EGUSphere-2024-665 – Response to referee 1

September 18, 2024

1 General comments from the referee

The manuscript “Viscoelastic mechanics of tidally induced lake drainage in the Amery grounding zone” by Zhang et al. explains the physics that drive a series of supraglacial lake drainage events in Antarctica with numerical models. Remote sensing data suggest that the extensional stress regime of the background ice flow is not enough to trigger the lake drainage events. They conduct a series of targeted numerical experiments to show that tidal flexure provides the necessary extensional stresses to drive hydrofracturing, depending on the depth of the supraglacial lake. This essentially confirms the hypothesis in the observational study [Trusel et al., 2022], that detailed these drainage events. While I have some comments primarily related to clarification and discussion, my judgment is that this would be an excellent contribution to TC.

We thank the reviewer for their constructive and insightful comments that have helped us improve the manuscript. Below we provide a detailed discussion of the comments and proposed changes. We use blue colour to indicate **comments**; our **replies** are in black.

2 Specific comments from the referee

In the last line of the abstract and conclusions you mention calving. While I understand that supraglacial lakes might play a role in ice-shelf breakup [Banwell et al., 2019], I was not exactly sure how the results in this study related to the calving front. Clearly there are similar physics because you are modelling fracture and flexure, but additional clarification would be helpful if you want to include this statement.

It was meant that the developed hydrofractures, if they remain open, can be advected downstream and destabilise the ice shelf by causing rifting. To avoid the confusion, we will remove the word "calving" and replace "rifting" with a general description "crevassing".

Introduction: Maxwell time of "approximately 9 hours in our estimation", this needs some more context for how you calculated this or a reference.

We will revise the paragraphs related to the Maxwell constitutive law. A detailed context for the revision can be found in the response to Referee 2. Below we provide details on how we choose the new set of rheological parameters and calculate the tidal stress.

Regularized flow law: It is good that you explicitly discuss the regularization because this is often not the case. I think it might be worth noting, with proper references, that the ice viscosity can vary over several orders of magnitude, and that the upper bound you have set seems to be on the lower end of the spectrum? Also suggest adding a statement here that you later test sensitivity to the Maxwell time.

We apologize for the typo related to the Maxwell time. The Maxwell time of ice in our model should be less than 40 hr instead of 9 hr. We recognise that the viscosity and Maxwell time are lower than their typical values. To address this, we will select the flow-law parameter $A_0 = 1.2 \times 10^{-25}$ Pa^{-3} s^{-1} with $n = 3$ at $T = -20$ °C in Glen's flow law [Cuffey and Paterson, 2010], as shown in Table 1. The modelled in-situ temperature at the lake region is between -10 °C and -20 °C [Wang et al., 2022], indicating that the real viscosity might be smaller. With these adjustments, the new Maxwell time is less than 5 d. In Figure 1 we provide results from a case with the new set of parameters. The tidal stress and grounding-line migration remain similar to the previous reference case, with only a slight change in magnitude.

Thanks for the constructive comment that helps us improve the model. A more detailed context about the rheology can be found in our response to Referee 2.

Physical property	Notation	Value
Glen's Law exponent	n	3
Viscosity coefficient	A_0	3.5×10^{-25} Pa^{-n} s^{-1}
Shear modulus	μ	0.30×10^{9} Pa
Viscosity regularisation parameter	δ_ν	10^{-18} s^{-2}
Upper bound of the viscosity	$2^{-(n+1)/2n} A_0^{-1/n} \delta_\nu^{-(n-1)/2n}$	1.2×10^{14} Pa s
Maxwell time	τ	≤ 5 d

Table 1: Rheological parameters used in numerical model and their reference values.

Section 2.1, last paragraph, "In B" should be "In Appendix B"?

