Spurious numerical mixing under strong tidal forcing : a case study in the South East Asian Seas using the Symphonie model (v3.1.2)

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Abstract. The role of mixing between layers of different density is key to how the ocean works and interacts with other components of the Earth System. Accounting correctly for its effect in numerical simulations is therefore of utmost importance. However, numerical models are still plagued with spurious sources of mixing, originating mostly from the vertical advection schemes in the case of fixed coordinates models. As the number of phenomenon explicitly resolved by models increases, so does the amplitude of resolved vertical motions and the amount of spurious numerical mixing; and regional models are no exception to this. This papers provides a clear illustration of this phenomenon in the context of simulation of the South East Asian Seas is provided, along with a simple way to reduce it. This region is known for its particularly strong internal tides and the fundamental role they play in the dynamic of the region. Using the Symphonie ocean model, simulations including and excluding tides and using a pseudo third-order upwind advection scheme on the vertical are compared to several reference datasets, and the impact on water masses in assessed. The high diffusivity of this advection scheme is demonstrated, along with the importance of accounting for tidal mixing for a correct representation of water masses. Simultaneously, an improvement of this advection scheme to make it more suitable for use on the vertical is provided. Simulations with the new formulation are added in the comparison. We conclude that the use of a higher order numerical diffusion operator greatly improves the overall performance of the model.

1 Introduction

Diapycnal mixing plays a fundamental role in the ocean, and precise quantification, localisation and understanding of its mechanism are still active topics of research (Meredith and Garabato, 2021). Considering its major role in the heat uptake of the ocean, its correct representation in climate models is of utmost importance. Yet, a major flaws of ocean models is the tendency of numerical methods used to solve the equations on discrete grids to produce numerical errors resulting in an excess of dissipation of tracer fields often referred to as *numerical mixing*. Considering the sensitivity of numerical models outputs to vertical mixing (Bryan, 1987), this spurious phenomenon becomes particularly troublesome when it occurs across isopycnals, since the physical mixing is much lower in this direction than along the isopycnals and numerical mixing can therefore easily dominates. In this context, the discretization of the vertical tracer advection equation in fixed coordinates model - being sigma

or geopotential coordinates - and the resulting spurious diapycnal fluxes have long been identified as a major source of spurious
25   numerical mixing (Griffies et al., 2000); and it is still considered as a major issue (Klingbeil et al., 2019).

Due to its advective origin, this spurious mixing is strongly linked to the strength of the flow, and is therefore more sensitive
in situations of strong vertical displacements. If this is troublesome in the current class of global ocean models (Megann, 2018;
Holmes et al., 2021), it might become even worse as the resolution increases and more physical processes are being resolved
explicitly - such as submesoscale and the strong vertical speeds anomalies associated with e.g. frontogenesis (Siegelman
30   et al., 2020); or tides and the associated internal waves field, especially in regions of strong internal tide activity. In this sense,
regional ocean models are particularly at risk, especially when run over long periods of time.

If the issue is known in the community of model developers, it is unsure whether the fast growing community of model *users*
is fully aware of this issue. Since the early 2000's, a number of studies have indeed focused on diagnosing spurious numerical
mixing in academic simulations (see for instance Burchard and Rennau, 2008; Klingbeil et al., 2014; Ilıcak et al., 2012; Gibson
35   et al., 2017). However, the literature on spurious mixing in realistic simulation is still sparse (e.g. Lee et al., 2002; Megann,
2018; Holmes et al., 2021), is focused mainly on relatively coarse resolution global models and relies on rather complex indirect
estimates of mixing through water mass transformation analysis. Furthermore, none of the models used included explicit tidal
forcing. Yet, owing to the advances in simulating tides in ocean models over the last decades (see e.g. Arbic, 2022) and
the role they play in setting the global state of the ocean (Wunsch and Ferrari, 2004), more and more realistic simulations
40   account for tidal forcing and the first few baroclinic modes are now routinely resolved in most regional applications. There is
therefore a need to explicitly demonstrate the spurious numerical effect tides can have on tracer fields, even in relatively high
resolution models forced at their lateral boundaries. Also, though less precise, comparison against observations - made possible
for instance by the growing amount of ARGO profiles in the ocean and availability of high-resolution satellite products - and
parameter sensitivity study can help to provide a clear picture of the effect of spurious mixing in realistic simulations.

45   Now, with the steady increase in available computing power, one can expect the spurious vertical mixing to be eventually
reduced to acceptable levels (Holmes et al., 2021), especially since advection schemes are rarely of first order. The number of
vertical levels has indeed increased by up to a factor of 10 over the last decades. This however relies on the assumption that
the range of vertical speeds resolved by the model does not depend too much on the vertical grid spacing. Furthermore, always
increasing the resolution might not be desirable nor achievable in a foreseeable future for every research team and projects
50   around the world, and therefore cannot be regarded as a definitive solution to the issue. Finally, the cost-effectiveness of an
increase in resolution depends largely on the order of the schemes used, and both still have to be considered simultaneously
(Sanderson, 1998).

Considering the eulerian nature of the problem, moving toward lagrangian or semi-lagrangian coordinates seems to have
been a preferred option in recent years. Such approaches seem promising and have indeed demonstrated capabilities in reducing
55   spurious diapcynal mixing (Gibson et al., 2017; Megann et al., 2022). In a recent paper, Megann (2024) demonstrated the
ability of so called $z-$tilde coordinates (Leclair and Madec, 2011) to effectively reduce spurious numerical mixing in a global
eddying simulation forced with tides. However, if they formally cancel the issue of spurious vertical mixing inherent to fixed
coordinates models, these approaches come at the cost of a whole range of new issues and limitations that remain a topic of

active research, owing to their relative youth (Fox-Kemper et al., 2019; Griffies et al., 2020). Moreover, their implementation
60 into already existing numerical codes requires sustained effort by model developers over years in order to be used in realistic
simulations.

There is therefore a need to expand the range of tools available for fixed coordinates ocean models. The goal of this paper is
twofold : a new, less diffusive, formulation for advection schemes based on a scheme already implemented in the Symphonie
model is proposed; validation of the method through the case study of simulations of the South East Asian Seas is then
65 proposed, providing a clear illustration of both the spurious numerical effect of tides in a regional model and their relevance in
the accurate depiction of the region.

2   Model configuration

2.1   The South-East Asian Seas

Many locations have been shown to exhibit intense internal tides activity (Zaron, 2019), correlated with the existence of large
70 areas of strong topographic gradients. Amongst them are the South East Asian (SEA) Seas (see Fig.1), a large region of
numerous deep marginal seas and shallow straits connecting the Western Pacific to the Indian Ocean through an on-average
westward circulation know as the Indonesian Throughflow (ITF) (Sprintall et al., 2019). Due to this complex bathymetry, the
region has long been known as a hotspot of internal tides generation (e.g. Apel et al., 1985), and their dissipation in the various
semi-enclosed basins has been recognized as a major driver of the intensified mixing observed there (Ffield and Gordon, 1996).
75 It leads to a sensible transformation of water masses on their way from the Pacific to the Indian Ocean, forming a unique water
mass that can be tracked across the basin and beyond, up to the Agulhas Current (Gordon, 2005). This intensified mixing also
acts to cool the SST in the region (Susanto and Ray, 2022) and modify the atmospheric deep convection, which in turn impacts
the regional and global climate (Koch-Larrouy et al., 2010; Sprintall et al., 2019). Interannual and climatic projections therefore
need an accurate modelization of the ocean in the SEA region; and this cannot be achieved without an accurate representation
80 of tidal mixing.

