

I cannot remember when I have had more helpful reviews than those given by the two people who carefully went through my paper. The fact that they both noted some of the same issues made it certain that I had to fix a few things. I describe the planned changes in detail after each of the related reviewer comments which are italicized.

I did have to abandon my effort to make the paper totally analytic in that I had to do numerical integrations to show the effect of non-linear temperature profiles. Since this took considerable effort, I may not have addressed all the other comments as well as I should. The most important changes are:

- 1) To try to make the model idea better I plan to add a description of viscous bending to the introduction and in section 3.3 on the thin plate approximation of the flexural wavelength.
- 2) To illustrate the asymptotic behavior of the analytic model I added a panel to Figure 5 showing predicted internal bending moments versus the log of the e-folding length for viscosity variations.
- 3) I analyzed the errors in the analytic approximation by carrying out numerical integration of the stress differences for the assumed ice flow law. The difference between the full and approximate solutions for the internal bending moment depend on the assumed flow law parameters and surface temperature, but are less than 3% for the most extreme cases illustrated in the figures.
- 4) A new section on “Effects of nonlinear temperature variation with depth” will include (1) a figure showing temperatures from 3 boreholes on the Ross Ice Shelf and 1 from the Amery Ice shelf, and (2) calculation of steady-state temperature profiles for a range of rates of surface accretion and basal accretion or melting (after Robin, 1955) and results of numerical analysis of internal bending moments for those temperature profiles. The Figures are given below.

Reviewer1

This manuscript presents a novel analysis of flexure at the terminus of a freely floating ice shelf. It addresses observations of upward flexure of the Ross Ice shelf near Roosevelt island.

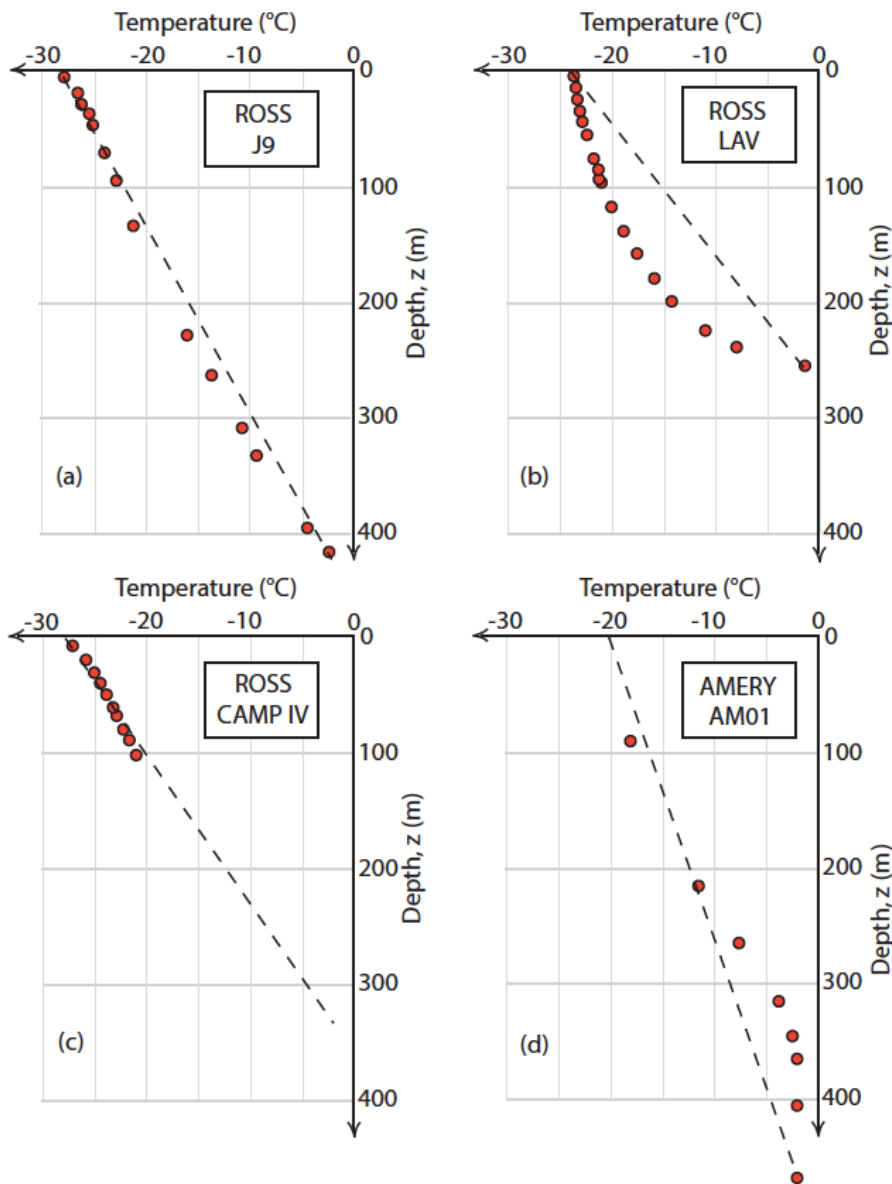
I plan to note that the upward flexure is seen not only near Roosevelt Island but along ~74% of the front of Ross Ice Shelf (Becker et al., 2021).

