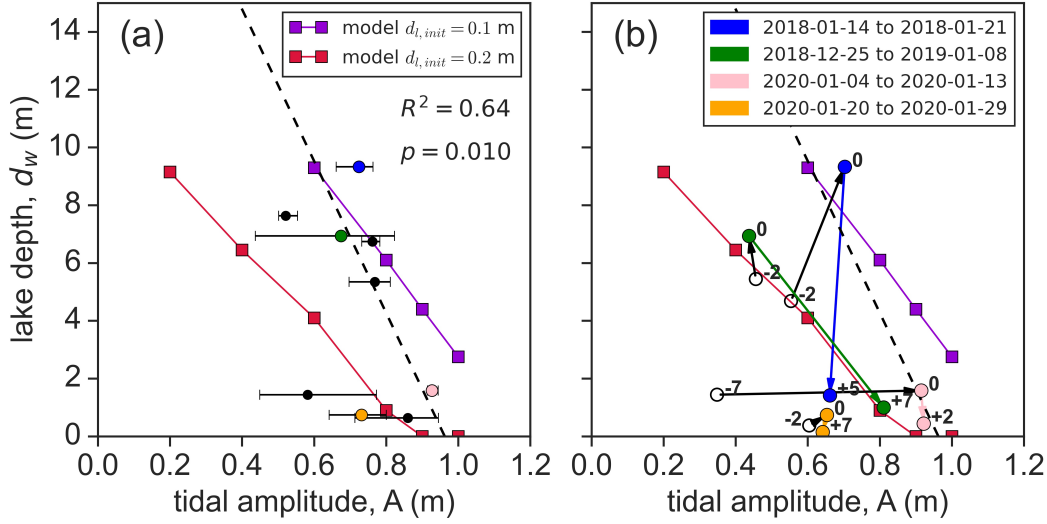
Figure 6. (a) A comparison between the model criterion and the drainage data. Each circle represents one drainage event from Trusel et al. (2022). The horizontal coordinate is the time-averaged daily maximum tidal amplitude during the drainage, with an error bar representing the range of the daily maximum tidal amplitude. The vertical coordinate is the pre-drainage lake depth. The dashed black line is a weighted linear regression of the observations. The red and violet lines are model-based criteria with different initial crack lengths, with squares representing the numerical experiments. The four coloured circles represent drainage events with best-constrained temporal evolution of lake depth and tidal amplitude. **(b)** Temporal evolution of coloured events in panel (a). The points labelled "0" represent the day of the drainage. The negative and positive values represent the days before and after the drainage, respectively.

3.3 Drainage criteria in terms of tidal amplitude and lake depth

As shown above, at low tides for an initial fracture with a given length $d_{l,init} = 0.1$ m, the prediction of hydrofracturing is controlled by two factors: tidal stress and water pressure, measured by tidal amplitude A and lake depth d_w. When the combined effect of tidal flexure and lake-water pressure can overcome the ice fracture toughness and ice-overburden stress, the fracture propagates, reaches the ice–bedrock interface, and drains the lake.

We construct a model-based drainage criterion in terms of A and d_w in a 2-D parameter space. The criterion is defined as the marginal conditions at which the initial fracture can reach the ice–bedrock interface. In Fig. 6a, we show two criteria with different initial crack lengths $d_{l,init}$ of 0.1 and 0.2 m. When (A, d_w) of a lake crosses the criterion from bottom-left to top-right, the total tensile stress is large enough that the hydrofracture can reach the bottom of the ice sheet. As shown in Fig. 6a, the increasing tidal amplitude can reduce the lake depth required for hydrofracturing. This provides a measure of the relative importance of tidal flexure to water pressure for lake drainage. Meanwhile, there exists a maximum tidal amplitude above which fracturing can be induced entirely by tidal stress, without a water supply. Supraglacial lakes would not be able to form under such large tidal stress.

In Fig. 6a, we compare the model-based criterion with observations from Trusel et al. (2022). Each circle represents a drainage event. To leading order, the model agrees with the data. The data cluster close to the model-based criterion for realistic values of $d_{l,init}$. A weighted linear regression of the observations also suggests that higher tidal amplitude reduces the lake depth required for drainage. However, the steeper slope of the regression line suggests the dependence of drainage on tidal amplitude is stronger than predicted by our model.

