Authors' response to reviewers report

Response to Reviewer 1

Clayton et al. derived analytical equations and conducted LEFM analysis to study the influence of firn-layer material properties (depth-varying density and modulus) on surface crevasse propagation in glacier and ice shelves. They found that the firn layer has a stabilizing effect on grounded glaciers (free slip boundary condition), whereas a destabilizing effect on ice shelves, with regard to fracturing and calving. The study has important implications for assessing the stability of ice sheets or ice shelves.

We thank the reviewer for their comments and positive assessment of the manuscript.

However, there are two major limitations in the assumptions of the models: i) Poisson ratio is assumed to be depth-invariant; ii) firn is assumed to be impermeable when evaluating the depth of meltwater-driven hydrofracture, neglecting the fact that meltwater will penetrate the porous firn layer instead of fracturing it. I suggest the authors reconsider the model assumption, or at least highlight the limitations, before it can receive further consideration.

Please see the responses below addressing these individual points. Paragraphs that have been added to the manuscript as a result of reviewer comments are indicated with italics in this response. Additionally, a new section and additional appendix have been added to the manuscript following reviewer feedback to showcase that the model remains appropriate for viscous/incompressible ice rheologies, and we have attached these new sections as appendix to this response.

Comment # 1

1) Why do the authors neglect depth-variations in Poisson ratio? Shouldn't Poisson ratio and Young's modulus both strongly depend on the density? Will a depth-varying Poisson ratio (which is more realistic) significantly affect the results? Below attach some references on the depth variations in Poisson ratio [1, 2, 3]. One possible way to represent the depth varying mechanical properties could be developing empirical relationships between Poisson ratio/Young's modulus, and the firn density.

[1] Schlegel, R., Diez, A., Löwe, H., Mayer, C., Lambrecht, A., Freitag, J., ... & Eisen, O. (2019). Comparison of elastic moduli from seismic diving-wave and ice-core microstructure analysis in Antarctic polar firn. Annals of Glaciology, 60(79), 220-230.

[2] Smith, J. L. (1965). The elastic constants, strength and density of Greenland snow as determined from measurements of sonic wave velocity (Vol. 167). US Army Cold Regions Research & Engineering Laboratory.

[3] King, E. C., & Jarvis, E. P. (2007). Use of shear waves to measure Poisson's ratio in polar firn. Journal of Environmental and Engineering Geophysics, 12(1), 15-21.

The reviewer raises an interesting point regarding the use of a depth invariant Poisson ratio ν. In response to this, we conducted a study using the LEFM model for a dry surface crevasse subject to various ocean-water heights in a grounded glacier, considering a depth dependent Poisson's ratio and a homogeneous Poisson's ratio of $\nu = 0.35$ to determine its effect on crevasse propagation. As no analytic expressions could be derived for the depth-dependent Poisson ratio case, stress profiles used within the LEFM study were obtained through numerical finite element simulations. Results for the subsequent LEFM study are presented in Fig. 1. An exponential distribution with depth z is assumed - similar to that of the density and Young's modulus distributions - with the tuned constant $D = 32.5$ and a surface Poisson's ratio $\nu_\mathrm{f} = 0.07$ based on the data from Schlegel et al. (2019), giving the Poisson ratio as:

$$\nu(z) = \nu_\mathrm{i} - (\nu_\mathrm{i} - \nu_\mathrm{f})\exp(-(H - z)/D). \quad (1)$$

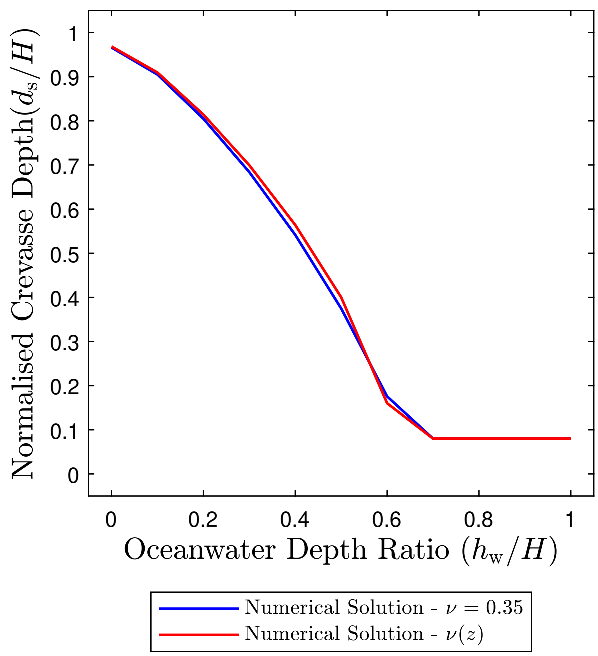

Figure 1: Normalised crevasse depth predictions versus oceanwater height ratio for a single isolated dry crevasse in a linear elastic ice sheet, considering homogeneous and depth-dependent Poisson Ratio.

Using this depth-dependent Poisson ratio, the largest difference in crevasse depth was observed for intermediate ocean-water levels, with an increase of 6% in crevasse depth with respect to the homogeneous case for an oceanwater height of $h_w = 0.5H$ (see Fig. 1 in this response/Figure E1 in the updated paper). This is in stark contrast with the findings for depth-dependent density and depth-dependent Young's modulus which predict a reduction in crevasse penetration depth. The effect of Poisson ratio is less influential compared to depth dependent density and Young's modulus, as depth dependent density resulted in a reduction of 20% of the crevasse depth and depth dependent Young's modulus resulted in a reduction of 45% of the crevasse depth.