Whereas these were previously explained in terms of an eroded ice bench, this manuscript shows that a vertical variation in ice viscosity, arising from a linear variation in ice temperature and acting on a vertically uniform rate of extension, gives rise to a bending moment that can explain the observed flexure. This is an appealing hypothesis because ice benches are not observed at Ross (using the same sensor that has observed them elsewhere). The argument is supported by a clear and relatively simple mathematical theory that is consistent with a classical analysis of ice shelves (Weertman 1957). The manuscript is well written and well illustrated, easy to follow, and makes a novel and significant contribution to our understanding of ice-shelf dynamics. It has important implications for our understanding of ice-shelf calving. It should be published with minor revisions.

I see no major problems with the manuscript as written. The author has cleverly applied insights from plate flexure derived in the context of tectonics to ice shelves. Amusingly, the author

highlights that his analysis was tee'd up by Reeh 1968, whose "mathematical troubles" are relieved by the simplifying assumption of a linear temperature variation through the ice shelf. This leads to a Taylor series expansion and truncation at leading order, making the moment integral tractable and unlocking a solution. This context prompts two relatively minor suggestions. The first is to better discuss and justify the linear temperature assumption, as this is crucial to progress. There are borehole measurements by Mike Craven et al (e.g., *J. Glaciology*, Vol. 55, No. 192, 2009) and likely others. Plotting their data in comparison to a linear fit might be nice.

Here is a figure and caption I plan to add to an added section 3.4 Effects of nonlinear temperature variation with depth:



New Figure 9. 4 sets of borehole temperature measurements that constrain the temperature profiles for parts of two Antarctic ice shelves. (a, b and c) are for the Ross ice shelf re-plotted from Taylor and MacAyeal (1979)

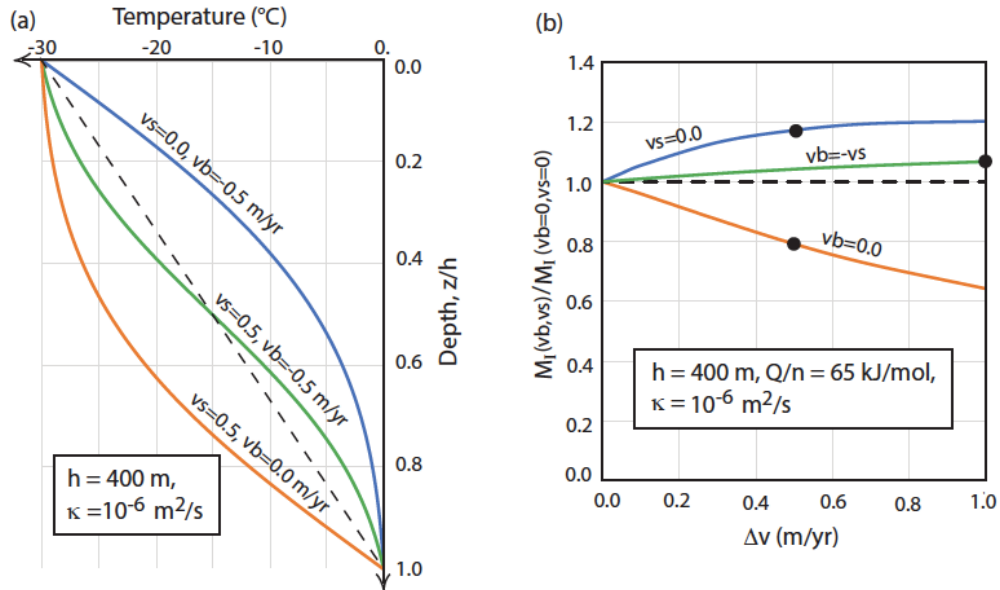
while (d) is from the Amery Ice Shelf re-plotted from Craven et al. (2009) and locations of the boreholes are given in those references.