To better demonstrate the drainage process, in Fig. 6b we show four events with the best observational constraint on temporal evolution of lake depth and tidal amplitude. We show measurements from before, during and after the drainage. Note that we simply assume the drainage occurs at the highest water level observed, which is the minimum pre-drainage water level due to the time interval between satellite images. The two events dated 2018-12-25 and 2020-01-04 cross the criterion with $d_{l,init} = 0.2$ m in the observational interval of several days. The event dated 2018-01-04 crosses both criteria. The post-drainage states are below the criterion with $d_{l,init} = 0.1$ m, representing the end of drainage due to insufficient water supply. Therefore, we believe our model captures the essential physics that controls hydrofracturing in the grounding zone.

4   Discussion

We use a viscoelastic model to predict GL migration and tidal stress of a marine ice sheet. By considering tidally-induced hydrofracturing in an LEFM model, we construct a model-based drainage criterion that accounts for the tidal amplitude and lake depth required for drainage. The criterion agrees with observations to leading order, thus supporting the hypothesis from Trusel et al. (2022) that "*hydrofracture is assisted by tidally forced ice flexure inherent to the ice shelf grounding zone.*" Furthermore, the criterion suggests that, in grounding zones, ocean tides might set a maximum volume for supraglacial lakes. We hypothesise that lakes tend to remain at a smaller volume than this critical value due to regular drainage events. The model also suggests a critical tidal amplitude beyond which tidal flexure alone can initiate fracturing, independent of lake pressure. Such fractures may act as vertical conduits, transporting surface water to the bed and preventing meltwater accumulation that forms supraglacial lakes. These hypotheses could be tested by a statistical study of more supraglacial lake drainage events with local ocean tides.

The tidal-flexure model and drainage criterion can be applied to other marine ice sheets with different rheological properties and bed topography. In the following, we perform a sensitivity analysis to explore the dependence of GL migration and tidal stress on ice rheology and bedslope angle. From this point forward, we use $\sigma_{xx,max}$ to denote the low-tide maximum tensile stress within the lake region, which is assumed to directly contribute to hydrofracturing. Finally, we discuss the limitations of our model by exploring missing factors that might affect GL dynamics and hydrofracture.

4.1   Sensitivity to ice Maxwell time

The Maxwell time of ice λ controls the viscoelastic tidal response of a marine ice sheet. Depending on the Deborah number De, the ratio of λ to the tidal period T, the tidal response can be dominated by either elasticity ($De \ll 1$) or viscosity ($De \gg 1$). Here, we explore the tidal response with a varying τ, representing a transition from viscoelastic to viscous rheology.
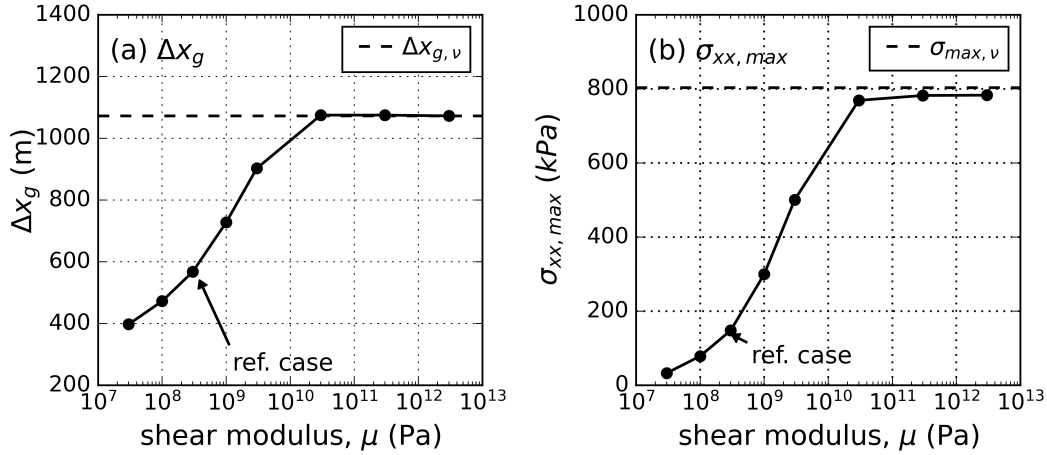
Figure 7. (a) The grounding-zone width Δx_g (solid line), defined as $\Delta x_g = \max\{x_r\} - \min\{x_l\}$ as a function of shear modulus $\mu = 3 \times 10^7$ to 3×10^{12} Pa, with x_l and x_r denote the left and right GL, respectively. Here for both the viscous case and the viscoelastic case, $x_l = x_r$. The dashed line shows $\Delta x_{g,v}$, the grounding-zone width in the viscous limit ($\mu \to \infty$). **(b)** Maximum tensile stress $\sigma_{xx,max}$ versus μ. The numerical reference case in Sect. 3 is labelled.