The results discussing a depth-dependent Poisson ratio have been added to Appendix E. While the influence is comparatively minor, we conclude that indicating that small variations in Poisson ratio observed within ice/firn do not play a significant role, but it nevertheless an interesting finding. The full text added as appendix E reads:

"*For the crevasse propagation studies previously presented, a depth invariant Poisson ratio of* $\nu = 0.35$ *was assumed. However, it has been suggested that Poisson ratio also exhibits a linear dependency on ice density and therefore leads to a depth-dependent profile (Smith, 1965). Furthermore, Schlegel et al. (2019) and King and Jarvis (2007) provides a depth-dependent Poisson ratio profile based on seismic velocity measurements on ice cores. To study the effect of this depth-dependent Poisson ratio, a linear elastic fracture mechanics study is performed. We assume an exponential distribution of Poisson's ratio with depth, similar to the density and Young's modulus distributions.*

$$\nu(z) = \nu_i - (\nu_i - \nu_f)e^{-(H-z)/D} \tag{2}$$

where $\nu_f = 0.07$ *is the Poisson ratio of firn in the upper surface,* $\nu_i = 0.35$ *is the Poisson ratio of fully consolidated ice and* $D = 32.5$ *is the tuned constant. This profile approximates the observations from Schlegel et al. (2019), where we have scaled the length parameter* D *to match our density and Young's modulus profiles as this profile was obtained at a different location (with significantly different ice-sheet*

and firn thickness). As it is not possible to derive a fully analytic expression for the stress profiles with this depth-dependent Poisson ratio, the longitudinal stress profiles are obtained numerically through the finite element model. Once obtained, the stresses are used to drive the propagation of the surface crevasse in the linear elastic fracture mechanics study.
We consider a dry (air-filled) crevasse, with different values of oceanwater height h_w *and plot the normalised crevasse penetration depth versus oceanwater height* h_w *in Fig. 1. This figure shows that the effect of including variations in Poisson ratio have a more limited effect compared to density and Young's modulus variations. The largest percentage difference in crevasse depth was observed for intermediate ocean-water levels, with an increase of* 6% *in crevasse depth with respect to the homogeneous case when considering a depth dependent Poisson ratio, for an oceanwater height of* $h_w = 0.5H$. *This is in contrast to the inclusion of firn density and Young's modulus, which predict a reduction in stabilised crevasse depth for surface crevasses in grounded glaciers. The effect of a including a depth-dependent Poisson ratio is less influential compared to density and Young's modulus, as depth-dependent density resulted in a reduction of* 20% *of the crevasse depth and depth-dependent Young's modulus resulted in a reduction of* 45% *of the crevasse depth. We therefore conclude that the inclusion of variations in Poisson ratio does not play a significant role in crevasse propagation. "*

Comment # 2

2) The longitudinal stress was derived for compressible linear elasticity (Eqn.1 in the manuscript), why not viscous model? I think that a common approach, when looking at calving for example (e.g. Benn et al 2007, Ann [4]), is to calculate the background stresses from a viscous model (associated with long- term creep of the ice, and estimated perhaps from satellite-derived estimates of strain rate) instead of using an elastic model to calculate that background state. The authors might need some explanation justifying why they use linear elasticity to calculate the longitudinal stress.

[4] Benn, D. I., Cowton, T., Todd, J., & Luckman, A. (2017). Glacier calving in Greenland. Current Climate Change Reports, 3, 282-290.

The model presented is also capable of capturing this incompressible stress state by setting the Poisson ratio to $\nu \approx 0.5$. For example, for constant Young's modulus and Poisson's ratio case with $\nu \approx 0.5$, the derived analytical solution in Eq. 1 exactly matches with that of the Weertman (1957) solution [1]. As discussed in Lipovsky (2020) [2], "This initial volumetric contraction does not occur in real ice shelves because at timescales longer than the Maxwell time ice is well approximated as being incompressible." We have added a remark to the paper clarifying this:
"The Poisson ratio ν *used within our results represents ice as a linear-elastic compressible solid, which is a common assumption for rapidly propagating cracks. If the crevassing process occurs on a time-scale well below the Maxwell time-scale, ranging from hours to days depending on the strain-rate due to nonlinear viscous nature, the assumption of compressibility would be valid. Instead, if the crevassing process occurs slowly, over the span of weeks, the assumption of incompressibility would be valid; so a Poisson ratio of* $\nu = 0.5$ *will allow for the model derived here to be applicable over longer time-scales. "*

If we use a Poisson ratio of $\nu = 0.5$, the crevasse depths obtained for a dry crevasse subject to different values of oceanwater height h_w are presented in Fig. 2/Figure 9. in the updated paper. This can be compared directly to the results presented in the paper for $\nu = 0.35$ (included in Fig. 2/ Figure 9. in the updated paper as dashed lines). For surface crevasses in glaciers subject to low levels of oceanwater, the penetration depth is unaffected by firn density due to crevasses stabilising in fully consolidated strata. This is the case for both linear elastic and non-linear viscous rheologies. For intermediate oceanwater heights ($0.3H < h_\mathrm{w} < 0.6H$), crevasses propagate deeper when considering a non-linear viscous rheology, since stresses are more extensional. Thus, significant reductions in crevasse depths are only observed for oceanwater heights $h_\mathrm{w} > 0.6H$. The largest reductions in crevasse depth are observed at $h_\mathrm{w} > 0.8H$ for the non-linear viscous rheology, with a maximum percentage difference of 64% between depth dependent and homogeneous results. These findings complement the results found in our paper, and follow similar trends to the compressible stress state results. A new section has been added to the main text of the paper, including the results for crevasse propagation considering a non-linear rheology. This section provides the stress profiles resulting from our analytic expressions, compared with stress profiles from numerical finite element simulations using a visco-elastic rheology, and shows that setting $\nu = 0.5$ indeed obtains the incompressible/viscous stress state.