In that new section I propose to discuss the effect of “Robin-type” temperature profiles on the calculated internal moments. To do that I also had to use a numerical integration of the stress differences predicted by the full flow law using the non-linear temperature profiles. I would add something like this:

Several effects, including accretion or melting of the surface or base of an ice shelf, can contribute to non-linearity of temperatures with depth and this will affect stresses and so internal moments. Following Robin (1955) we can estimate the effects of surface accretion and basal accretion or melting on temperatures in an ice shelf ice shelf. Pure shear thinning of the layer maintains a uniform shelf thickness, h , while the velocity of the surface is v_S and the velocity of the base is v_B . The surface is maintained at T_S and the base at T_B . The steady-state temperature T as a function of depth below the ice surface, z , can be written:

$$T(z^*) = T_S + (T_B - T_S) \left\{ \frac{\text{erf}(\xi z^*) - \text{erf}(-\xi z_{ref})}{\text{erf}[\xi(1-z_{ref})] - \text{erf}(-\xi z_{ref})} \right\} \quad (\text{to be added as equation 23})$$

where $z^* = z/h - z_{ref}$, $z_{ref} = \frac{v_S}{(v_S - v_B)}$, $\xi = \sqrt{\frac{(v_S - v_B)h}{2\kappa}}$ and κ is the thermal diffusivity, here taken to be $10^{-6} \text{ m}^2/\text{s}$. Resulting temperature profiles for a layer 400 m thick and several combinations of v_S and v_B are shown in the figure below (proposed new Figure 10). Equation (11) was then used with the calculated temperatures with depth to give the stress differences needed for numerical integration of equations 5 and 10 to get the predicted internal moment of the horizontal stress distribution (M_I).



New Figure 10. Effect of surface or basal accretion on ice shelf temperature profiles and internal bending moments. (a) Examples of three steady-state temperature profiles for the indicated values of surface and basal velocities (v_s and v_B) compared with a linear temperature profile for the same surface and base temperatures. (b) Numerically calculated normalized internal bending moments for a range of temperature profiles calculated with the indicated parameters. Black dots indicate the temperature profiles shown in part (a). Moments are divided by the moment for a linear temperature profile.

In describing this in the text I plan to note that freezing onto the base of an ice shelf should act to increase the amplitude of the internal moment and so the expected upward bending of the shelf edge. Accretion to the surface and melting of the base act to diminish the internal moment.

The second is to use the second-order term in the Taylor series as a means to estimate the truncation error in equation (14). My quick calculation gives a multiplicative factor of $\exp[(T'/T_s z)^2]$. Taking $z=h$ as an upper bound, this gives $\exp(\Delta T/T_s)^2 \sim \exp((30/240)^2) \sim 1.02$. So a maximum 2% error in viscosity due to Taylor expansion. This could be propagated through the calculation to obtain the error on M_I (but in fact the linear temperature assumption must be a larger source of error).

Rather than do that I used the full assumed flow law (Equation 11 in my text) and numerically integrate the resulting stress differences based on equation (12) to calculate the internal moment using equation (10). As noted above, the difference between the full and approximate solutions for the internal bending moment depend on the assumed flow law parameters and surface temperature, but are less than 3% for the most extreme cases illustrated in the figures. In going through this exercise, I realized that I had not updated the last version of the equation that I used to calculate the e-folding length for viscosity variations in equation (15). So now equation (15) becomes:

$$z_0 = \frac{nRT_s T_B}{Q \frac{dT}{dz}} \quad \text{or} \quad \frac{z_0}{h} = \frac{nRT_s T_B}{Q(T_B - T_s)} \quad (15)$$

Changing the “ T_s^2 ” term in that equation with “ $T_s T_B$ ” forces the log-linear approximation to pass equal the viscosity or stress difference values at the top and base of the layer as given by the full flow law. This significantly reduces the errors of the moment calculated analytically. I had done that in the cases illustrated in the submitted text (as can be seen in Figure 4(b)) but had not updated equation (15).

My third suggestion is to more carefully discuss the time-dependence of viscoelastic flexure. Although the details will vary between problems, the scaling with time/(Maxwell time) should not. How does this affect the comparison with the Ross ice shelf? What is the age of that edge? Is it fresh (i.e., age/Maxwell $\ll 1$)? This relates to the approximate of stresses as, close to the shelf edge, they will be modified with time since calving. In this regard, the bi-metallic strip analogy is somewhat misleading, as it is in mechanical equilibrium at a fixed temperature.