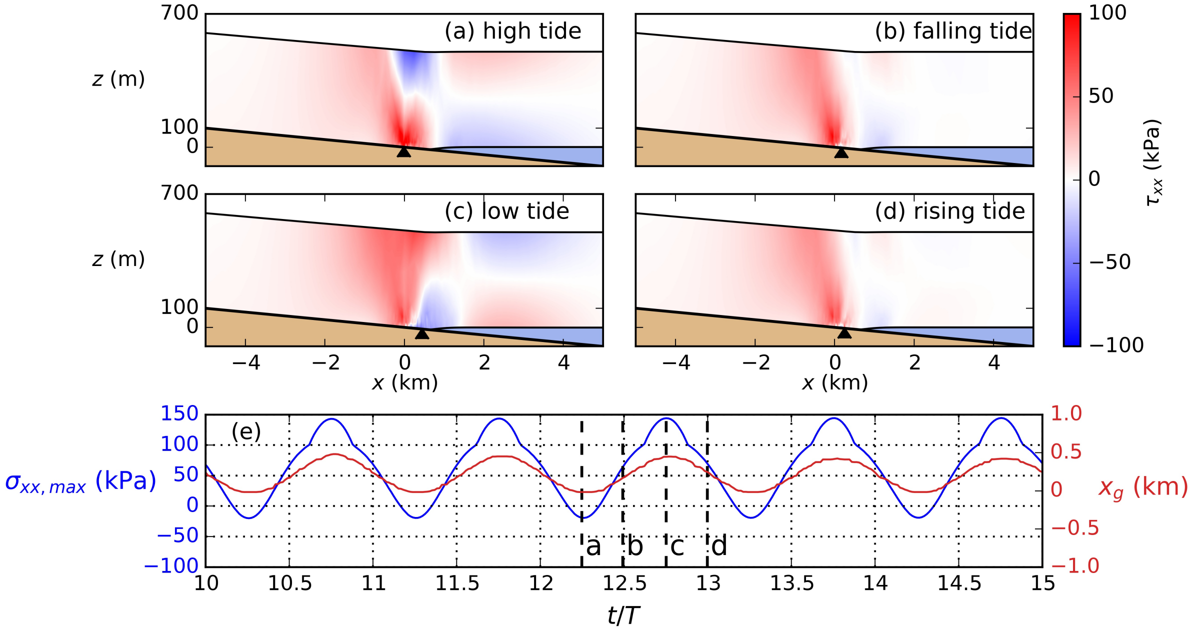

Figure 1: Tidal response of a marine ice sheet at different tidal phases, using the new set of parameters: $A_0 = 1.2 \times 10^{-25}$ Pa^{-3} s^{-1}. **(a)–(d)** Deviatoric tensile stress τ_{xx} in one tidal period. **(e)** The maximum tensile stress $\sigma_{xx,max}$ (blue) on the top boundary within the lake region ($\bar{x_g} - 0.5$ km $\leq x \leq \bar{x_g} + 0.5$ km) and the GL position x_g (red) versus time (scaled by the tidal period T) with positive values representing downstream migration. Vertical dashed lines show the time of panels (a)-(d).

Yes, we will change it.

When you introduce the upper-convected derivative, you should add a reference and probably provide some motivation (i.e. objectivity). The following review is excellent: Snoeijer J. H., Pandey A., Herrada M. A. and Eggers J. (2020). The relationship between viscoelasticity and elasticity. Proc. R. Soc. A. 47620200419

We will add the suggested reference and introduce the motivation to use the UCM model.

Comment somewhere on how the modelled grounding zone widths compare to width estimated from interferometry (Chen et al., 2023)?

Thanks for the suggestion for this model-data comparison. Chen et al. [2023] discussed the grounding zones of the feeding glaciers of the Amery Ice Shelf. Although the location is different from the lake position we have discussed, Chen et al. [2023] reported that near the grounding line with prograde bedslopes, the observed grounding-zone width is one order of magnitude larger than predicted from hydrostatic equilibrium that gives [Tsai and Gudmundsson, 2015]

$$\Delta x_g = \Delta h \left[\beta + \frac{\rho_i}{\rho_w} (\alpha - \beta) \right]^{-1}, \tag{1}$$

where Δh is the tidal range, α and β are surface slopes and bed slopes along the flowline. In our reference case, the calculated grounding zone width $\Delta x_g \approx 500$ m is about 5 times the estimation from hydrostatic equilibrium (100 m). In our calculated width of the grounding zone, we neglect the effect of subglacial drainage system by assuming hydrostatic water pressure and rigid bedrock on basal ice.