In addition, we also conduct linear elastic fracture mechanics studies for water-filled surface crevasses in ice shelves of height $H = 125$m and length $L = 5000$m, considering a non-linear viscous ice rheology. Similarly to the linear elastic compressible case, we consider surface crevasses at the horizontal position $x = 4750$m (250 m from the ice shelf terminus) and extract the numerical longitudinal stress profile from the finite element analysis. We plot the stabilised crevasse depth versus meltwater depth ratio for the non-linear viscous (NLV) rheology in Fig. 5 / Figure 10. of the updated paper, along with the results for linear elastic compressiblity (LE).

When comparing the stabilised crevasse depths close to the front, we note that the penetration depth is independent of ice rheology, which is in contrast to the grounded glacier. For the homogeneous density, minimal crevasse propagation is observed for meltwater depth ratios below $h_\mathrm{s}/d_\mathrm{s} < 0.6$, with full thickness propagation only occurring when fractures are close to saturation. The inclusion of the depth-dependent density results in deeper crevasse penetration depths, with minimal differences in penetration depth between the linear elastic and non-linear viscous rheology. This likely indicates that for crevasses close to the front, fracture is driven by the flotation height. For depth-dependent density, the reduction in flotation height leads to an increase in tensile stress in the upper surface, due to reductions in R_{xx} and increased bending stress. In addition, the lithostatic component of longitudinal stress is reduced, leading to deeper crevasse propagation when including firn density.

We also consider the propagation of an isolated surface crevasse located in the far field region of a floating ice shelf, with results presented in Fig. 4 / Figure 11. of the updated paper. As stated previously, for the linear elastic compressible rheology the stress state is fully compressive for both the homogeneous and the depth-dependent density case, thus no crevasse propagation is observed regardless of meltwater depth ratio. By contrast, when considering the non-linear viscous rheology of ice, surface crevasses may propagate in the far field region if there is sufficient meltwater pressure present. Large increases in crevasse penetration depth are observed for meltwater depth ratios greater than $h_\mathrm{s}/d_\mathrm{s} = 0.50$, with full thickness propagation being observed close to crevasse saturation at $h_\mathrm{s}/d_\mathrm{s} = 0.95$. Similar to crevasses near the front, the inclusion of depth dependent density results in increased crevasse penetration depths compared to the homogeneous density scenario. These results have been added as Section 5 in the paper.

[1] J. Weertman, 'Deformation of Floating Ice Shelves', Journal of Glaciology, vol. 3, no. 21, pp. 38–42, Jan. 1957, `https://doi.org/10.3189/S0022143000024710`.

[2] B. P. Lipovsky, 'Ice shelf rift propagation: stability, three-dimensional effects, and the role of marginal weakening', The Cryosphere, vol. 14, no. 5, pp. 1673–1683, May 2020, `https://doi.org/10.5194/tc-14-1673-2020`.

Comment # 3

3) Once the authors start to consider meltwater within the crevasse, it confuses me that the porous nature of firn is completely ignored. LEFM no longer holds for porous material and poromechanics [5] should be considered. Could the authors at least highlight the limitations of current results (Figure 5&7 in the main text)?

[5] Coussy, O. (2004). Poromechanics. John Wiley & Sons.

The reviewer raises an intriguing point regarding the porous nature of firn with regards to meltwater pressure. In this study we have assumed that meltwater pressure is restricted solely to the fractured region. For crevasses in colder ice this is a realistic assumption. During crevasse propagation, water will seep into the firn surrounding the crevasse and start freezing, forming a thin ice layer. This thin ice layer then prevents further water from leaking into the surrounding firn. A similar effect happens near the

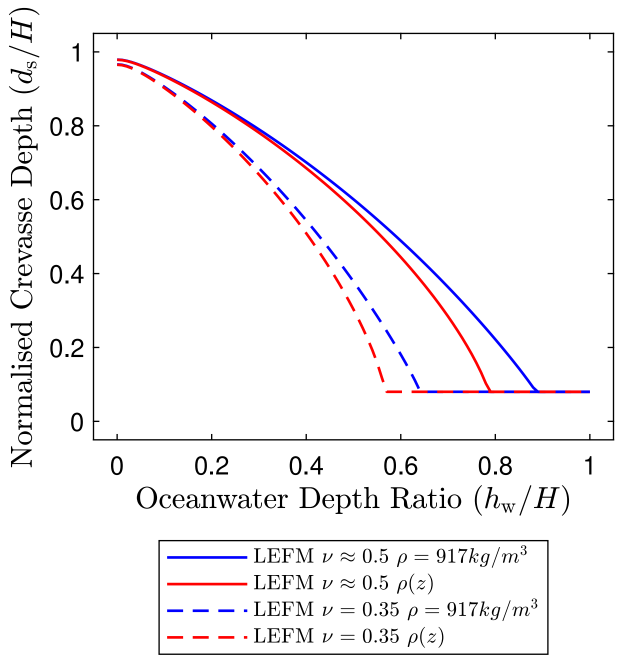

Figure 2: Normalised crevasse depth predictions versus oceanwater height ratio for a single isolated dry crevasse in a linear elastic ice sheet, considering homogeneous and depth-dependent mechanical properties for linear elastic incompressible ice ($\nu = 0.5$)