In response to this suggestion and a similar one made by reviewer 2 I plan to add a paragraph to sections 2 “Conceptual Model” describing viscous effects.

Also, in explaining viscous bending effects more thoroughly in section 3.3 on “Topographic variation...” I will replace the paragraph beginning on line 262 with:

Reeh (1968) and Olive et al. (2016) find that for a viscous or viscoelastic plate with a uniform viscosity η the wavelength of the flexure changes with time as:

$$\alpha(t)/\alpha(t = 0) \sim \left[\frac{\tau_M}{t} \right]^{1/4} \text{ and}$$

$$\alpha(t = 0) \sim \left[\frac{Eh^3}{\rho_w g (1 - \nu^2)} \right]^{1/4}. \quad (21)$$

where t is time, E is Young's Modulus and ν is Poisson's ratio and where $\tau_M = \frac{\text{Young's Modulus}}{\text{average viscosity}}$ is a measure of the Maxwell time of the layer. Combining equations 20 and 21 suggests that the amplitude should increase with time roughly as: $e_0^M(t) = e_0^M(0) \left[\frac{t}{\tau_M} \right]^{1/2}$. Eventually, as the flexure parameter approaches the layer thickness, the two-dimensional nature of the problem means that the thin-plate approximation is no longer valid. For layer of a few hundred meters thickness this should take about 1000 Maxwell times. For an average layer viscosity of a few times 10^{14} Pas and a layer thickness of 300 m this should take about 4 years. Reeh (1968) came to this conclusion and estimated that the long-term flexure parameter can be a bit smaller than the layer thickness. A more thorough study was done by Olive et al. (2016) who compare fully two-dimensional viscoelastic models of flexure to the thin plate solution and find the best fitting thin plate flexure parameter evolution to match the 2D results. They find that after many Maxwell times that the effective flexure parameter is smaller than the layer thickness. Thus, for figures 6 and 7 α is set to 250 m while the ice layer thickness is taken to be 400 m.

Finally, I plan to add a sentence or two to section 4 "Discussion and Conclusions" describing how IceSat II lidar observations analyzed by Sartore et al., (2024) show that after a calving event on part of the front of the Ross Ice Shelf that the moat and rampart takes several years to develop and grow. This could be explained in terms of the expected viscous change in the flexural wavelength and the resulting increase in bending deflections.

Broadly, I think the author should draw more attention to the assumptions made and the caveats and cautions that they introduce. This would not detract from the importance of the manuscript, but would better promote further research to build and test the ideas introduced here.

I tried to do this mainly with the new section on temperature profiles describe above.

Some detailed points, by line number in the manuscript:

[32] where ice shelf serves as an adjective, it should have a hyphen. E.g., ice-shelf edges

Will change!

[Fig 1] expand the figure caption to explain the lines in these figures. Improve the resolution to clarify that the hashing are ascending and descending track lines.

It is worth noting that Reviewer 2 also wanted a better explanation. I plan to add this to the figure caption: “The grey lines are estimated streamlines of ice flow while the red lines show both ascending and descending IceSat II track lines analyzed by Becker et al. (2021).”

[76--78] These two sentences say the same thing, which is confusing. Only one is needed.

I will delete the second sentence.

[98--99] The sentence starting with "Imagine" is important but the reader hasn't yet been adequately informed about why. Somewhere above (maybe the introduction) there should be a brief discussion of how visco-elastic bending is time dependent.

As noted above I will add a couple of sentences to the conceptual model and a revised paragraph in section 3.3.

[103] "To do this" grammatical issue here.

I plan to find a proper grammarian who can tell me what is wrong here.

[163] The result here appear to be positive but represents downward flexure (line 124 states that upward bending corresponds to positive total applied moments). Please check signs.

Correct, I left out the minus sign and will put it in. I also have to add a minus sign to equation (19).

[175] Spelling of MacAyeal.

Will fix.

[188] The assumption regarding stresses evaluated at large distance from the edge of the ice is somewhat sketchy so I think a bit more emphasis and discussion would be relevant here.