335 With the viscosity governed by Glen's Flow law, we consider cases with varying shear modulus μ, thus varying Maxwell time. All other parameters are set to their reference values. We define a grounding-zone width, Δx_g, as the range of GL in a tidal period. In Fig. 7a, Δx_g is plotted versus μ. With the bedslope $\theta = 2 \times 10^{-2}$, only one GL is found to migrate with the tides, giving a grounding-zone width of about $0.5 - 1$ km. As $\mu \to \infty$, the tidal response becomes purely viscous and the grounding zone widens and reaches its viscous limit $\Delta x_{g,v}$. In Fig. 7b, $\sigma_{xx,max}$ increases with increasing μ and converges to

340 the viscous stress calculated by the viscous model (Stubblefield et al., 2021).

 The Maxwell time of ice is crucial in determining GL migration and tidal stress, because viscous rheology tends to increase the width of a grounding zone and produce larger tensile stress than viscoelastic rheology. Therefore, our model indicates the importance of using viscoelastic rheology with an accurate ice Maxwell time to predict the magnitude of tidal stress. Given the dependence of Δx_g on μ (Fig. 7a), it may be possible to infer ice mechanical properties from observations on the range of GL

345 migration.

4.2 Sensitivity to bedslope angle

The above discussion shows how tidal response varies with shear modulus, with a characteristic bedslope $\theta = 2 \times 10^{-2}$ at the Amery Ice Shelf GL. Here we extend the results to different bedslopes and explore how the tidal response of a GL would change with local bathymetry. We consider three marine ice sheets with bedslope $\theta = 2 \times 10^{-4}$, 2×10^{-3}, and 2×10^{-2}, with

350 all other parameters set to be the same as the reference case.

 For simplification, we focus on the effect of θ while keeping the inflow velocity u_0 and the basal friction coefficient C in Eq. 17 fixed. Because of this, the ice adjusts to the changing bedslope through either thinning or thickening. The modelled
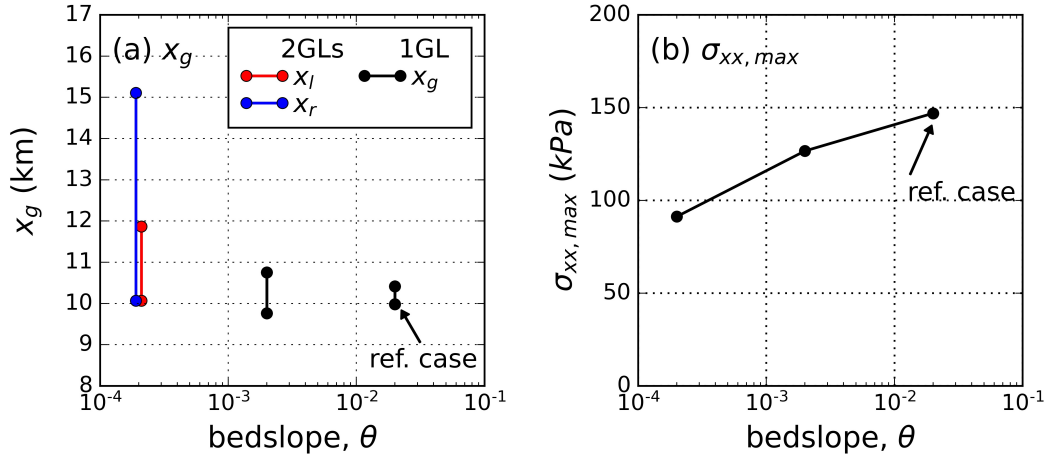
Figure 8. (a) The range of the GL position in one tidal period as a function of bedslope angle $\theta = 2 \times 10^{-4}, 2 \times 10^{-3}, 2 \times 10^{-2}$. When $\theta = 2 \times 10^{-4}$, there are two GLs. The left and right GLs are denoted x_l and x_r, respectively. The other two cases give single GL x_g shown by the black line. **(b)** Maximum tidal stress $\sigma_{xx,max}$ versus θ.

surface velocity near the grounding line deviates from the observed value u_0, but maintains the same order of magnitude. The GL migration is shown in Fig. 8a. Different from the single GL shown above, the low-bedslope regime $\theta = 2 \times 10^{-4}$ is

355 characterized by double GLs at low tides. Between the left GL at x_l and the right GL at x_r, the ice sheet is lifted due to a water layer trapped underneath (Stubblefield et al., 2021), forming a "low-tide grounding zone." For the other two cases, only a single GL is found, with the range of the GL decreasing with increasing θ. Moreover, the maximum tidal stress monotonically increases with increasing θ (Fig. 8b). For a specific grounding line, the local basal topography and characteristic bedslope angle can be constrained by observations (Fretwell et al., 2013). Thus, the uncertainties of the modelled tidal GL migration and stress

360 mainly come from the rheological model.