In average, the vertical diffusivity over the region is indeed increased by more than one order of magnitude when compared
with open-ocean estimates (Ffield and Gordon, 1992), with a high spatial variability (Purwandana et al., 2020); and it has
been demonstrated in many studies that models not accounting for tides-induced mixing fail at representing correctly the water
masses along the course of the ITF (Koch-Larrouy et al., 2007; Jochum and Potemra, 2008; Nugroho et al., 2018; Sasaki et al.,
85 2018; Katavouta et al., 2022). Parameterization of tidally driven mixing has been successfully implemented in regional models
(see e.g. Koch-Larrouy et al., 2007), but still exhibit some weaknesses (Iskandar et al., 2023). Also, such an approach misses
the dynamical effect of tides, which is suspected to play a non-negligible role in the horizontal mixing of water mass properties
via the interaction of tidal currents with the complex topography of the region (Hatayama et al., 1996; Nagai et al., 2021),
especially for high resolution models. With the increase in computational capacities allowing for simulations to be run on finer
90 grids, thus enabling the explicit resolution of more baroclinic modes, explicit tidal forcing is therefore being used more and
more in regional models of the SEA region (Castruccio et al., 2013; Tranchant et al., 2016; Katavouta et al., 2022).
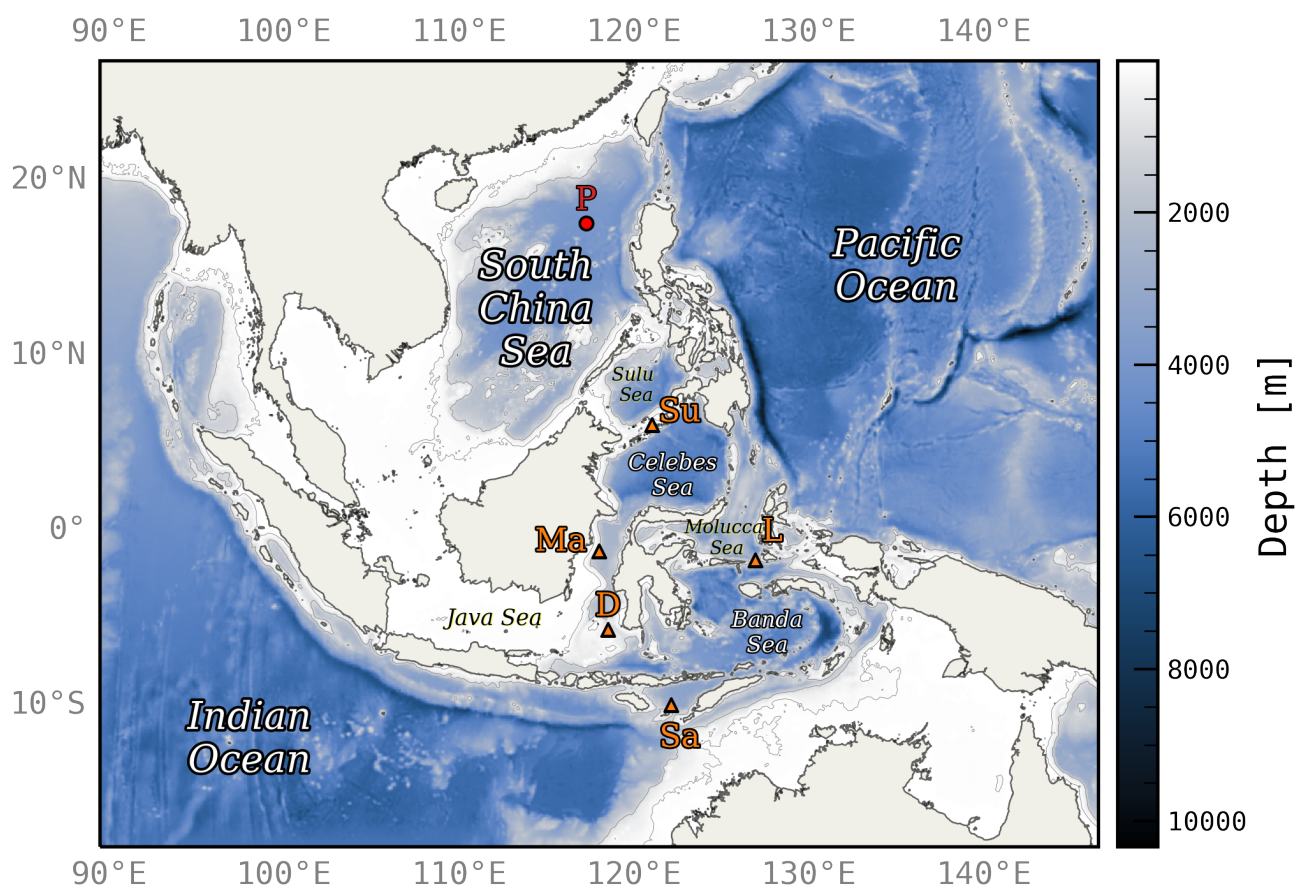
Figure 1. Bathymetry of the SEA configuration. The 100 and 1000 m isobaths are represented in fine grey lines. The triangle between the Sulawesi and Java Sea indicates the location of the Makassar Strait. Locations Su, Sa, L, D, Ma B, C, D corresponds respectively to the Sulu islands chain, the Savu Sea, the Lifamatola Passage and the Dewakang Sill. Point P refers to the location of the profile which evolution is shown in Fig.2.

2.2 Model configuration and overview

The model employed in this study is the Symphonie ocean model (Damien et al., 2017), a three dimensional ocean circulation model solving the primitive equations under Boussinesq and hydrostatic equation. Symphonie has been used in the past over
95 Southeast Asia at several spatial scales, from the Gulf of Tonkin shelf scale (Piton et al., 2021; Nguyen-Duy et al., 2021) and South Vietnam upwelling coastal scale (To Duy et al., 2022; Herrmann et al., 2023) to the South China Sea regional scale (Trinh et al., 2024). Turbulent closure is achieved through the $k - \epsilon$ scheme (Burchard and Bolding, 2001). Integration in time is peformed using a leapfrog time-stepping algorithm together with a Robert-Asselin filter to damp out spurious numerical modes. Barotropic and baroclinic modes are handled separately following a mode splitting procedure (Blumberg and Mellor,
100 1987). Momentum is advected via a fourth-order central differencing scheme along with an explicit biharmonic diffusion term, with a viscosity derived from the one developed in Griffies and Hallberg (2000). Since tracer advection schemes are the focus of this paper, the one used in Symphopnie are discussed in Sec.2.3.

Owing to the complex bathymetry of the region, equations are solved on a staggered Arakawa C-grid covering the entire South-East Asian region (see Fig.1) using a rather high regular resolution of about 5 km on the horizontal. This resolution is
105 relatively high compared to previous regional studies (Nugroho et al., 2018; Katavouta et al., 2022) and allows for a better representation of the many narrow straits in the region, especially along the Sunda Islands that separate the Indonesian Seas from the Indian Ocean. To reduce truncature errors of sigma-coordinates while maintaining an accurate description of the complex bathymetry, vanishing quasi-sigma (VQS) coordinates are used on the vertical (Dukhovskoy et al., 2009; Estournel et al., 2021), with 60 levels.

110 Currents and tracers are initialized using the $1/12° \times 50$ fields from the Global Ocean Physics Reanalysis from the Copernicus Marine Service (GLORYS12V1, https://doi.org/10.48670/moi-00021), hereafter called GLORYS, interpolated on our higher resolution grid. Daily averages of temperature, salinity, sea surface height, and currents are then provided by the same model and applied at lateral boundaries. Tides are forced by harmonics M2, N2, S2, K2, K1, O1, P1, Q1 and M4 from the tide atlas FES2014 (Lyard et al., 2021). Along with the 5 km resolution, the configuration should be able to resolve between
115 80 to 90% of the barotropic to baroclinic conversion rate, following Niwa and Hibiya (2011), though the exact location of energy dissipation is still unclear. Surface fluxes are computed from the bulk formulae of Large and Yeager (2004) using the European Centre for Medium Range Weather Forecasts (ECMWF) operational forecasts at 1/8° horizontal resolution and 3 hours temporal resolution (https://www.ecmwf.int/).

Given the relatively low amount of validation data in the region (Sprintall et al., 2019), simulations have been chosen to coin-
120 cide with the peak of ARGO floats profiles in the region, and are therefore run on the 2017-2018 period. No ARGO floats were indeed available in two major passages of the ITF before these years, namely the Makassar Strait (linking the Sulawesi Sea to the Java Sea) and the Banda Sea. See Fig.A1 for locations of ARGO floats in the region. Daily averages of each variables are saved for analysis. In total, four simulations have been run - T0, T1, NT0 and NT1 - which details will be described in Sec.3

2.3 Advection

125 In the following, we briefly describe the advection scheme used in Symphonie and the method implemented to improve the diffusive properties of the vertical component. We begin by describing the general framework in which the discussion is set and define the notations used. We then present the advection scheme already in use in Symphonie. Its numerical diffusivity is estimated and compared to physical mixing. A method is finally proposed to improve its diffusive properties and make it usable on the vertical.

