EGUSphere-2024-665 – Response to referee 2

September 18, 2024

We thank the reviewer for their constructive and insightful comments that have helped us improve the manuscript. Below we provide a detailed discussion of the comments and proposed changes. We use blue colour to indicate **comments**; our **replies** are in black.

1 Question on lateral shear

It shows that the flow of the outlet glacier is strongly affected by the lateral shear. The size of the arrows depicting the ice velocity are much lower outside of the centreline. My hand-drawn thin red line meant to illustrate how the along the flow ice-velocity component changes across the outlet glacier. This is a typical velocity profile of ice flow strongly affected by the lateral shear caused by the presence of the lateral confinement (e.g. Raymond, 1996). Its effects cannot be ignored. Consequently, they need to be accounted for either by having a three-dimensional model that includes the second horizontal dimension transverse to the ice flow and imposing the relevant conditions on the lateral boundaries, or by parameterizing them in the momentum balance eqn.(3). These effects of the lateral shear will substantially alter the model results.

Figure 1 (in this response) shows the velocity and strain rate within a 40 km $\times$ 20 km region centred at the lake, which is a zoomed-in version of Fig. 1 in our manuscript, using the modified rheological parameters (discussed below) of Table 1. We agree with the referee that the ice flow can be subject to lateral shear. However, the background ice-flow rate in this area is small, ~ 20 m/yr as shown in Figure 1(f). The transverse shear stress $|\tau_t| < 50$ kPa and along-flow extension $|\tau_p| < 100$ kPa in this region are smaller than the modelled tidal variations in Figure 2. That is why we have focused on tidally induced, along-flow extensional stress and have neglected the transverse shear when modeling tidal flexure. However, we acknowledge that this simplification could introduce error into the estimated background tensile stress at the grounding line, which in turn affects the fracturing criterion. We will discuss the limitation in our manuscript, and highlight this as an important direction for future research.