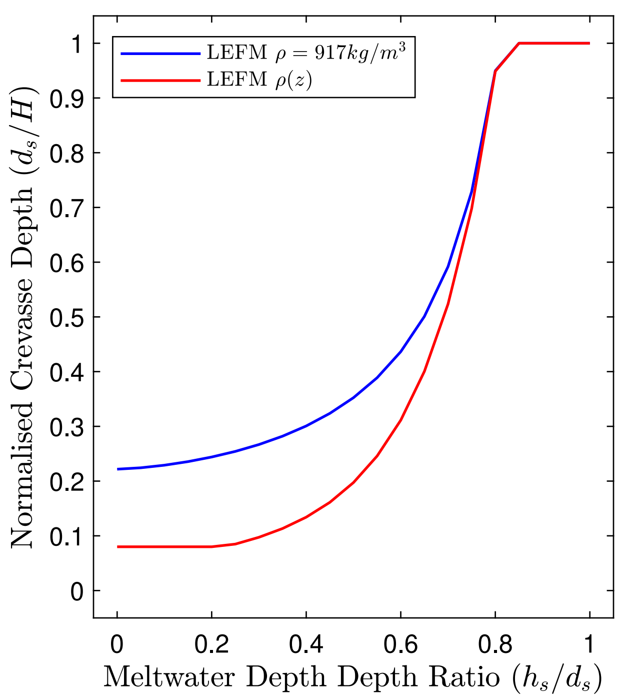

Figure 3: Normalised crevasse depth predictions versus meltwater depth ratio for a single isolated crevasse in a linear elastic ice sheet for an oceanwater height of $h_w = 0.8H$, considering homogeneous and depth-dependent density for linear elastic incompressible ice ($\nu \approx 0.5$)

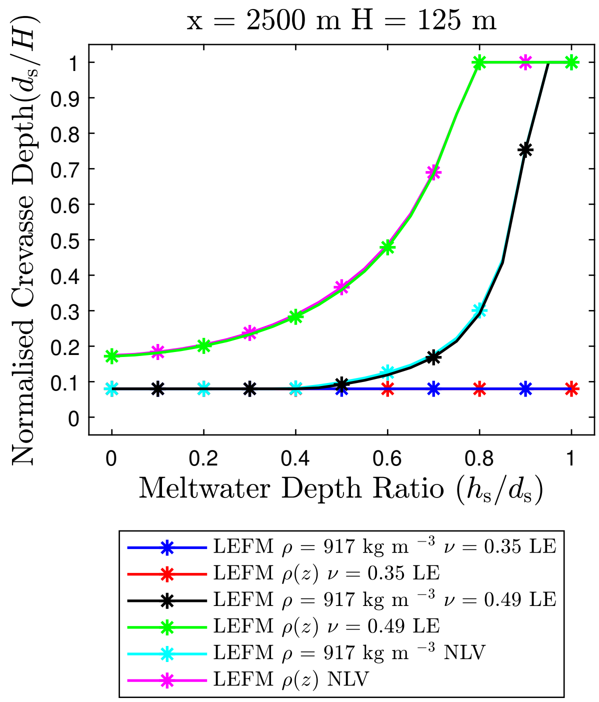

Figure 4: Normalised crevasse depth predictions versus meltwater depth ratio for a single isolated surface crevasse located in the far field region ($x = 2500$m) considering a linear elastic (LE) and non-linear viscous (NLV) rheology.

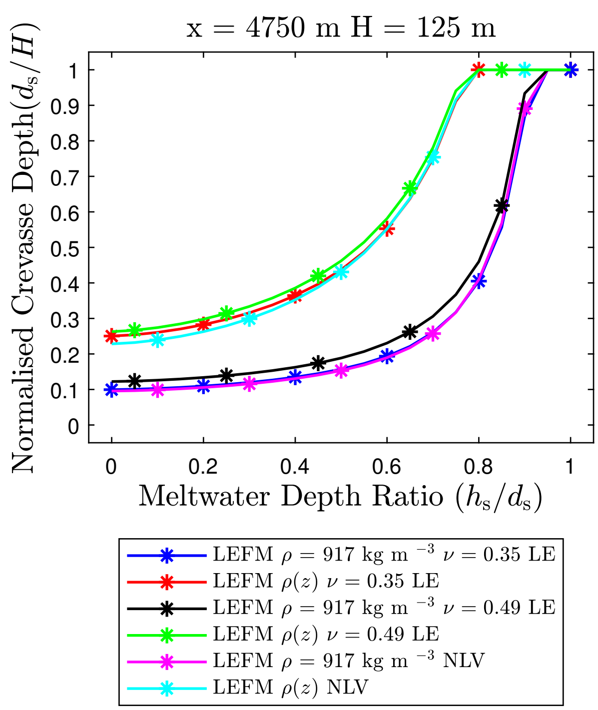

Figure 5: Normalised crevasse depth predictions versus meltwater depth ratio for a single isolated surface crevasse located close to the ice shelf front ($x = 4750$m) considering a linear elastic (LE) and non-linear viscous (NLV) rheology.