After equation (18), "...variational formulation weakly converges...". To my knowledge, this is less obvious for the UCM model because it cannot be cast directly as a minimization problem like those dealt with in. Nevertheless, approximating the contact conditions by $\sigma_e = max\left(0, \sigma_e + \frac{1}{\varepsilon} u_n\right) \approx max\left(0, \frac{1}{\varepsilon} u_n\right)$, for small ε (where $\sigma_e = \sigma_n - p_w$) still makes sense for UCM and motivates the use of a penalty term, as long as there aren't singularities in σ_e.

Thanks for raising this issue. We will state the difference between the viscous model and the UCM model, as well as the approximation used in the contact conditions.

Last paragraph of Section 2.5, "In A" should be "In Appendix A"?

Yes, we will change it.

Section 2.6 seems kind of random at first glance and needs more context... e.g., say what are you going to do later with the lake depths? Also, it might be better to place this after Section 2.1 rather than after the modelling material.

We will provide more context in Section 2.6 that gives our motivations for estimating lake depths in relation to the data-model comparison that follows. We will move this section up to follow Section 2.1.

Table 1: Units on viscosity and friction regularization parameters?

Units will be added.

Section 3.1: Specify that $\sigma - A$ relationship is for $\sigma_{xx,max}$.

When introducing the "$\sigma - A$ relationship", we will add a new sentence "Note that the 'σ' here refers to the maximum tensile stress $\sigma_{xx,max}$ calculated above."

Figures 5 and 6 needs to label panels (a) and (b).

We will add panel labels.

Figure 5: clarify why the dashed lines go into the positive region? I thought they were compressive at high tide so not contributing to fracture, and I became confused.

Here the stress intensity factor depends on the vertical distribution of the net stress σ_{xx} and the length of the fracture. On high tides, the bending stress is compressive on the top and tensile on the bottom. Meanwhile, the water pressure increases more quickly than the ice overburden stress with depth. If the fracture tip is located deeper in the ice, the propagation can be promoted by the water pressure, as well as the tensile bending stress in the lower part of the ice sheet.

Section 3.2: Not many details are provided about the weight function method. I presume that you are doing something with $\sigma_{xx,max}$, but some more context would be helpful.

We will add an appendix showing how we use the weight function method to calculate the stress intensity factor.

I don't think you say what is the value of fracture toughness K_C?

Thanks for pointing that out. We assume that the ice toughness $K_C = 100$ kPa m$^{1/2}$ [Rist et al., 1996], which is an estimate widely used in ice-fracture problems. We will add it to the table.

Section 3.3: All of this is in terms of the stress intensity factor, but I was wondering what are the stress thresholds associated with fracture propagation so that you can relate these to the background extensional stress ($<$ 40 kPa), which you say is not enough to cause fracture propagation on its own? Something to verify this claim would be good.

Thanks for raising this issue about demonstrating the LEFM model. We will rewrite the criterion in terms of the stress threshold and compare it with the observed background stress. In Fig. 6 of the manuscript, we have investigated the fracture propagation with the modelled background extensional stress when $A = 0$ m. The results suggest that with only the modelled background stress, the initial fracture will never propagate under any lake depth.

Section 3.3: "Supraglacial lakes would not be able to form under such large tidal stress." Does this limit correspond to the zero water depth in Figure 6 (state if so)? This is also an interesting point that you could revisit in the discussion.

Thanks for the constructive comment. Yes, this corresponds to the zero water depth where the criteria intercept the horizontal axis. In this case, the tidal stress is sufficiently large to induce crevassing without lake water supply, forming surface crevasses [Hulbe et al., 2016]. We will state this point explicitly and add a paragraph about this to the discussion.

Section 4: You are using present tense “We use” / “We construct” but you have already done these things at this point in the paper so maybe “We used” or “We have used”?

We will change it.

Section 4.1: Change “Ice Maxwell time” to “The Maxwell time of ice”?

We will change it. Thank you again for your helpful review.

References

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