2.3.1 Notations

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As the issue we are dealing with arises from the vertical component of the tracer transport, we will restrict our analysis to a one dimensional situation and use notations considered standard for the vertical, even though horizontal advection will also be discussed. The continuous advection of a tracer $\tilde{s}$ by an incompressible flow of velocity field $\tilde{w}$ is expressed by the following equation :

135 $$\partial_t \tilde{s} = -\partial_z (\tilde{w}\tilde{s}) \tag{1}$$

Tilde are used to distinguish continuous fields from their discrete counterparts. On a staggered regular grid $\{z_j\}$ with constant spacing Δz, discretization of the equation in space in a finite volume formulation for the discrete field $s = \{s_j\}$ advected by a velocity field $w = \{w_j\}$ writes

$$\frac{ds_j}{dt} = -\frac{(F_{j+} - F_{j-})}{\Delta z} \tag{2}$$

140 where F_{j+} (resp. F_{j-}) conceptually represents the value of the flux field $F = \tilde{w}\tilde{s}$ at the interface between cells j and $j+1$ (resp. cells $j-1$ and j). In such a formulation, w_j represents the velocity field $\tilde{w}$ sampled at location $z_j - \Delta z/2$ while s_j represents the amount of tracer $\tilde{s}$ in grid cell j, ie.

$$s_j(t) = \frac{1}{\Delta z} \int_{z_j - \Delta z/2}^{z_j + \Delta z/2} \tilde{s}(t,z)dx \tag{3}$$

Hence, since F represents the flux at the interface, an estimate s_{j+} of $\tilde{s}(t, z_j + \Delta z/2)$ will have to be reconstructed from

145 the values of $\{s_j\}$ via a given *scheme*. Advection schemes used nowadays in ocean models usually originate from standard interpolations procedures, or weighted (often with non-linear weights) combinations of several other interpolations in order for the computed solution to satisfy certain desired properties.

2.3.2 Advection of tracer in Symphonie

Horizontal advection of tracer is carried out using a pseudo-QUICKEST scheme, inspired from the QUICKEST scheme by

150 Leonard (1979), and hereafter referenced to as QKE. Its flux formulation writes formally

$$F^{\text{QKE}} = (1 - (2n_c)^2)F^{\text{UP3}} + (2n_c)^2 F^{\text{UP1}} \tag{4}$$

where n_c is the Courant number on the corresponding direction and F^{UP1}, F^{UP3} the fluxes for the first order and third order upwind advection schemes, respectively. Following Webb et al. (1998), the UP3 scheme is split into a fourth-order centered advection part (C_4) and a bilaplacian diffusive part (D_4), and the D_4 part is evaluated at time step $t-1$ during the leapfrog integration, so as to ensure conditional stability. The UP1 part is fully evaluated at time step $t-1$.

The scheme behaves like a UP3 scheme for low Courant numbers, while for Courant numbers close to 0.5 (i.e. $2n_c \sim 1$), the scheme turns into the forward integration of a first order upwind discretization integrated with time step $2\Delta t$, known to transport perfectly in those conditions. This hybrid behaviour increases the range of stability of the model and allows it to handle more robustly the few hotspots of strong Courant numbers. This scheme has demonstrated its robustness in the past applications of Symphonie, where vertical motions were either relatively low or the size of the domains allowed for a rapid regeneration of water masses by forcing at the boundaries, two assumptions that do not hold anymore in the present configuration.

Vertical advection of tracer in Symphonie is traditionally carried out with a second order centered advection scheme. However, under strong vertical motions, dispersive errors can have spurious effects on the density field, especially on the long run (see e.g. Griffies et al., 2000; Hecht, 2010) and some kind of numerical diffusion is therefore required to prevent small scales density instabilities from developing, either by introducing linear diffusion operators (often implicitly included in up-wind formulations) or by using non-linear limiting procedures such as TVD schemes (Cushman-Roisin and Beckers, 2011) or their stronger version, FCT schemes (Zalesak, 1979), in order to enforce some kind of monotonicity in the evolution of the field. Such non-linear schemes are however computationally costly and still exhibit a sensible amount of spurious numerical diffusion, comparable to the physical one (see e.g. Megann, 2018), with the additional drawback that it cannot be explicitly diagnosed. We therefore opted for building on a linear scheme already implemented in Symphonie and to try to improve its diffusive properties through a lightweight formulation to make it usable on the vertical at rather low additional computational cost.

2.3.3 Excessive diffusion on the vertical

We can simplify the analytical discussion by considering that along the vertical, Eq.4 essentially boils down to the formulation of a UP3 scheme in the regions of interest of the model. Indeed, using typical numerical values $W \approx 10^{-3} - 10^{-4}$ m.s^{-1}, $\Delta z \approx 10^1 - 10^2$ m and a time-step $\Delta t = 180$ s $\approx 2 \times 10^2$ s, we get $n_c \approx 10^{-2} - 10^{-5}$, so that $n_c \ll (1 - (2n_c)^2) \sim 1$ and the role played by the UP1 part can be neglected, at least in the ocean interior. Though of higher order than the UP1, the UP3 scheme is nonetheless known to be still excessively diffusive (see e.g. Madec et al., 2022).

To quantitatively illustrate this behaviour, we derive the dispersion relation for a single Fourier mode $s_j \propto s_k e^{i(kj\Delta z - \omega t)}$, linking ω to k and expressing how a given harmonic wave with wavenumber k evolves over time. In a such a spectral framework, the diffusive aspect of a scheme is expressed by the addition of a damping term in the corresponding dispersion relation. More specifically, writing this dispersion relation in its canonical form $\omega(k) = \Omega(k) - i\gamma(k)$, with Ω and γ real valued functions of

k, the scheme is said to be diffusive when the damping coefficient $\gamma(k)$, that quantifies the typical time $\tau = 1/\gamma$ at which a given wavelength is being damped out, is strictly positive.

Let's consider the damping coefficient obtained for the UP3 scheme. Taking for instance results in Webb et al. (1998), assuming for simplicity the vertical speed W to be constant and using the normalized form $\theta = k\Delta z$ for the wavenumber, we have in the most compact form

$$\gamma^{\text{UP3}}(\theta) = \frac{W}{3\Delta z}(1 - \cos\theta)^2 \tag{5}$$

For the largest wavelengths, i.e. $\theta \to 0$, $\gamma^{\text{UP3}}(\theta) \to 0$, which is expected since we do not want the largest scales to be damped out; while for the smallest wavelengths, i.e. $\theta \to \pi$, the damping is maximal and equal to $\frac{4W}{3\Delta z}$: the smallest, noisy, wavelengths are damped out. Comparing this to the damping coefficient obtained for the simplest physical diffusion term $\kappa\partial_{zz}s$, κ being the vertical diffusion coefficient computed from the turbulence closure scheme, discretized in its usual form $\kappa(s_{i+1} - 2s_i + s_{i-1})/(\Delta z)^2$ that leads to a physical damping coefficient

$$\gamma^{\text{phy}}(\theta) = \frac{2\kappa}{\Delta z^2}(1 - \cos\theta) \tag{6}$$

the ratio of numerical to physical damping $\Gamma_{\text{UP3}} = \gamma^{\text{UP3}}/\gamma^{\text{phy}}$ writes

$$\Gamma_{\text{UP3}} = \frac{W\Delta z}{6\kappa}(1 - \cos\theta) = \frac{P_e}{6}(1 - \cos\theta) \tag{7}$$

where P_e is the grid Peclet number. Typical values of P_e found in the model can be obtained using the same numerical values as before and considering a value of $\kappa \approx 10^{-5}$ m^2.s^{-1} typical for vertical diffusivity in the ocean interior (e.g. Polzin et al., 1997; Alford et al., 1999). This leads to $P_e \approx 10^2 - 10^4$. For intermediate wavelengths $N = 2\pi/\theta$ of a ten of grid points, that is $\theta = 2\pi/10$, typical of the number of grid points used to represent a thermocline, we have then $\Gamma_{\text{UP3}} \approx 3 - 300$. This means that the numerical mixing resulting from the discrete vertical advection is at least three times larger than the physical mixing at this scale, even under relatively conservative assumptions about the quantitative values, and can even be two orders of magnitude larger. The numerical diffusion is thus not *selective enough*, in the sense that it spuriously damps physical scales. In the following, we propose a method to improve this selectivity.