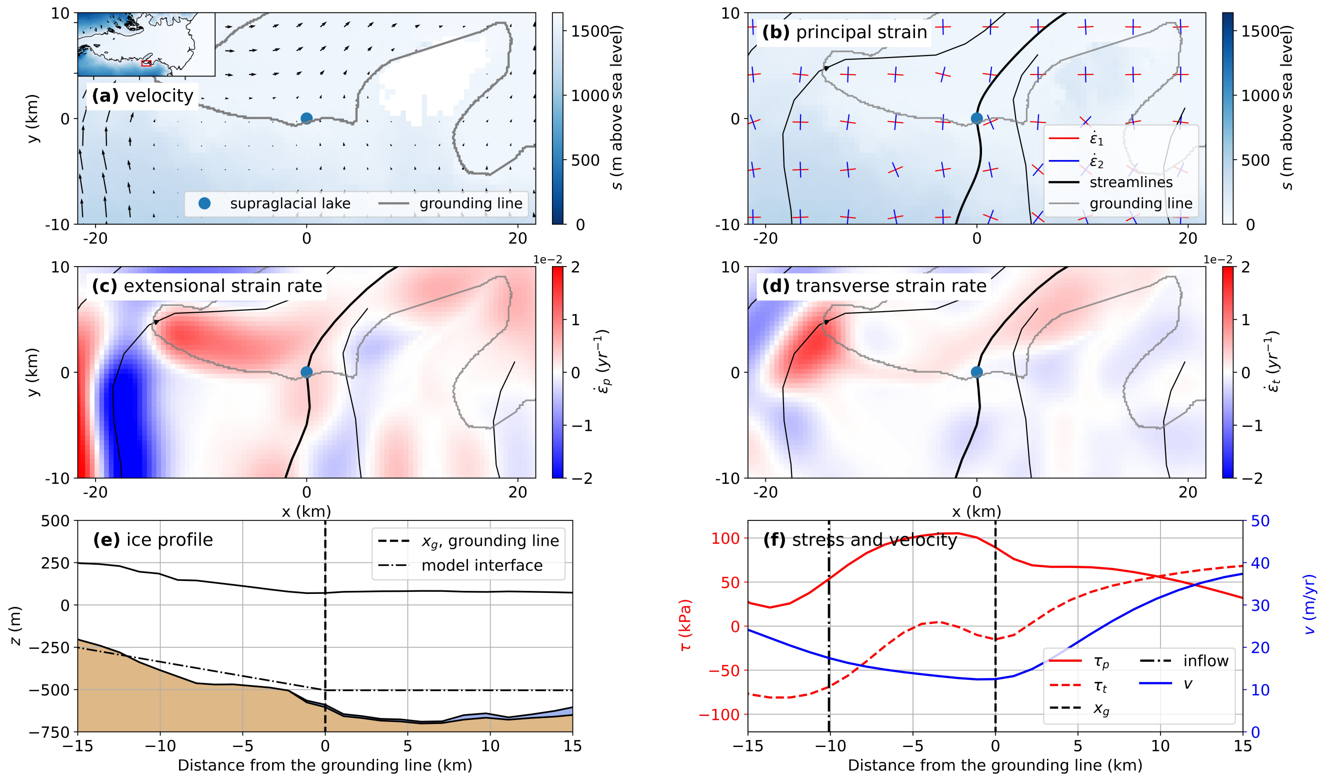

Figure 1: Ice surface velocity, strain rate and stress in the 40 km × 20 km region centred at the supraglacial lake, using the modified rheological parameters. **(a)** Velocity field near the grounding line (grey), where the supraglacial lake is denoted with the blue dot. The color represents the ice-sheet surface elevation above sea level. The map inset on the top left corner shows the full Amery Ice Shelf topography, with the plotted region outlined with a red box; **(b)** principal strain rate and streamlines. The streamline that crosses the lake is marked with the bold line; **(c)** along-flow strain rate; **(d)** transverse strain rate; **(e)** local ice-sheet geometry and bed topography; **(f)** For the streamline that crosses the lake, along-flow deviatoric extension σ_p (solid red line) and shear stress σ_t (dashed red line), and speed v (blue). Note that $x = x_g = 0$ is the position of the supraglacial lake as well as the grounding line.

2 Question on ice rheology

Secondly, the quoted magnitude of the observed velocity imposed at the inflow boundary is low, 9 m/yr; so are the magnitudes of velocity shown in fig. 1f (a minor comment: it is unclear whether this velocity profile is computed or observed). Using parameters listed in table 1 and the Shallow Ice Approximation one could estimate the ice surface velocity resulted from the internal deformation only, assuming no-slip at the ice bed interface. That value is 20 m/yr, which is larger than the observed surface velocity by a factor of two. This suggests that (a) either the chosen parameters are off (specifically the ice stiffness parameter A0, which I will come back to) or (b) the ice flow is dominated, or strongly influenced, by the vertical shear, and the focus on the longitudinal stress τ_{xx} is unwarranted, or both.

Thank you for raising this issue with the background velocity. The 9 m/yr inflow velocity is the observed velocity averaged across the entire lake region from the MEaSUREs InSAR-Based Antarctica Ice Velocity Map [Rignot et al., 2016, Trusel et al., 2022]. We will make this clear by modifying the first sentence of section 2.1 in our manuscript.

On the flow line past the lake centre, the observed velocity is about 12 m/yr at the lake centre, and is about 17 m/yr at 10 km upstream of the lake centre. The estimate from Shallow Ice Approximation (20 m/yr) would better match the upstream surface velocity, instead of the lake region close to the grounding line. To improve the modelled background flow regime near the lake, instead of using the 9 m/yr velocity as the inflow velocity, we will use 17 m/yr at the inflow boundary in our model. We discuss the issue with ice viscosity in detail below.

Thirdly, the chosen value of A0 is very high. The ice-stiffness parameter is a function of the temperature of ice through its column. The chosen value would correspond to ice temperatures of the range from -5°C to -7°C, which is very warm. Although summer temperatures can exceed freezing point from time-to-time, as indicated by the supraglacial lakes, the annual mean surface temperature is around -20°C (e.g., Kittel et al., 2021). With ice flow primarily driven by the internal deformation, the ice temperature through the most of the ice column is not substantially warmer; it is only in the fairly narrow band near the bed it is warmer due to the geothermal heat flux. The very high chosen value of the ice stiffness parameter leads to a very low ice viscosity, of the order of 10^{13} Pa·s, which is at least an order, or more likely two orders of magnitude lower than the typical values of ice viscosity.

This brings me to the second problem with the study — the choice of the ice rheology. The authors have estimate it 9 hrs (the penultimate line on page 2) and 40 hrs (the penultimate line of section 2.3 page 6). For more realistic values of ice viscosity it is of the order 5–15 days, which is substantially longer than the period of diurnal tides that cause the ice flexure. This fairly unambiguously indicates that ice responds to diurnal tides as elastic medium. Two questions that immediately comes to mind — is it worth the effort the authors have gone through and complexity of the viscoelastic rheology? Can't one simulate it with much simpler elastic rheology?