surface, while water is able to permeate through the top layer of firn (which is typically around 0°C), ice layers (referred to as ice lenses) that prevent further water inflow form as soon as the water permeates deeper and reaches sub-zero temperatures [1,2]. We have added the following text to the paper, clarifying the implications of not including a poromechanics formulation for the firn:

" One final limitation of our analytic models is related to water-filled crevasses. While we investigate the effects of considering depth-varying firn layer density and Young's modulus, we acknowledge that both these effects may be arising from the porosity of the firn. It is possible that water leaking from the crevasse into the surrounding porous firn. For colder ice-sheets and deeper crevasses, such that the water column is surrounded by ice of sub-zero temperatures, this assumption is reasonable, because any water that seeps into the surrounding ice/firn will freeze, creating an impermeable ice layer (i.e. ice lenses) surrounding the crevasse, which will prevent water from permeating further into the firn (Buzzard et al.,2018; Amory et al., 2024). As these ice lenses are typically very thin, these do not alter the stress state at the glacier scale. However, if temperate glaciers are considered, or conditions where water-filled crevasses do not penetrate to considerable depth, the firn/ice surrounding the crevasse might not be sufficiently cold to cause the ice lens to form. In such circumstances, the presented model will overestimate the crevasse depths obtained, as the saturated firn will reduce the effects of the water pressure within the crevasse by re-distributing this pressure over a larger region surrounding the crevasse. In future studies, we will consider the application of poro-damage phase field models (Sun et al., 2021; Clayton et al., 2022) to study fracture of saturated and unsaturated porous ice materials, and compared them with LEFM models. "

[1] S. C. Buzzard, D. L. Feltham, and D. Flocco, 'A Mathematical Model of Melt Lake Development on an Ice Shelf', Journal of Advances in Modeling Earth Systems, vol. 10, no. 2, pp. 262–283, Feb. 2018, https://doi.org/10.1002/2017MS001155.

[2] C. Amory et al., 'Firn on ice sheets', Nat Rev Earth Environ, vol. 5, no. 2, Art. no. 2, Feb. 2024, https://doi.org/10.1038/s43017-023-00507-9.

case, the influence of the depth-varying density is reduced when the ice shelf thickness increases, due to a larger proportion of ice being fully consolidated. However, there are still some differences in penetration depth compared to the homogeneous case for thick ice shelves ($H = 1000$ m), with a percentage difference of 14.0% for the dry crevasse, and 19.2% for $h_s/d_s = 0.75$.

Including the effects of both depth-varying density and depth-varying modulus highlights that density is the more prominent property influencing surface crevasse propagation in ice shelves. It is observed in Fig. 7 that the majority of results for depth-varying density and modulus (green lines) overlap the depth-varying density results (red lines). The exception to this is for dry crevasses in thin ice shelves, where the stabilised penetration depth is $0.194H$ compared to $0.25H$ when considering solely depth-varying density.

5 Non-linear Viscous Incompressible Rheology

The above analysis has considered ice to behave as a linear elastic compressible solid, with a Poisson ratio of $\nu = 0.35$. This is a common assumption if crevasse propagation occurs in a rapid brittle manner, such that the cracking occurs on a timescale well below the Maxwell time (typically in the order of hours-days for glacial ice). If the slow development of crevasses is to be considered, with crevasses stabilising over a span of weeks, then ice should be considered as an incompressible solid. This can be achieved by setting the Poisson ratio to $\nu \approx 0.5$ (using $\nu = 0.49$ in our studies to prevent numerical issues). In addition, we conduct a finite element simulation for a grounded glacier, including the viscous contributions of ice flow, modelled through Glen's flow law and extract numerical values of the longitudinal stress. To illustrate the influence of ice rheology, we plot the longitudinal stress profile for a land terminating ($h_w = 0$) grounded glacier, considering linear elastic compressibility ($\nu = 0.35$), linear elastic incompressibility ($\nu \approx 0.5$) and a non-linear viscous rheology in Fig. 8.

Firstly, we note that when ice is considered as linear elastic incompressible ($\nu \approx 0.5$), a stress solution is obtained which matches the steady state creep stress state derived by Weertman (1957) for a depth-independent density, and matches stress profiles obtained through simulations using a visco-elastic rheology. We observe that stresses are more extensional in the upper surface and more compressive at the base when considering incompressibility and that stress is independent of ice rheology (Glen's law creep coefficients). For the homogeneous case, the longitudinal stress varies linearly with depth and is symmetrical about the centre line $z = H/2$. Similarly to the linear elastic compressible case, the inclusion of depth-dependent density results in a reduction in both the lithostatic stress contribution σ_{zz} and the resisitive stress R_{xx}, for both material rheologies, a point which was neglected by van der Veen (1998b) who considered R_{xx} to be independent of depth-varying density.