Our sensitivity analysis is based on the assumption of a fixed basal friction parameter. To estimate the tidal stress at a specific geographical location, the tidal response should be modelled with a basal sliding law that matches observed surface velocities. Then, a drainage criterion can be set up to estimate the local tidal contribution to supraglacial lake drainage.

4.3 Limitations

365 Deviations between the model-based criterion and data regression indicate that we may be underestimating the tidal contribution to hydrofracturing (Fig. 6). Currently, we assume a stress-free top surface and constant ice properties in the viscoelastic flow model. However, the supraglacial lake can induce additional stress in the surrounding ice, particularly on floating portions of the grounding zone that allow downward flexure (MacAyeal and Sergienko, 2013). Meanwhile, the existence of ice fatigue due to stress oscillations can weaken ice strength and promote hydrofracturing (Borstad et al., 2012; Lhermitte et al., 2020). A

370 better approach may be to directly consider the supraglacial lake and the ice damage within the 2-D viscoelastic model.

Another limitation arises from our assumption of hydrostatic water pressure on the ice–ocean interface. The pressure gradient induced by tidally modulated subglacial water flow can cause elastic flexure in ice sheets close to the grounding line (Warburton et al., 2020). Furthermore, ocean tides can change the effective pressure at the bed and lubricate the ice–bedrock interface, leading to a variation of basal friction that is not accounted for by the sliding law in our model (Gudmundsson, 2011). Thus, it is important to incorporate the subglacial hydrology in simulating the tidal response of a marine ice sheet.

Limitations also come from the data availability. Relative to the tidal period and lake-drainage period, the lower temporal resolution of the remotely sensed observations might obscure the true lake depth and tidal amplitude at the time of drainage. Field measurements and satellite images of supraglacial lake drainage with a higher observational frequency could improve our understanding of tidally-induced drainage.

5 Conclusions

Our study of tidally-induced stress and hydrofracture propagation in a viscoelastic marine ice sheet grounding zone supports the hypothesis on tidally-induced supraglacial lake drainage proposed by Trusel et al. (2022). We further propose a model-based criterion for lake drainage that indicates how ocean tides and lake depth together determine whether supraglacial lakes will drain via hydrofracture at the Amery Ice Shelf GL. Importantly, the criterion indicates that grounding-zone lakes tend to remain at a smaller volume than a critical value controlled by tidal flexure. For similar, tidally-modulated marine ice sheets, our results suggest that ocean tides can generate significant stress near the grounding line, potentially increasing the vulnerability of ice sheets to hydrofracturing in grounding zones where lakes form. Our work serves as an initial attempt to analyse the tidal effect on hydrofracturing, which helps to better explain the role of ocean tides in driving supraglacial lake drainage, calving events, and grounding line migration in marine ice sheets.

Code and data availability. The code and data used for the reference case in Sect. 3 is available at the online GitHub repository https://github.com/HwenZhang/TidalHydroFrac.git. It can be modified to reproduce the results presented in Sect. 4 using the values of parameters provided in the text.

