1 General

The manuscript "Improve iLOVECLIM (version 1.1) with a multi-layer snow model: surface mass balance evolution during the Last Interglacial" by Thi-Khanh-Dieu Hoang and Co-autors introduces the BErgen Snow Simulator (BESSI) as a potential coupling interface for ice sheets in the Earth system model iLOVECLIM. BESSI is envisioned to replace the previous iLOVECLIM interface, the less complex, empirical insolation-temperature melt equation (ITM). In contrast to ITM, BESSI is physics based and as such might be applicable for a larger range of background climates (e.g. Greenland Ice Sheet during the last interglacial under orbitally changing insolation; or the Antarctic Ice Sheet with only sporadicly occurring melt and precipitation dominating SMB variability) without the need to retune parameters. The authors first evaluate BESSI for the modern Greenland Ice Sheet using 1979-2021 climate forcing from the regional climate model MAR and comparing it to the respective MAR surface mass balance output. Then BESSI is forced with climate output from a iLOVECLIM simulation of the last interglacial period (LIG, 135000-115000 years before present) and compared to a ITM simulation of the surface mass balance using the same climate forcing.
In my opiniion, the evaluation could go to greater depth. The manuscript is mostly easy to read but the language is sometimes imprecise (anomaly, bias, difference, gap are genorously used quite exchangeable) or sometimes appears to be non-idiomatic. Figures are mostly of good quality but axes labels are often too small. Nonetheless, the implementation of more physics based schemes is surely a desirable improvement of this Earth System Model of intermediate complexity and will be a valuable innovation and I recommend publication after consideration of the following concerns:

2 Major concerns and general comments

In the Introduction the authors formulate the aim to "answer the question of whether a physics-based scheme can improve the representation of SMB for paleo timescale", but I don't see that the presented results allow to do so. The BESSI surely is more complex and also provides additional information about SMB components, but the results of the interglacial simulation are not necessarily better (possibly only different) than results of the ITM simulation. As a direct comparison to SMB observations for past climates is not possible, I would recommend to add some in depth analysis of the MAR based simulation with respect to the response to qualitatively different constellation (e.g. showcases for early summer versus late summer melt, bare ice in comparison to high accumulation zone, clear sky vs. overcast conditions...). These constellations are probably quite differently represented by BESSI and ITM and might provide insight into the applicability of the individual models for different background conditions.
For the above analysis it would be nice to add a 1979-2021 MAR-ITM simulation which would also allow to directly compare the skill of the two interfaces for present day climate.

At several places the authors claim that ITM is more sensitive to temperature than BESSI - it would be helpful to illustrate this with for example a scatter plot of MELT(BESSI)-MELT(ITM) against temperature and insolation.

It is remarkable that ITM, with $c = -25 Wm^{-2}$ exhibits a similar sensitivity to interglacial climate change as BESSI with bias correction. With three tuneable parameters it will probably be possible to find ITM parameters which would be in general agreement with BESSI both for preindustrial and last interglacial climate and difference in the behaviour of the two schemes maybe depending on parameter choices in ITM. Also the choice of $c = -25 Wm^{-2}$ might represent a first order bias correction of the iLOVECLIM climate. Please discuss.

I recommend to evaluate the melt, refreezing and sublimation separately in the figures, as biases are compensating and might be masked by strong precipitation contribution to the signal and e.g. in Fig. S1 the color scale does not resolve differences in sublimation.

Finally I am wondering if this paper would be better placed in a more method-focused journal such as *Geoscientific Model Development*.

3 Some specific comments

title: "Improve iLOVECLIM"... => "Improved iLOVECLIM" ?
l.42: rephrase, maybe: the albedo feedback being absent in the simulation.
l.43: maybe better: The first option is to use dedicated snow pack models coupled to RCMs...
l.61: maybe: the model's performance => the model's behaviour
l.77: please reformulate more carefully as LIG SMB is not known.
l.78: also here: I don't see how the advantage can be evaluated. Maybe "evaluate the effect..."
sect. 2.1.1: Please highlight changes with respect to the earlier published model version.
sect. 2.3: Maybe include a table of all experiments with some climte characteristics (mean JJA temperature and insolation range) and a figure of the topographies used (15km, 40km, T21).
Fig 2: Maybe include sublimation,refeezing and melt in the pannels in the middle. Increase font size.
l.290ff: a bit clumsy, please rephrase.
table 2: Typo: "Greeland", maybe also include bias corrected BESSI.
l. 305: the "negtive SMB zone" is somewhat quite comparable to the more extensive ablation zone in MAR. Please discuss.
l. 324: "North of AIS" is no good orientation here, maybe Eastern Weddell Sea sector.
l. 355: avoid "overestimation" because this would imply that BESSI serves as the reference here- rather use something like "more sensitive than"
Figure 9, upper panel: maybe include mean summer temperature for the two ice sheets.
l. 365: albedo feedback should be discussed in greater detail.
l. 374-380: a bit confusing, please rephrase.
l. 400: correct: south-western part of the GrIS
l. 406: "cheaper cost" specify the computational cost for orientation (e.g. wall clock time/100 model years) of BESSI-iLOVECLIM and ITM-iLOVECLIM, maybe here, maybe somewhere else...
l. 415: more processes do not always increase reliability- additional, poorly constrained feedbacks might actually increase uncertainty...
l. 420: insolation is a common forcing for both BESSI and ITM.
l. 446-447: check grammar.
Fig. B2: Caption does not seem to belong here.

Fig 1,2,4,5,6,7,8...: Increase font size.

References

Fettweis, X., Hofer, S., Krebs-Kanzow, U., Amory, C., Aoki, T., Berends, C. J., Born, A., Box, J. E., Delhasse, A., Fujita, K., Gierz, P., Goelzer, H., Hanna, E., Hashimoto, A., Huybrechts, P., Kapsch, M.-L., King, M. D., Kittel, C., Lang, C., Langen, P. L., Lenaerts, J. T. M., Liston, G. E., Lohmann, G., Mernild, S. H., Mikolajewicz, U., Modali, K., Mottram, R. H., Niwano, M., Noël, B., Ryan, J. C., Smith, A., Streffing, J., Tedesco, M., van de Berg, W. J., van den Broeke, M., van de Wal, R. S. W., van Kampenhout, L., Wilton, D., Wouters, B., Ziemen, F., and Zolles, T.: GrSMBMIP: intercomparison of the modelled 1980–2012 surface mass balance over the Greenland Ice Sheet, The Cryosphere, 14, 3935–3958, https://doi.org/10.5194/tc-14-3935-2020, https://tc.copernicus.org/articles/14/3935/2020/, 2020.