2.3.4 Filtered diffusion

As discussed previously, UP3 can be written as the sum of a purely dispersive (C$_4$) and a purely dissipative (D$_4$) components. Since the damping originates from D$_4$, we now aim at making this term more scale-selective. This can be achieved through a filtering process : assuming a high-pass filter Φ that applies on the tracer field s, the idea is to apply the diffusive operator on the filtered field only, i.e. turn D4 into D$_4[\Phi[s]]$, so that only the high spatial frequencies (smaller scales) are dissipated. In spectral space, since D$_4$ is linear, the damping coefficient for this scheme becomes formally

$$\gamma(\theta) = \frac{W}{\Delta z}\Re(\hat{\text{D}}_4(\theta))\hat{\Phi}(\theta) \tag{8}$$

Therefore, for large scales where $\hat{\Phi}(\theta) \sim 0$, $\gamma(\theta)$ will be almost zero, while for small scales where $\hat{\Phi}(\theta) \sim 1$, $\gamma(\theta)$ will be unchanged. We assumed here $\hat{\Phi}(\theta)$ to be real valued, an assumption justified by the fact that it would otherwise introduce

spurious lag in the filtered signal. Theoretically, many filters could fit. However, we now focus on the specific filter with which

215   the method has been primarily developed and other formulations will be discussed in Sec.4. Inspired by a formalism proposed

in Juricke et al. (2020b) in the context of the momentum equation, we define a filter $\Phi = \mathbf{1} - \phi$ where ϕ is a *low*-pass filter

and $\mathbf{1}$ the identity operator. Formally, we rewrite the formulation of the UP3 in what will be referred to later as the *filtered*

formulation :

$$\text{UP3}[s] = \text{C}_4[s] + \text{D}_4[s] - \text{D}_4 \cdot \phi[s] \tag{9}$$

220   Eq.8 then becomes

$$\gamma(\theta) = \frac{W}{\Delta z}\Re(\hat{\mathcal{D}}(\theta))(1 - \hat{\phi}(\theta)) \tag{10}$$

Note that the original formulation of Juricke et al. (2020b) makes use of a more general filtering, namely

$$\Phi_\alpha = \mathbf{1} - \alpha\phi \tag{11}$$

where $\alpha > 0$ is a parameter. α allows to control the strength of the filtering, $\alpha = 0$ corresponding to a no-filter situation while

225    $\alpha = 1$ corresponds to a fully active filter. The latter corresponds to our filtered formulation. The interesting aspect of the original

formulation lies in the use of a value $\alpha > 1$. Such a regime could be interesting for the tracer equation, but since it comes with

several issues, we restrict ourselves in this paper to the simpler case $\alpha = 1$.

For our practical analysis, we introduce the three-points filter $\phi_3[s]_j = (s_{j+1} + 2s_j + s_{j-1})/4$. If any low-pass filter can be

chosen in theory, we have used this one for several reasons : it is the same as the one used in Juricke et al. (2020a); its simplicity

230   allows for straightforward theoretical developments; and finally its limited stencil size does not increase the computational cost

of the scheme too much, which may be desirable if the model is to be run over long time periods. This choice is further

discussed in Sec.4. Its spectral representation writes $\hat{\phi}_3(\theta) = (1 + \cos\theta)/2$. This leads to the following damping coefficient for

the filtered UP3 scheme (hereafter UP3-F) :

$$\gamma^{\text{UP3-F}}(\theta) = \frac{W}{3\Delta z}(1 - \cos\theta)^2(1 - \frac{1}{2}(1 + \cos\theta)) = \frac{W}{6\Delta z}(1 - \cos\theta)^3 \tag{12}$$

235   The relative damping for the filtered formulation $\Gamma_{\text{UP3-F}} = \gamma^{\text{UP3-F}}/\gamma^{\text{phy}}$ becomes therefore

$$\Gamma_{\text{UP3-F}} = \frac{P_e}{12}(1 - \cos\theta)^2 \tag{13}$$

Considering again a wavelength of 10 grid points and using the same numerical values as before, we have $\Gamma_{\text{UP3-F}} \approx 0.3 - 30$.

The damping at large scales as been reduced by one order of magnitude, and for the range of P_e of concern, the numerical

dissipation can be now as high as or lower than the physical one. Numerical diffusion will now exceed physical diffusion

240   were the vertical grid spacing is the coarsest, that is at depth. Indeed, $\Delta z = 100$ m is reached at 1000 m depth in our model,

and at this depth, tracer profiles are already smooth enough so that their second or higher order derivatives are almost zero

and diffusion, even numerical, is not really active at all. We therefore consider those values as being *a priori* acceptable. The

scheme finally used for vertical tracer equation will be referred to as the QKE-F scheme. The flux evaluation in Eq.4 is thus

finally modified as such :

245    $$F^{\text{QKE}} = (1 - (2n_c)^2)F^{\text{UP3-F}} + (2n_c)^2 F^{\text{UP1}} \tag{14}$$

3   Results

A simplified one-dimensional framework such as the one presented above is essential for gaining insight into the actual be-
haviour of a numerical method (Griffies et al., 2000). However, this theoretical development relies on limiting assumptions
intended to maintain analytical simplicity and, therefore, cannot be the only basis for designing practical advection schemes.
250   This section provides a real case study for the validation of the method, illustrated as a way to improve water masses representa-
tion in simulations of the SEA Seas using the Symphonie model. We start by discussing briefly the way spurious mixing will be
diagnosed in the model. A presentation of the results obtained from simulations with and without filtering is then presented and
compared to their counterparts without tidal mixing. Doing so, we illustrate clearly the negative impact of numerical mixing
and the improvement offered by our method in terms of representations of the water masses.

3.1   Diagnosing numerical mixing

Several frameworks for diagnosing numerical mixing have been proposed in the literature (e.g. Griffies et al., 2000; Lee et al.,
2002; Burchard and Rennau, 2008; Gibson et al., 2017), each with their own strengths and weaknesses. These frameworks are
however designed for academical inter-comparisons of numerical methods for which the analytical computation of diffusion is
not always possible, such as for non-linear schemes. As we focus here on the practical application of the method, we choose
260   here to proceed in a similar way as in Marchesiello et al. (2009) and to use anomalies in tracer fields (especially salinity, which
exhibits a complex vertical structure with local maxima and minima) as a proxy for spurious mixing.

Several datasets are used for validation, namely GLORYS reanalysis, ARGO profiles (Wong et al., 2020) and OSTIA SST
gridded products (Donlon et al., 2012). Details on the data and processing are given when first used.

3.2   Effect of the QUICKEST scheme without the filtered formulation

265   To explicitly illustrate the excessive diffusivity of the QKE scheme on the vertical when used along with tidal forcing as well
as the improvements brought by the filtered formulation, two reference simulations are first run with the filtered formulation
turned off ($\alpha = 0$, equivalent to using QKE in its simplest formulation): one with (T0) and one without tides (NT0).

3.2.1   Salinity and temperatures profiles

Comparison to GLORYS data is first carried out to assess the ability of the model to maintain the salinity field in the South
270   China Sea (SCS) over time (Fig.2). Considering the higher availability of ARGO floats in this basin (see Appendix A), it is
assumed that GLORYS data offer a good and easy-to-handle reference for the state of the ocean. To allow for a more compact
representation, the focus is laid on the $50 - 350$ m layer, were differences between simulations are the most salient. Simply
using the QKE scheme leads to an expected unrealistical erosion of the salinity maxima observed in Pacific waters in the
northern South China Sea over the course of the first months of simulation in the T0 simulation (forced with tides). The salinity
275   maximum of about 34.7 psu in the $100 - 200$ m layer inherited from the initialization with GLORYS and visible at the very
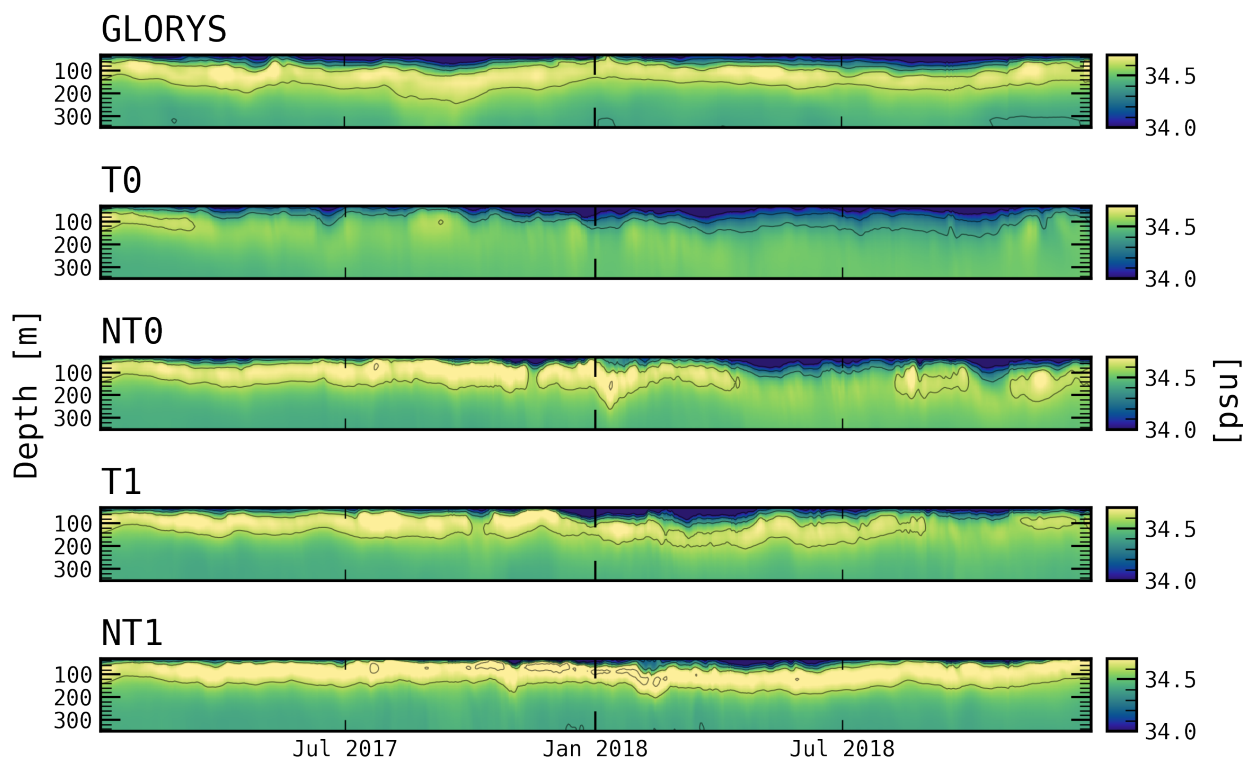
Figure 2. Temporal evolution of a salinity profile ($50 - 350$ m) averaged over a $1°$ box in the South China Sea, at location P in Fig.1, in both GLORYS reanalysis and several Symphonie simulations. Contours are plotted every 0.2 psu