Appendix A: Convergence test

The convergence test shows the results are mesh-independent. Here, we apply the same flow law parameter values with $A_0 = 3.2 \times 10^{-24}\text{Pa}^{-3}\,\text{s}^{-1}$ and $n = 3$ as (Stubblefield et al., 2021). Considering a marine ice sheet with bedslope $\theta = 10^{-3}$ and friction coefficient $C = 7 \times 10^5$, we use the fine-grid solution $x_{g,e}$, σ_e ($\Delta x = 6.25$m) as the exact solution. All other parameters are kept same as the reference case. Here $x_{g,e}$ denotes the time series of the exact GL position, and σ_e denotes the time series of the exact maximum tensile stress on the ice-sheet surface within the lake region $|x - \bar{x}_g| \leq 0.5$ km. As Δx decreases, the GL position x_g and maximum tensile stress $\sigma_{xx,max}$ linearly converge to the fine-grid solution (Fig. A1).
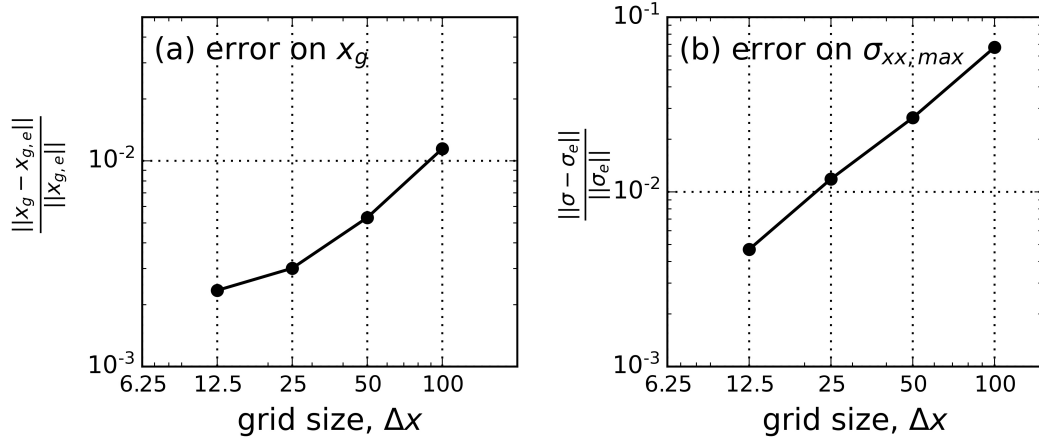
Figure A1. Convergence of (a) GL position and (b) maximum tensile stress $\sigma_{xx,max}$ with decreasing element size Δx (12.5 m, 25 m, 50 m, 100 m). For simplicity, we denote $\sigma_{xx,max}$ by σ without causing any confusion. Here $x_{g,e}$ and σ_e denote the exact solution to the GL position and maximum tensile stress $\sigma_{xx,max}$, respectively. $||\cdot||$ is the L^2 norm.

Appendix B: Simulation with real bed topography

In Fig. B1 we present model results using the real bed topography shown in Fig. 1e (Morlighem et al., 2017, 2020). For comparison, the physical properties of ice and basal slipperiness C are kept the same as in idealised models with a linear bed. The jagged variation of $x_g(t)$ is a consequence of the use of coarse grids near the grounding line for convergence. While the tidal stress and grounding-zone width are modified by bed undulation, the results have an equivalent order of magnitude to the case with linear bed topography, indicating the importance of tidal stress regardless of bed roughness. Therefore, we use the linear bed topography in the model.

Appendix C: Weight function method

In this appendix, we introduce the weight function method to calculate the stress intensity factor K_I of a vertical surface hydrofracture of length d_l. Fig. C1 shows the schematic of the fracture and weight function method. The stress contributing to hydrofracturing is denoted as $\sigma_f = \sigma_{xx} + p_w$, which combines the net stress σ_{xx} from the model with the internal water pressure $p_w(z)$ within the fracture. Assuming $p_w(z)$ is hydrostatic, it is given by $p_w(z) = \rho_w g (d_l + d_w)$, where d_w is the water depth of the lake. Here the ice stress $\sigma_{xx}(z)$ represents the vertical stress distribution at the location where the maximum surface horizontal tensile stress $\sigma_{xx,max}$ is observed.

The factor $K_I(d_l)$ is then computed by integrating σ_f along the fracture using a weight function G_1 (Tada et al., 2000),