The longitudinal stress profiles presented in Fig. 8 are used to drive crevasse propagation in the linear elastic fracture mechanics study. Values of crevasse penetration depth for an isolated dry crevasse in a grounded glacier, subject to different values of oceanwater height h_w are presented in Fig. 9. The solid line curves consider incompressible ice, whilst the dashed lines represent compressible ice of Poisson ratio $\nu = 0.35$. Considering ice as an incompressible solid leads to deeper crevasse penetration depths compared to linear elastic compressibility, but these crevasses follow a similar trend as observed for the compressible case: For surface crevasses in glaciers subject to low levels of oceanwater, the penetration depth is unaffected by firn density due to crevasses stabilising in fully consolidated strata. However, as the oceanwater height increases, crevasses

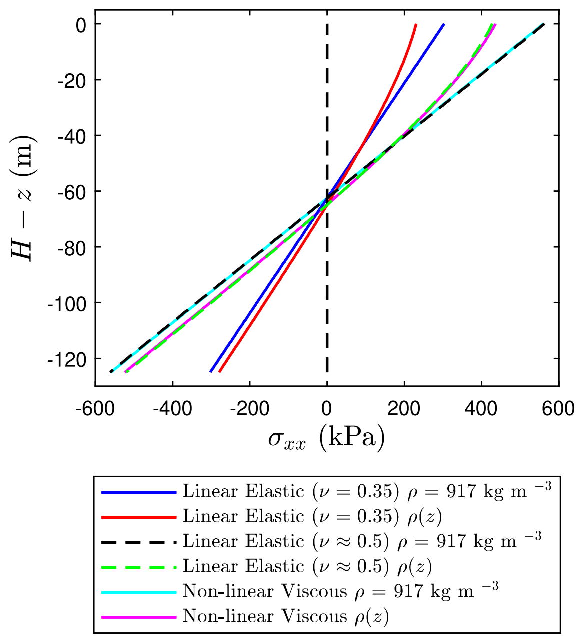

Figure 8. Far field longitudinal stress σ_{xx} throughout the depth of a land terminating glacier ($h_\mathrm{w} = 0$), showing the effects of depth-varying density $\rho(z)$; considering linear elastic compressibility ($\nu = 0.35$), linear elastic incompressibility ($\nu \approx 0.5$) and a non-linear viscous rheology.

become shallower, and as a result, the inclusion of firn density becomes more prevalent. Comparing the effects of assuming an incompressible/viscous rheology, the percentage difference in penetration depth when considering depth-dependent density, for a dry crevasse of oceanwater height $h_\mathrm{w} = 0.5H$ reduces to 4%, compared to 20% for linear elastic compressibility. The ocean-water height required to prevent any development of dry crevasses differs, with values of $h_\mathrm{w} = 0.55H$ being sufficient for compressible depth-dependent density cases whereas oceanwater levels of $h_\mathrm{w} = 0.8H$ are required for the incompressible case. Comparing this to the cases in which no density variations are considered still shows a similar trend, with higher oceanwater needed to stabilise crevasses when density variations are not considered.

Finally, we consider water-filled surface crevasses in floating ice shelves of height $H = 125$m and length $L = 5000$m, using a non-linear viscous ice rheology. Similarly to the linear elastic compressible case, we consider surface crevasses at the horizontal position $x = 4750$m (250 m from the ice shelf terminus) and extract the longitudinal stress profiles from the finite element analysis. We plot the stabilised crevasse depth versus meltwater depth ratio for the non-linear viscous (NLV) rheology in Fig. 10 along with the results for linear elastic (LE) compressibility ($\nu = 0.35$) and incompressibility ($\nu = 0.49$). When comparing the stabilised crevasse depths close to the front, we note that the penetration depth is independent of ice rheology, which is in

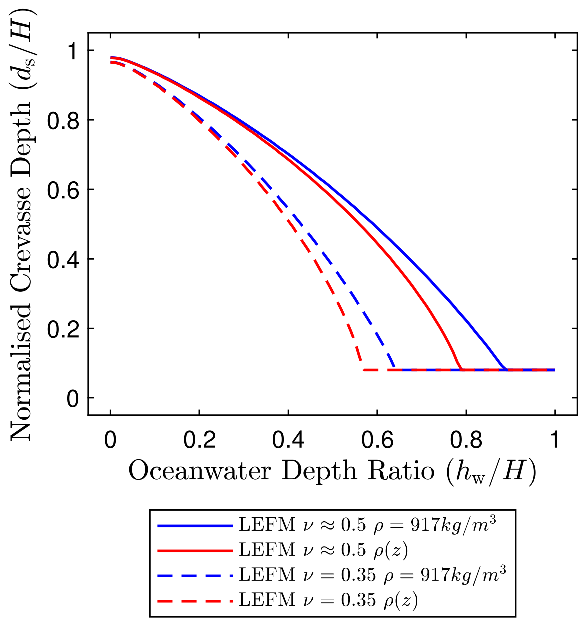

Figure 9. Normalised crevasse depth predictions versus oceanwater height ratio for a single isolated dry crevasse in a grounded glacier, considering compressible ($\nu = 0.35$) and incompressible ($\nu \approx 0.5$) ice homogeneous and depth-dependent mechanical properties

contrast to the grounded glacier case. For the homogeneous density, minimal crevasse propagation is observed for meltwater depth ratios below $h_s/d_s < 0.6$, with full thickness propagation only occurring when fractures are close to saturation. The inclusion of the depth-dependent density results in deeper crevasse penetration depths, with minimal differences in penetration depth between the linear elastic cases and non-linear viscous rheology. This likely indicates that for crevasses close to the front, fracture is driven by the flotation height and the bending stresses due to the floating condition. For depth-dependent density, the reduction in flotation height leads to an increase in tensile stress in the upper surface, due to increases in R_{xx} and increased bending stress. In addition, the lithostatic component of longitudinal stress is reduced, leading to deeper crevasse propagation when including firn density.