beginning of the simulation disappears to form a nearly isohaline profile below 100 m after a few months, with salinity below 34.6 psu. On the other hand, NT0 simulation is able to maintain the salinity peak, comparable to GLORYS reference.

Secondly, comparison of mean temperature and salinity profiles to ARGO data available in the region over the period of interest are carried out (Fig.3). For each simulation, a dataset of simulation profiles collocated to the ARGO profiles in the

280 zone - in terms both of space and time - is built. The profiles are split in several clusters corresponding roughly to the major basins sampled in the zone (see Fig.A1). They are then interpolated on the same grid over the first 500 m and a mean profile is finally computed for each simulation and each cluster for comparison. The focus is laid on the first 500 m where the temperature and salinity profiles exhibit their richest spatial structure, and as the ITF is also mostly located in this layer. In Fig.3, we display only profiles in the Molucca Sea for clarity (see Fig.1), since the results in this basin are representative of the overall trend.

285 However, the same analysis has been conducted in each basin of the Southeast Asia region and will be discussed later on in the light of other simulations.

As suggested by the drift observed for the South China Sea profile, the mean salinity between 100 m and 300 m in T0 (light grey line, panel (b)) is strongly underestimated with respect to ARGO data, with a bias around -0.15 psu at 125 m depth. The
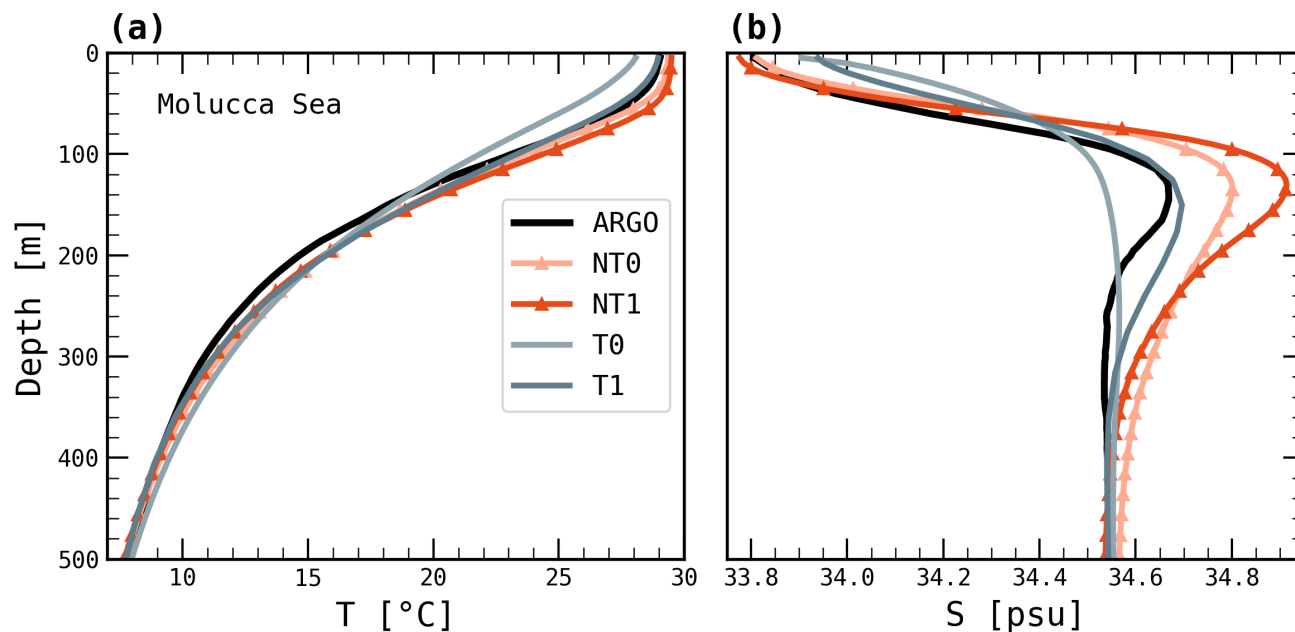
Figure 3. Mean (a) temperature and (b) salinity profiles over the first 500 m in the Molucca Sea for simulations T0, NT0, T1, NT1 and ARGO profiles.

salinity maxima found between $100 - 200$ m in observations and characteristic from Pacific Waters has completely disappeared.

290 In panel (a), the temperature gradient in the 10-300 m is also slightly less steep, and the temperature in the upper 100 m in the T0 simulation also displays a negative bias of up to $2°$ C at the surface.

Conversely, the salinity bias in the thermocline for the simulation without tides NT0 (light orange) with respect to observations is positive, and the salinity maximum at 120 m is overestimated by about $0.1 - 0.2$ psu. Differences in the temperature profiles are less obvious than for T0, even though a slight positive bias can be observed at the surface. Those differences result

295 from an underestimation of physical mixing in NT0, that results in a misrepresentation of water masses in the region : as tides and the associated internal waves field are not taken into account, highly saline waters coming from the Pacific are not well mixed, which biases the model output.

3.2.2 Comparison to satellite sea surface temperature data

Finally, the *Operational Sea Surface Temperature and Sea Ice Analysis* (OSTIA) Level 4 Sea Surface Temperature daily

300 product is also used for comparison. This product has been obtained by mapping existing observations from several instruments on a regular 1/20° grid with optimal interpolation, and daily averaging results. Its resolution is therefore comparable in both space and time to our simulations output. As with every highly processed product, care has to be taken not to put excessive confidence in such data, since biases can exist, especially on small coastal features (see e.g. To Duy et al., 2022). Large scale

trends are however well captured by OSTIA, and comparison is therefore carried out by computing the mean bias over the
entire simulation period between simulations and OSTIA. Results are displayed in Fig.4. Since errors in the Pacific and Indian
Ocean are more likely caused by open-boundaries forcing, the focus is laid on the interior basins, where most of the differences
are concentrated.

A cold negative bias of up to $2°$ C spans the entire Indonesian Seas in simulation T0 (bottom left panel), especially marked
over the Savu Sea, the Dewakang Sill as well as along the islands south of the Sulu Sea (see Fig.1, top left panel for details
on the locations). Conversely, NT0 displays a warm positive large scale SST bias of up to $1°$ C with respect to the reference
dataset, with maximum positive biases at the same locations where maximum negative biases are observed in T0. Both locations
are known for being strong internal waves generation sites (Apel et al., 1985; Nagai and Hibiya, 2020) and exhibiting strong
SST variations due to internal tides activity (Ray and Susanto, 2016). This confirms the conclusions obtained by examining
salinity profiles : the (numerical) mixing is too strong in the presence of tides in T0, bringing colder water from below to the
surface, thus cooling the simulated SST; conversely, the (physical) mixing is overly too weak in NT0 due to the absence of
tides-induced mixing, therefore preventing warmer surface waters to be mixed with colder waters at depth.

3.3   Effect of the QKE-F scheme

To assess the effect of the filtered formulation, two new simulations are run with the filtered formulation turned on ($\alpha = 1$) :
one including tides (T1), along with its counterpart without tides (NT1). Differences with T0 and NT0 simulations on the same
diagnostics as described above are presented in the following section.