$$K_I(d_l) = \int\limits_{h_0-d_l}^{h_0} \sigma_f(z) G_1\left(\frac{d_l}{H}, \frac{h_0-z}{d_l}\right) dz, \tag{C1}$$
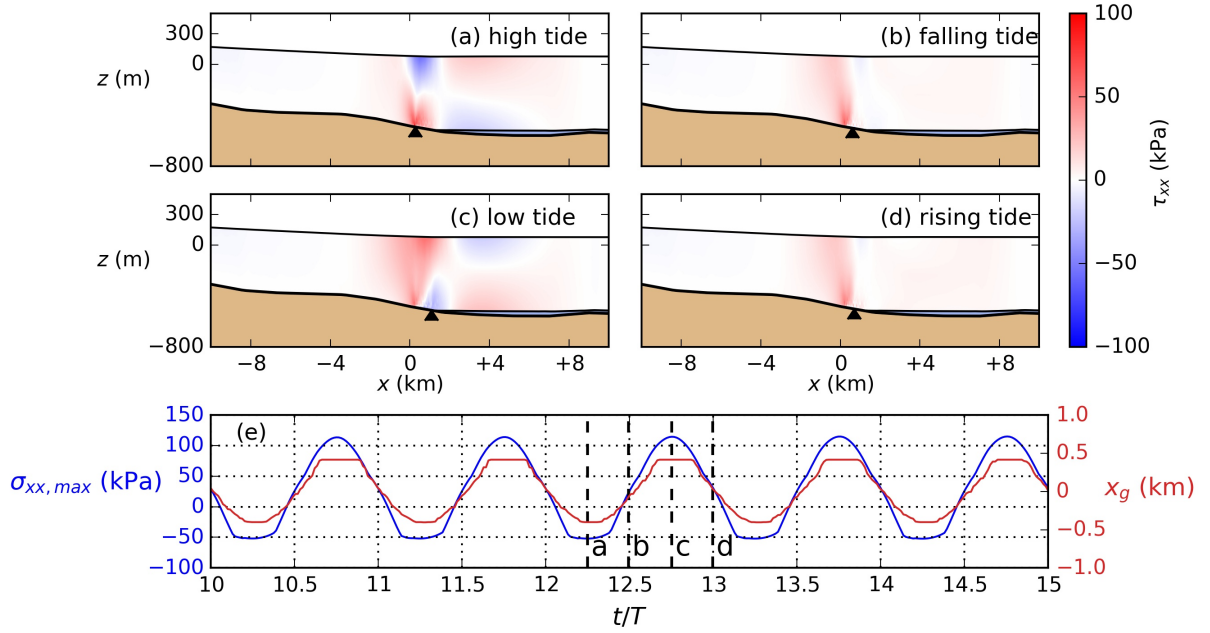
Figure B1. Tidal response of the Amery Ice Shelf with real bed topography. **(a)–(d)** Deviatoric tensile stress τ_{xx} in one tidal period. **(e)** The maximum tensile stress $\sigma_{xx,max}$ (blue) on the top boundary within the lake region ($\bar{x}_g - 0.5$ km $\leq x \leq \bar{x}_g + 0.5$ km) and the GL position x_g (red) versus time (scaled by the tidal period T) with positive values representing downstream migration. Vertical dashed lines show the time of panels (a)-(d).

where $h_0 = \Gamma_h - d_b$ is the lake basin elevation, d_l/H is the fracture length scaled by local ice thickness H and $(h_0 - z)/d_l$ denotes the depth below the lake basin, scaled by d_l. The weight function G_1 accounts for the surface hydrofracturing within an ice sheet located on a rigid bedrock (Jimenez and Duddu, 2018),

$$G_1(\lambda, \gamma) = \frac{2}{\sqrt{2H}} \sqrt{\frac{\tan\left(\frac{\pi\lambda}{2}\right)}{1 - (\cos\frac{\pi\lambda}{2}/\cos\frac{\pi\lambda\gamma}{2})^2}} \left\{ 1 + 0.3\sqrt{1 - \gamma^{5/4}} \left[0.5\left(1 - \sin\frac{\pi\lambda}{2}\right)\left(2 + \sin\frac{\pi\lambda}{2}\right) \right] \right\}. \tag{C2}$$

420   *Author contributions.* **Hanwen Zhang**: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data Curation, Writing - Original Draft, Writing - Review & Editing, Visualization. **Richard Katz**: Conceptualization, Software, Resources, Writing - Review & Editing, Supervision, Project administration, Funding acquisition. **Laura Stevens**: Conceptualization, Resources, Writing - Review & Editing, Supervision, Project administration.

Competing interests. The authors declare that they have no conflict of interest.
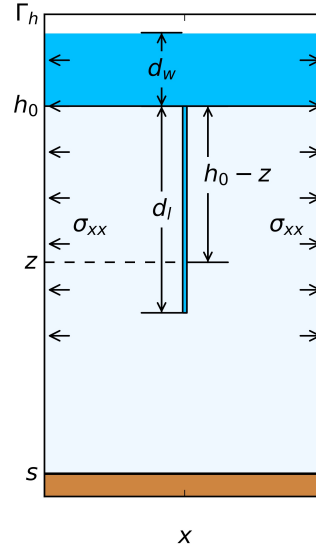
Figure C1. Schematic of the fracture geometry used in the weight function method.

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