Physical property	Notation	Value
Glen's Law exponent	n	3
Viscosity coefficient	A_0	1.2×10^{-25} Pa^{-n} s^{-1}
Shear modulus	μ	0.30×10^9 Pa
Viscosity regularisation parameter	δ_ν	10^{-18} s^{-2}
Upper bound of the viscosity	$2^{-(n+1)/2n} A_0^{-1/n} \delta_\nu^{-(n-1)/2n}$	1.27×10^{14} Pa s
Maxwell time	τ	≤ 5 d

Table 1: Rheological parameters used in numerical model and their reference values.

We apologize for the typo related to the Maxwell time and thank the referee for pointing out this issue, which has helped us improve the model. The Maxwell time of ice in our model should be less than 40 hr instead of 9 hr. We recognise that the viscosity and Maxwell time are lower than their typical values. To address this, we will select the flow-law parameter $A_0 = 1.2 \times 10^{-25}$ Pa^{-3} s^{-1} with $n = 3$ at the temperature $T = -20$°C [Cuffey and Paterson, 2010] in Glen's flow law, as shown in Table 1. The modelled in-situ temperature at the lake region is between -10 °C and -20 °C [Wang et al., 2022]. With these adjustments, the new Maxwell time is less than 5 d. In Figure 2 we provide results from a case with the new set of parameters. The tidal stress and grounding-line migration remain similar to the previous reference case, with only an increase in magnitude.

In section 4.1, we discussed the effect of rheology on tidal stress by plotting the tidal stress $\sigma_{xx,\max}$ and grounding-zone width Δx_g against the shear modulus μ. These results are included here as Figure 3. The grounding-zone width and tidal stress decrease with ice becoming more elastic ($\mu \to 0$), indicating the sensitivity of the modelled tidal stress to ice rheology. Therefore, we considered the viscoelastic rheology to obtain an accurate estimate of the tidal stress.

Applying the new rheological parameter A_0 to the observations, the modified in-situ tensile stress near the lake is approximately 90 kPa (Figure 1), which is larger than the model's estimate (i.e., the time-average value of $\sigma_{xx,max} \sim 60$ kPa in Figure 2(e)). The $\sim$ 30-kPa discrepancy in the background stresses could arise from lateral stress or non-uniform ice properties. Although this discrepancy magnitude is smaller than the tidal stress, it could still introduce error into the model-based criterion. We will discuss these limitations in the manuscript to acknowledge potential sources of uncertainty in our model.