3.3.1   Salinity and temperature profiles

Looking at the evolution of the salinity profile in the South China Sea in Fig.2, the salinity maximum in T1 has been re-
stored with respect to T0, and the evolution of the profile over the course of the T1 simulation lies much closer to GLORYS
reference dataset, but also to NT0. This implies first that tides do not play a significant role in the (trans)formation of water
masses at this location; and second that the difference between T0 and NT0 is caused by the excessive numerical diffusion
occurring when tides are added. Concerning simulations without tides, the salinity evolution for NT1 depicted in Fig.2 is
also qualitatively better than for NT0, implying that there is still spurious mixing happening in NT0 in the South China Sea.
This is to be expected, since even simulations without tides can display transient relatively high vertical velocities (on the order
of a few tens of meter per day) and therefore spurious mixing, especially at eddy-resolving resolution (see Megann et al., 2022).

Results in terms of mean profiles in the Molucca Sea are also reported in Fig.3 for T1 and NT1. T1 performs better than T0
and overall follows the ARGO reference more closely: the negative $\sim 1°C$ temperature bias at the surface in T0 and the nega-
tive ~ 0.1 psu salinity bias in thermocline waters disappears in T1. The implemented filtered formulation has thus effectively
reduced the spurious mixing developing in the context of strong tidal forcing down to acceptable levels. The picture is different
for simulations without tides, NT0 and NT1. In terms of salinity, NT1 performs worse than the other simulations, even when
compared to NT0, with an increased positive bias of 0.3 psu at 120 m depth, against ~ -0.15, 0.01 and $+0.2$ psu for T0, T1
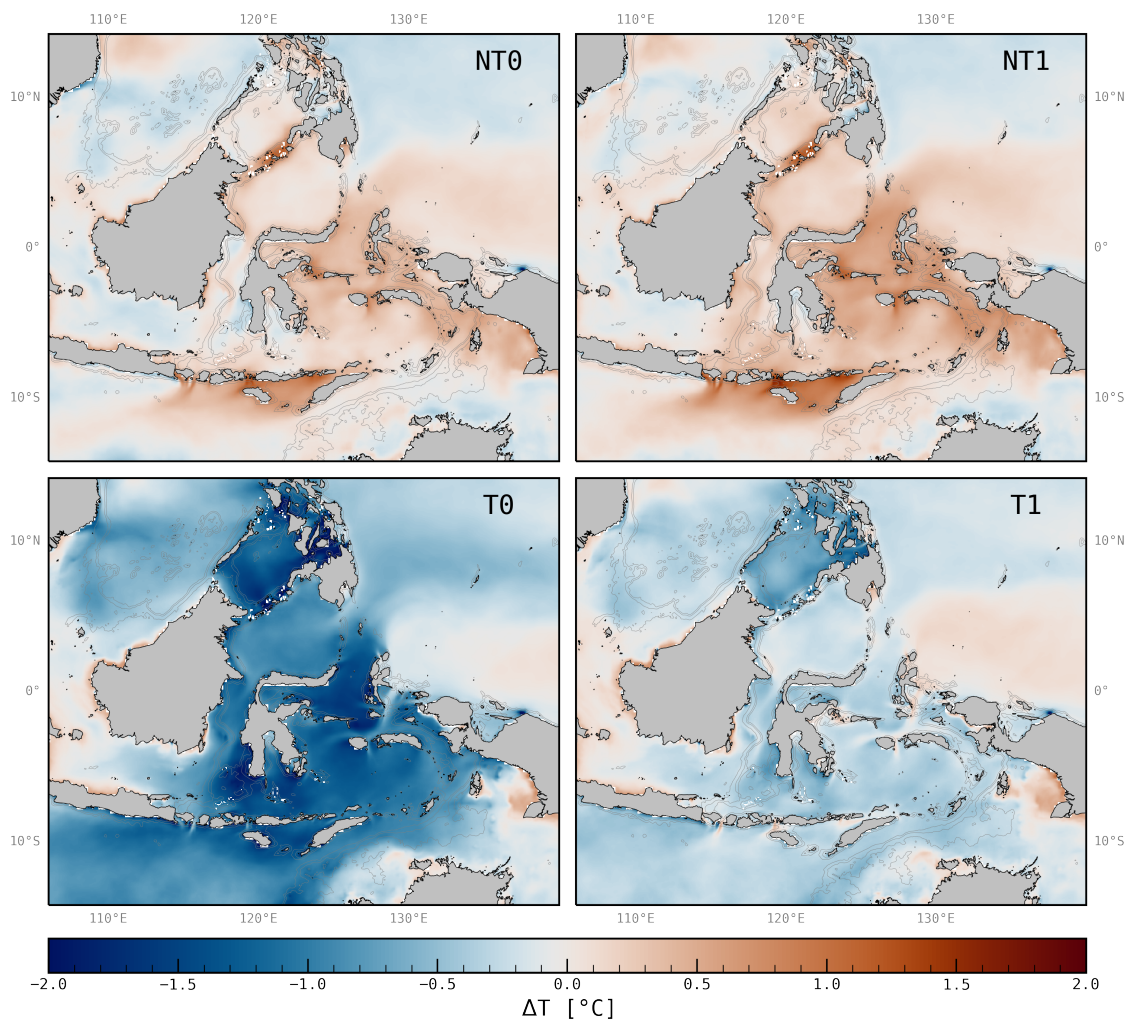
Figure 4. Mean SST bias with respect to the OSTIA dataset ($\Delta T = SST(SYMPHONIE) - SST(OSTIA)$) for simulations T0, NT0, T1, NT1, computed over years 2017-2018. Positive values correspond to an overestimation of model output. The 100, 500 and 1000 m isobaths are also displayed (fine grey lines).
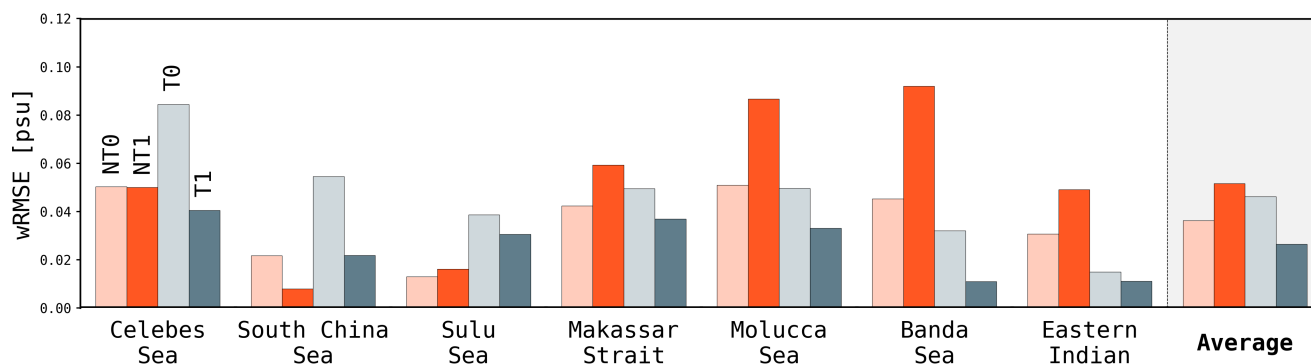
Figure 5. wRMSE between mean salinity profiles for simulations T0, NT0, T1 and NT1 (for each zone, left to right bar respectively) for each of the ARGO subgroups. The last column displays the metric averaged over all zones.

and NT0, respectively. Tidal mixing indeed takes place in the Molucca Sea and drives the transformation of water masses (see e.g. Koch-Larrouy et al., 2007). The weaker bias from NT0 in comparison to NT1 is due to the fact that the spurious numerical mixing in NT0 is higher than in NT1, thus making up for a part of the lacking tides-driven physical mixing. This shows again

340   that even without tidal motions, a sensible amount of spurious mixing is still taking place in NT0. With fewer numerical mixing in NT1, the underestimation of physical mixing is made all the more salient, which increases the bias and overestimation of the salinity maximum in NT1 compared to all other simulations.

To provide a more synoptic picture of the effectiveness of the filtered scheme, we carry out a comparison of all simulations in

345   every basins of the zone. For a given simulation and zone we define an integrated metric of the performances in terms of water mass representation, to allow for a more direct and quantitative comparison than through visual inspection solely. In each zone, a weighted root mean squared error between each simulation and the reference ARGO profile is computed for the mean profile. As the variations in the upper layers are of higher magnitude than in the deeper layers, we weight accordingly the error by reducing the importance given to values where the variation is higher. The resulting metric is referred to as wRMSE, and more

350   details on its computation are provided in Appendix B. Here, the focus is laid on salinity as the effect of numerical mixing on these profiles is more obvious (see Fig.3), but the same computations have been made for the temperature with similar results (not shown).

Results of wRMSE computations on the salinity profiles for all simulations output for the various zones are displayed in Fig.5. Values lie between 0.01 and 0.1 psu. Comparing the integrated wRMSE scores in the Molucca Sea with the profiles

355   provided in Fig.3, our metric indeed quantitatively translates the qualitative observation that in this zone, T1 performs the best and NT1 performs the worst while T0 and NT0 are in-between.