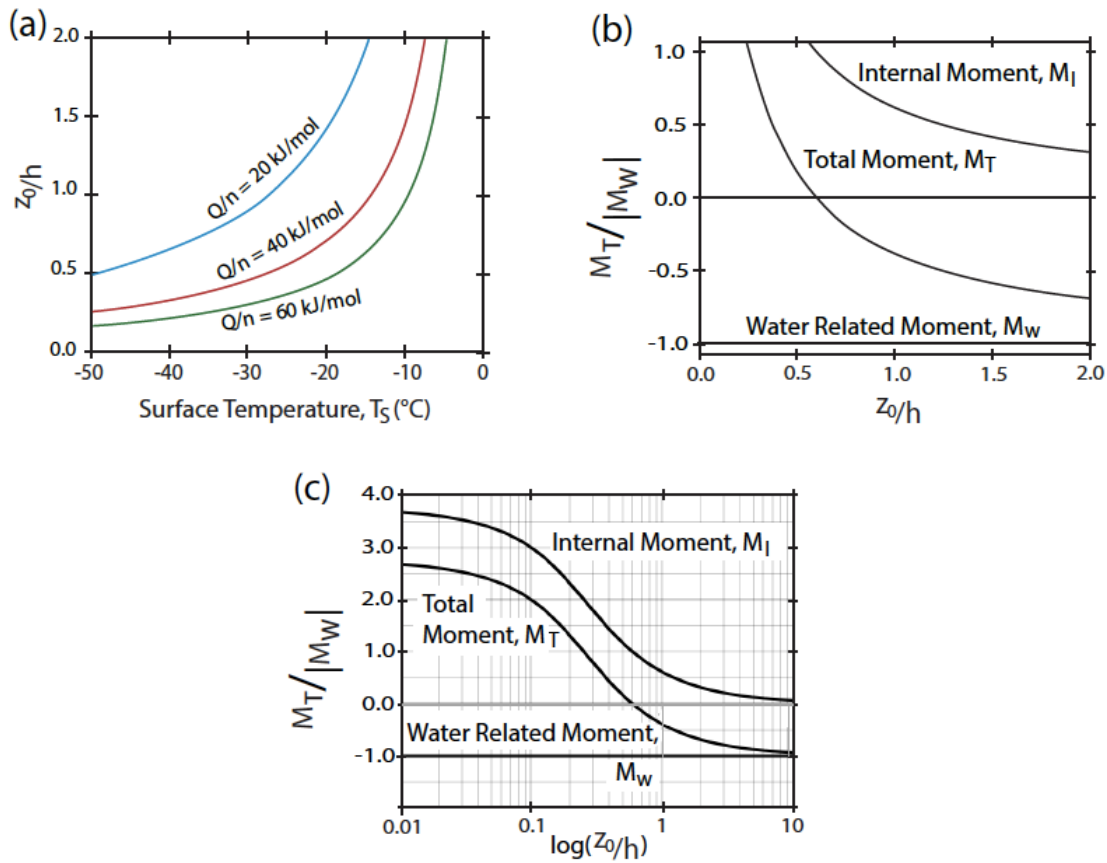
I will add reference to Weertman (1957) on this topic as suggested by reviewer 2.

[210] A reference here to Weertman 1957 or similar would be appropriate and helpful.

I will add this as well.

[Fig 5b] I think that a version of this plot with a logarithmic x axis (and an expanded domain and range) would be helpful in seeing the asymptotic behaviour of M_I at large and small z_0/h .

Good idea. In the text I will note that the analytic internal moment solution for small values of z_0 goes to -3.75 times the value of the water related moment. For large values of z_0 it goes to zero. Here is the new version of Figure (5) with an added plot (c):



New Figure 5c (to be added to present Figure 5) shows the variation of internal and total moments as functions of the logarithm of the e-folding scale if viscosity variations.

[294] "illustrates shows"

Will cut one.

[340] "places"

Will be "placed"

[throughout] mathematical notation should be italic but frequently appears as regular next.

Will change.

Added references:

Craven, M., Allison, I., Fricker, H.A., Warner, R.: Properties of a marine ice layer under the Amery Ice Shelf, Antarctica, J. Glaciol. 2009; 55: 717-728. doi:10.3189/00221430978947094

Sartore, N. B., Wagner, T. J. W., Siegfried, M. R., Pujara, N., and Zoet, L. K.: Calving of Ross Ice Shelf from wave erosion and hydrostatic stresses, EGUsphere [preprint], <https://doi.org/10.5194/egusphere-2024-571>, 2024.

Thomas, R. H. and MacAyeal, D. R.: Derived characteristics of the Ross Ice Shelf, Antarctica, *J. Glaciol.* 1982; 28:397-412. doi:10.3189/S0022143000005025