We also consider the propagation of an isolated surface crevasse located in the far field region ($x = 2500$ m) of a floating ice shelf, with results presented in Fig. 11. As shown previously, for the linear elastic compressible rheology the stress state is fully compressive for both the homogeneous and the depth-dependent density case, thus no crevasse propagation is observed regardless of meltwater depth ratio. By contrast, when considering the non-linear viscous rheology of ice, surface crevasses may propagate in the far field region if there is sufficient meltwater pressure present. Large increases in crevasse penetration depth are observed for meltwater depth ratios greater than $h_s/d_s = 0.50$, with full thickness propagation being observed close

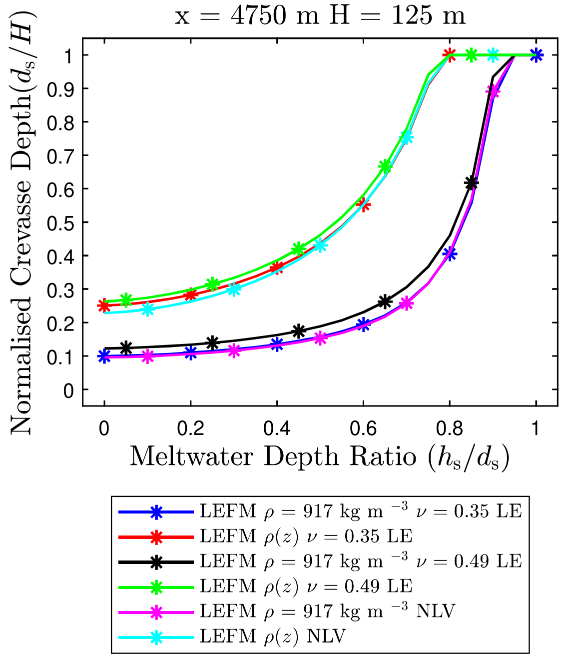

Figure 10. Normalised crevasse depth predictions versus meltwater depth ratio for a single isolated surface crevasse located close to the ice shelf front ($x = 4750$m) considering a linear elastic (LE) and non-linear viscous (NLV) rheology.

to crevasse saturation at $h_s/d_s = 0.95$. Similar to crevasses near the front, the inclusion of depth dependent density results in increased crevasse penetration depths compared to the homogeneous density scenario. Thus, similar conclusions can be drawn for both elastic and viscous rheologies.

6 Discussion

An important finding of this paper is that the inclusion of the depth-varying mechanical properties of unconsolidated ice strata results in a reduction in both the lithostatic compressive stress and the resistive tensile stress components. Contrary to the conventional understanding (van der Veen, 1998a), we find that accounting for depth-varying density and modulus can lead to an overall reduction in surface crevasse depths in grounded glaciers. This is because, in some scenarios, the reduction in resistive stress can hinder crevasse propagation more than the increase in crevasse propagation resulting from the reduction in lithostatic stress. Thus, our study suggests that firn layers can have a stabilizing effect by curtailing surface crevasse growth in grounded glaciers.

Assuming ice to be an linear elastic compressible material, we find that considering depth-varying Young's modulus has a greater influence on crevasse depths than density in thinner glaciers. For example, considering depth-varying density results

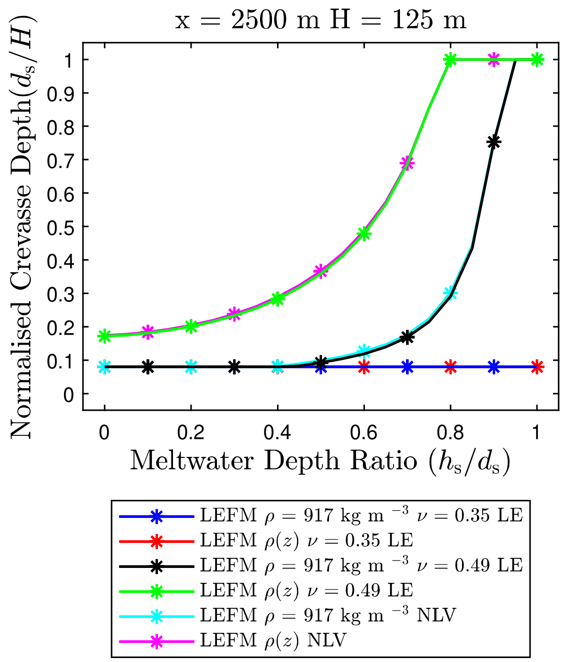

Figure 11. Normalised crevasse depth predictions versus meltwater depth ratio for a single isolated surface crevasse located in the far field region ($x = 2500$m) considering a linear elastic (LE) and non-linear viscous (NLV) rheology.

in a maximum percentage difference of 20% in the penetration depth of dry crevasses, compared to a maximum percentage difference of 45% when considering depth-varying Young's modulus. The largest reductions in crevasse depths are observed in thinner glaciers (depths of approximately 100–150 m), where the stabilizing effects of the firn layers seem to be more prominent. Larger meltwater depth ratios are required to propagate surface crevasses in thinner ice shelves; whereas in thicker glaciers, the influence of firn density is lesser (in some cases negligible), so surface crevasses propagate deeper into the fully consolidated strata. Thus, our study reveals that LEFM models assuming homogeneous ice properties are valid for crevasse depth estimation in thicker glaciers with ice thickness $H > 250$ m.