Comparison between T0 and T1 shows systematically improved wRMSE in T1 in each zone, with much greater difference in zones directly neighbouring the Pacific (Celebes and SCS) : around 0.04 psu in the SCS, and only 0.01 psu in the Makassar

Strait. Except for the Sulu Sea, T1 also improves the results with respect to NT0. The difference is however usually smaller,
except in the exit basins (Bandas Sea and Indian Ocean) where the difference is sensible : wRMSE for T1 is $0.02 - 0.03$ psu
lower than for NT0 in those basins.

Performances of NT1 relatively to the other simulations greatly vary with the location of the zone along the course of the ITF,
that is from the Pacific to the Indian Ocean through the Celebes Sea, the Makassar Strait and the Banda Sea. NT1 performances
in the basins of the downwind of the ITF (Makassar strait, Banda Sea, Eastern Indian), where tidal mixing plays an important
role, are by far the worse : around 0.08 psu between T1 and NT1 in the Banda Sea, and 0.4 psu in the Eastern Indian Ocean.
However, in the SCS, NT1 performs better than all the others. The reason behind this best performance of NT1 in the SCS is
still unclear, and might be linked to some other factors, including the details of the configuration. Similarly, best performances
of NT0 and NT1 in the Sulu Sea are somewhat unexpected, and this feature will be discussed below in the light of comparison
to satellite data.

Figure 5 also displays the average over all basins of the wRMSE for each simulation, with a uniform weighting for each zone.
Using this SEA average metric, T1 performs overall best than the others, while NT1 performs the worst. The wRMSE could
have also been computed for each simulation by comparing the average profile over all available profiles over the ensemble of
the basins to the same profile obtained with ARGO data; however, this would have led the results to be strongly biased towards
zones with the most data - in particular the SCS. The underlying assumptions here are that the amount of profiles available in
each basin is sufficiently high so that comparisons can be trusted equally and that the correct representation of each basin is of
equal importance.

3.3.2 Comparison to satellite data

Finally, we also validate the filtered formulation against OSTIA SST data. The mean SST in T1 is greatly improved compared
to T0 (Fig. 4) : the negative bias over the SEA Seas is lower by one order of magnitude, and is now on the same order as errors
made in the Pacific Oceans, that is a few tenths of Celsius. This further confirms that our filtered formulation improves the
representation of sea surface temperature in the context of strong tidal mixing.

The SST in the Sulu and Philippine Seas however still exhibits a strong cold bias in T1, up to $1°$ C locally, weaker than in
T0 but stronger than in NT0 and NT1. The explanation for this behaviour has been traced to an overestimation of tidal currents
in this basin due to an imperfect representation of the numerous islands and channels around it, especially in the Philippine
Archipelago. Manual edition of the land mask to improve the representation of the zone had been carried out, with sensible
improvements when compared to the first tests, but tidal amplitudes remain too strong there. This results in an overestimation
of the naturally strong internal tide activity in the area (Apel et al., 1985), which can only dissipate within the basin due to
its enclosed nature, thus increasing the mixing. As a result, tidal vertical mixing is overestimated there, leading to spurious
damping of the profiles and SST underestimation. This is slightly "corrected" when the filtering is activated (T1 vs. T0), and
even greatly "corrected" when tides are turned off (NT0 vs. T0, NT1 vs. T1).

Besides, the previously described positive SST biases in NT0 simulation shown in Fig.4 are slightly increased in NT1,
indicative of weaker vertical mixing in the interior basins - namely along the Sulu islands chain, the Savu Sea as well as around

the Lifamatola Passage. Those results are again related to a reduction in spurious numerical mixing induced by the filtered scheme in NT1 compared to NT0, that partly compensates for the lack of tides-induced *physical* mixing in NT0, and makes it
395 more salient in NT1.

4 Discussion

Comparisons of a set of four sensitivity simulations, with and without tides and with and without activation of the filtered formulation, with available ARGO temperature and salinity profiles and OSTIA sea surface temperature data showed that the filtered formulation brings numerical diffusion down to a level allowing for a correct representation of water masses in our
400 simulations over the Southeast Asia Seas, even under strong tidal motions. Through a comparison with the highly diffusive QKE scheme in its raw formulation and with simulations without tides, we have highlighted the spurious numerical effect that tides can have, while stressing their physical importance in transforming the water masses. Nevertheless, some questions remain open.

405   First of all, the choice of the filter in Eq.9 is still weakly constrained. For simplicity and to stick to the original formulation proposed in Juricke et al. (2020a), we chose a simple three-points average, and achieved satisfying results. We can however wonder if another filter could be better suited. Basically, we would like the filter to have its cutoff wavenumber θ_c to be equal to the lowest wavenumber that the advective part $\mathcal{A}$ of the scheme can effectively resolve, in order to ensure that all the noise is effectively dissipated while the well represented scales are protected from spurious damping. It is debatable whether such
410 a clear separation between noise and physics can be drawn, as it will always rely on the definition of an arbitrary condition (such as in Kent et al. (2014) or Winther et al. (2007)). Nonetheless, following computations made by Winther et al. (2007), the minimum wavelength accurately represented by a fourth-order central differencing scheme is approximately $N = 4.4$ grid points. Defining the cutoff wavenumber of a filter in a standard way as the wavenumber θ_c at which $\hat{\phi}(\theta_c) \approx 1/\sqrt{2}$ (as is commonly done in signal processing), we have, for ϕ_3, a cutoff wavelength $N_c = 2\pi/\theta_c \approx 5.5$ grid points. That is, the cutoff
415 of our filter is slightly greater than the actual accuracy of the advective part, ensuring that all the noisy wavelengths are effectively filtered out. On the contrary, for a higher order filter of the form

$$\phi_{5*}[s]_j = s_i + \frac{1}{16}(-s_{j+2} + 4s_{j+1} - 6s_j + 4s_{j-1} - s_{j-2}) \tag{15}$$

which transfer function writes

$$\hat{\phi}_{5*}[k] = 1 - \frac{1}{4}(1 - \cos\theta)^2 \tag{16}$$

420 the cutoff wavelength is approximately $N_c = 3.8$ grid points. This is slightly lower than, but comparable to the above defined accuracy of 4.4 grid points. Considering the rather approximative aspect of these computations, we could thus still consider this other filter valid. However, the stencil required to compute such a filter is wider than for ϕ_3 (five grid points for ϕ_{5*} vs. three for ϕ_3), increasing the computational cost and numerical complexity, especially when computing boundary conditions.

The ϕ_3 filter is therefore a rather good compromise, even though other filters have not been tested in the scope of this study; but we cannot exclude that other, possibly non-linear, filters might be better suited in other situations.

Second, the choice of this filter leads to an interesting result that raises the question of choosing the UP3-F vs. UP5 scheme. Looking at Eq.12, it turns out that except for the multiplying coefficient $1/6$, the damping coefficient for the filtered formulation of the UP3 scheme $\gamma^{\text{UP3-F}}$ is equal to the damping coefficient for a fifth-order upwind biased scheme (UP5), namely :

$$\gamma^{\text{UP5}} = \frac{8W}{60\Delta z}(1 - \cos\theta)^3 \tag{17}$$

See for instance Soufflet et al. (2016) for the derivation of γ^{UP5}. Basically, this means that the diffusive part of our filtered formulation of the UP3 scheme is equivalent to the diffusive part of a UP5 scheme, that is, a trilaplacian operator. This result is similar to the one described in Juricke et al. (2020a), where the filtered harmonic diffusion boils down to a biharmonic diffusion when α is set to 1. We can therefore wonder if it would not be preferable to use directly a UP5 scheme instead of a filtered scheme, since it should achieve overall a higher order of accuracy. The answer is : most likely. There are however several aspects to be considered here. Though it might be negligible in comparison to the cost of computing other components of the model, the computational cost of the UP5 scheme is slightly higher than the one of the UP3-F formulation, since in UP3-F, only the dissipative component uses a larger stencil. But more importantly, the base scheme actually used in Symphonie is QKE. It should be possible to build a scheme similar to QKE on the UP5, but some additional work would be necessary and has not been carried out in the scope of this study. Moreover, we should keep in mind that Eq.12 is derived from Eq.11 in the particular case where $\alpha = 1$. Though other values of α have not been discussed here, formulations where $\alpha \neq 1$ (and especially $\alpha > 1$) are believed to have some potential in further reducing numerical mixing, and will be investigated in further studies. Taking a step back, this result nevertheless suggests that a bilaplacian diffusion is still too dissipative, while a trilaplacian operator could achieve better results, at least in the situation studied here. This result might be of importance for existing and upcoming numerical cores, since increasing the order of the diffusion might be a rather affordable option.