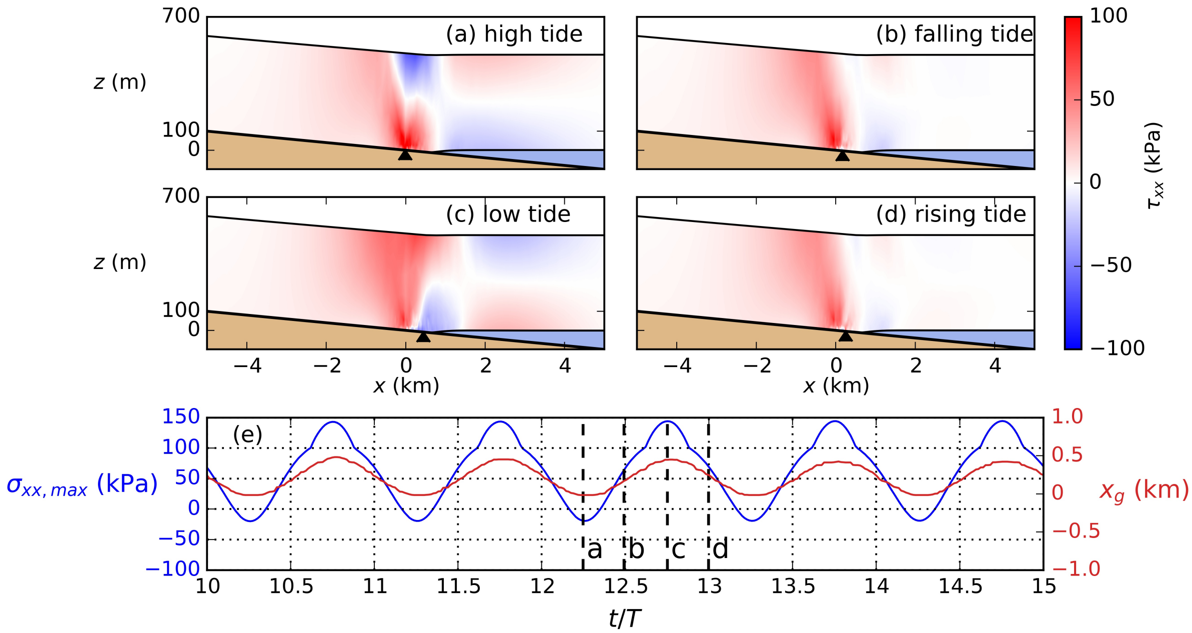

Figure 2: Tidal response of a marine ice sheet at different tidal phases, using the new set of parameters with $A_0 = 1.2 \times 10^{-25}$ Pa^{-3} s^{-1}. **(a)–(d)** Deviatoric tensile stress τ_{xx} in one tidal period. **(e)** The maximum tensile stress $\sigma_{xx,max}$ (blue) on the top boundary within the lake region ($\bar{x_g} - 0.5$ km $\leq x \leq \bar{x_g} + 0.5$ km) and the GL position x_g (red) versus time (scaled by the tidal period T) with positive values representing downstream migration. Vertical dashed lines show the time of panels (a)-(d).

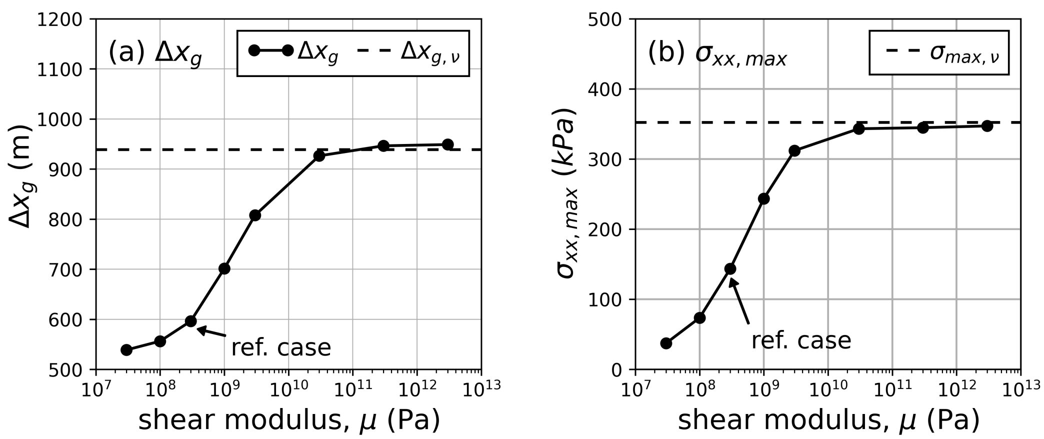

Figure 3: **(a)** The grounding-zone width Δx_g (solid line), defined as $\Delta x_g = \max\{x_r\} - \min\{x_l\}$ as a function of shear modulus $\mu = 3 \times 10^7$ to 3×10^{12} Pa, with x_l and x_r denote the left and right GL, respectively. The dashed line shows $\Delta x_{g,\nu}$, the grounding-zone width in the viscous limit ($\mu \to \infty$). **(b)** Maximum tensile stress $\sigma_{xx,max}$ versus μ.

References

Kurt M Cuffey and William Stanley Bryce Paterson. *The physics of glaciers*. Academic Press, 2010.

E Rignot, J Mouginot, and B. Scheuchl. Measures antarctic grounding line from differential satellite radar interferometry, version 2, 2016. URL https://nsidc.org/data/NSIDC-0498/versions/2.

Luke D Trusel, Zhuolai Pan, and Mahsa Moussavi. Repeated tidally induced hydrofracture of a supraglacial lake at the amery ice shelf grounding zone. *Geophysical Research Letters*, 49(7): e2021GL095661, 2022. doi: 10.1029/2021GL095661.

Yu Wang, Chen Zhao, Rupert Gladstone, Ben Galton-Fenzi, and Roland Warner. Thermal structure of the amery ice shelf from borehole observations and simulations. *The Cryosphere*, 16(4): 1221–1245, 2022.