Accounting for depth-varying density in the floating ice shelf case increases the penetration depth of surface crevasses close to the ice-ocean front, with this increase caused by reductions in buoyancy height and lithostatic compressive stresses. The effect of depth-varying density is dominant in thinner ice shelves, but it can still impact surface crevasse propagation in ice shelves as thick as $H = 1000$ m, although to a lesser extend. For instance, the crevasse depth ratio increases to $d_s = 0.91H$ (188% increase compared to homogeneous case) for thin ice shelves ($H = 125$ m); whereas, a 19% increase is observed for 1 km thick ice ($d_s = 0.45H$). Considering depth-varying Young's modulus in the floating ice shelf case slightly reduces surface crevasse depth for low meltwater depths, and the effect becomes less significant in thicker ice shelves. This study suggests as the

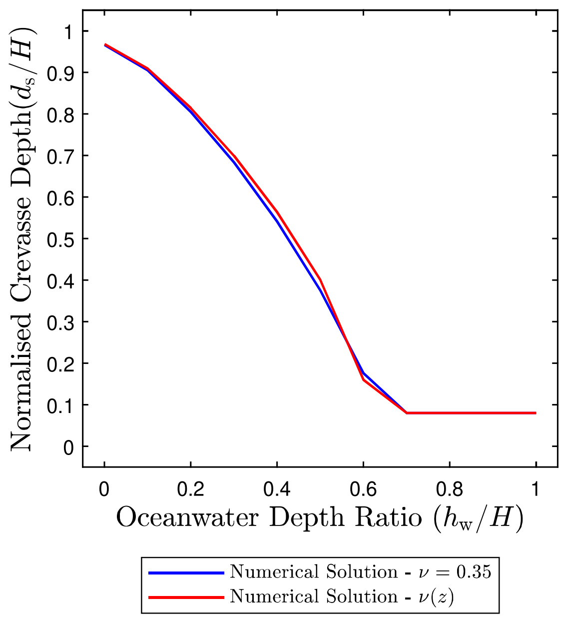

Figure E1. Normalised crevasse depth predictions versus oceanwater height ratio for a single isolated dry crevasse in a linear elastic ice sheet, considering homogeneous and depth-dependent Poisson Ratio.

Appendix E: Influence of Poisson ratio ν

E1 Depth-variable Poisson ratio

For the crevasse propagation studies previously presented, a depth invariant Poisson ratio of $\nu = 0.35$ was assumed. However, it has been suggested that Poisson ratio also exhibits a linear dependency on ice density and therefore leads to a depth-dependent profile (Smith, 1965). Furthermore, Schlegel et al. (2019) and King and Jarvis (2007) provides a depth-dependent Poisson ratio profile based on seismic velocity measurements on ice cores. To study the effect of this depth-dependent Poisson ratio, a linear elastic fracture mechanics study is performed. We assume an exponential distribution of Poisson's ratio with depth, similar to the density and Young's modulus distributions.

$$\nu(z) = \nu_i - (\nu_i - \nu_f)e^{-(H-z)/D} \tag{E1}$$

where $\nu_f = 0.07$ is the Poisson ratio of firn in the upper surface, $\nu_i = 0.35$ is the Poisson ratio of fully consolidated ice and $D = 32.5$ is the tuned constant. This profile approximates the observations from Schlegel et al. (2019), where we have scaled the length parameter D to match our density and Young's modulus profiles as this profile was obtained at a different location (with significantly different ice-sheet and firn thickness). As it is not possible to derive a fully analytic expression for the stress profiles with this depth-dependent Poisson ratio, the longitudinal stress profiles are obtained numerically through the finite element model. Once obtained, the stresses are used to drive the propagation of the surface crevasse in the linear elastic fracture mechanics study.

We consider a dry (air-filled) crevasse, with different values of oceanwater height h_{w} and plot the normalised crevasse penetration depth versus oceanwater height h_{w} in Fig. E1. This figure shows that the effect of including variations in Poisson ratio have a more limited effect compared to density and Young's modulus variations. The largest percentage difference in crevasse depth was observed for intermediate ocean-water levels, with an increase of 6% in crevasse depth with respect to the homogeneous case when considering a depth dependent Poisson ratio, for an oceanwater height of $h_{\mathrm{w}} = 0.5H$. This is in contrast to the inclusion of firn density and Young's modulus, which predict a reduction in stabilised crevasse depth for surface crevasses in grounded glaciers. The effect of a including a depth-dependent Poisson ratio is less influential compared to density and Young's modulus, as depth-dependent density resulted in a reduction of 20% of the crevasse depth and depth-dependent Young's modulus resulted in a reduction of 45% of the crevasse depth. We therefore conclude that the inclusion of variations in Poisson ratio does not play a significant role in crevasse propagation.

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Competing interests. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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