5 Conclusions

In this paper, we have presented a new way of formulating the vertical advection scheme in the Symphonie model that builds on a previously available scheme and aims at making its diffusive component more scale-selective, thus reducing spurious numerical mixing, especially in a context where tides are explicitly resolved. This method has then been validated in a regional model of the South-East Asian Seas, known to be the generation site of strong internal tides that dissipate in the semi-enclosed basins of the region. By running simulations with and without tides and with the new formulation turned on and off, we have shown that the numerical diffusion is reduced enough so that the model is able to satisfyingly represent observed water masses throughout the various seas. At the same time, we provided a clear illustration of the spurious effect that tides can have in a fixed coordinates model, even in a regional model forced at its lateral boundaries. This, although previously known, has, to our

knowledge, not been as explicitly reported before as in this study. We believe this to be helpful for the growing community of model users not necessarily well-versed in numerical modelling. We advocate for a better recognition of spurious vertical mixing, especially in the context of long simulations run with explicit tidal forcing, and of the importance of choosing carefully the proper numerical methods.

460

Some aspects however still remain uncertain and call for further work.

First, the method used here to identify the spurious effect of numerical mixing on tracer fields is rather coarse and qualitative. It builds on the fact that observational data are more and more available, and has been chosen to provide a clear illustration. A more thorough quantification might be however of valuable insight to further disentangle the numerical from the physical

465 mixing. The choice of such a method is however not straightforward, each exhibiting their own strengths and limitations (see e.g. Banerjee et al., 2023).

Focusing more specifically on the advection scheme presented in this paper, the choice of the filter to be used is still weakly constrained, thus calling for further investigations. A comparison to other standard schemes, though out of scope of this paper, could also be carried out. At the same time, owing to the variety of approaches in ocean modelling, defining a universal

470 advection scheme is still difficult, if not delusional. The choice of a method indeed always involves a compromise between the quality of the solution - which is by itself oddly defined and depends on the problem itself - algorithmic complexity - which might prevent the use of a particular method on a particular set of coordinates for example - and computational cost. This is also the reason why modern dynamical cores usually offer users the possibility to choose between several advection schemes, depending on their needs. As stated in Gerdes et al. (1991), as long as the theoretical basis of a scheme is justified, the realism

475 of the resulting simulation when compared to observations is the only way to define what's "best". The method presented here should therefore be seen more as a new element in a much wider toolbox of solutions aiming at reducing numerical errors in ocean models rather than a definitive solution to the issue of spurious vertical mixing.

Code and data availability. The code to plot the figures can be downloaded on the following github repository : https://github.com/Clapinet/ SpuriousMixingSEA. Preprocessed Symphonie and ARGO data as well as the source code of the model (v312) are accessible via the follow-

480 ing url https://doi.org/10.5281/zenodo.10715502. Considering their size, raw 3D model outputs are not stored publicly, but are available upon request. More information on the Symphonie model can be found on the SIROCCO gorup website : https://sirocco.obs-mip.fr/ .GLORYS data are accessible via https://doi.org/10.48670/moi-00021. OSTIA data are accessible via https://doi.org/10.48670/moi-00168. ARGO data have been downloaded from https://dataselection.euro-argo.eu/.

Appendix A: Location and amount of ARGO floats in the region

485 Figure A1 shows the distribution of ARGO profiles in the region. In the interior basins, all data iver the period of interest have been selected. In the Eastern Indian Ocean, considering the large amount of data available, only a subset of profiles sampling the exit of the ITF has been selected.
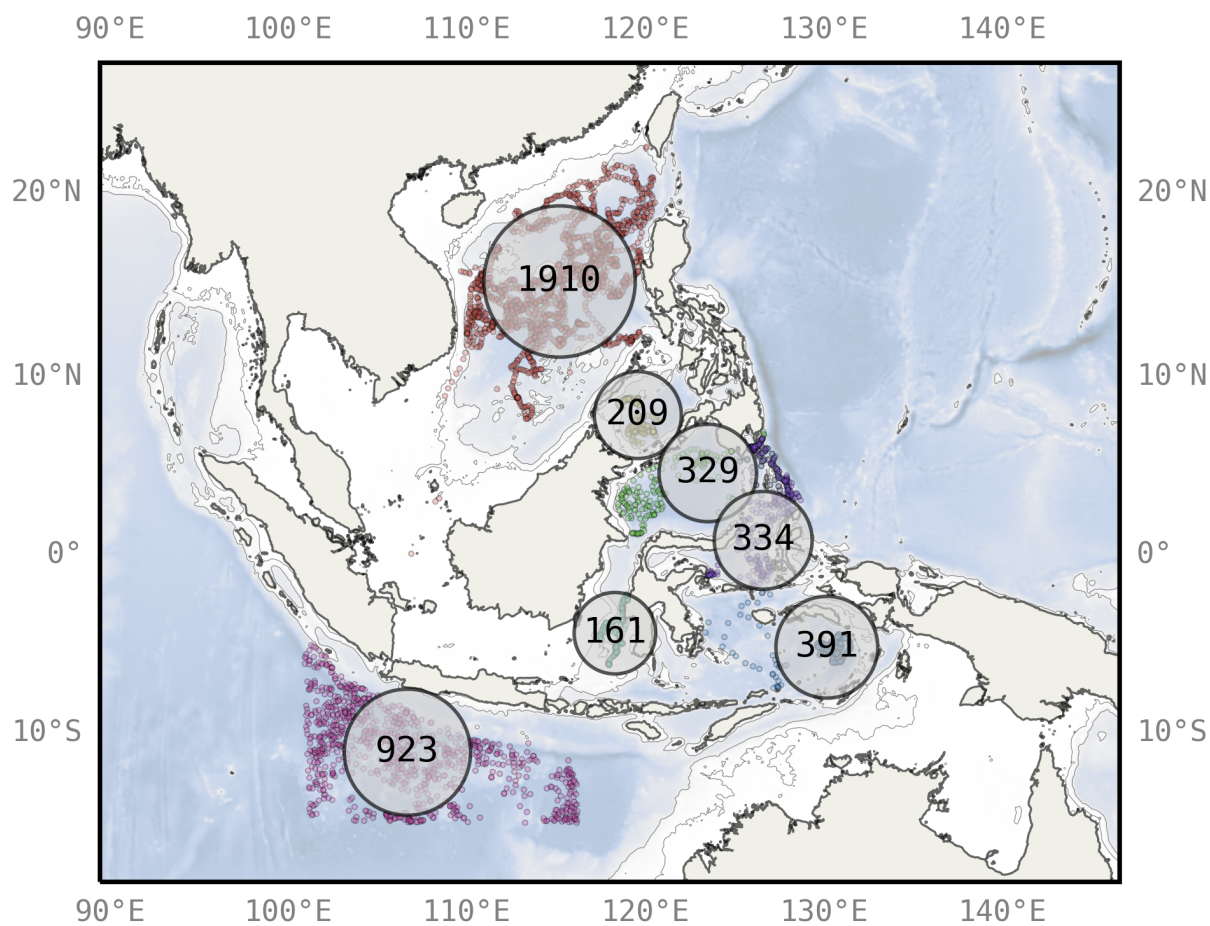
Figure A1. Locations and number of ARGO floats in each basin of the region over the 2017-2018 period. Individual profiles are represented by colored points, while the numbers in larger circles indicate the number of profiles in each basin over the period of interest. Basin names are provided in Fig.1.

Appendix B: Metric for profiles comparison

For each zone b and each depth level k, the standard deviation σ_k^b of the ARGO data in the zone is computed. The RMSE is
490 then computed by weighting each depth level k by $w_k^b = (1/\sigma_k^b)/(\sum_j 1/\sigma_j^b)$. This modified metric is referred to as *weighted* RMSE (wRMSE), and writes as follow :

$$\text{wRMSE}[X_b^s, X_b^r] = \sqrt{\sum_k w_k^b (X_b^s[k] - X_b^r[k])^2} \tag{B1}$$

where $\{X_b^s[k]\}_k$ is a profile sampled at m depth levels $\{k\}$ in zone b for a given simulation s and $\{X_b^r[k]\}_k$ a reference profile (e.g. ARGO). Choosing weights as such means that less importance, compared to classical RMSE, is given to depth
495 levels where the *observed* variations quantified by σ_k are greater; and conversely more importance is given to levels with less variation between the various profiles.

Author contributions. AG, PM and MH contributed to the design of the study. MH, JP and PM designed the configuration. MH ran the simulations. PM implemented the methods. AG analysed the simulations, carried out the theoretical work and wrote the manuscript with contributions from all coauthors.

500 *Competing interests.* The authors declare no competing interests.

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