In the following, the reviewer's comments are in blue, and our response is in black

1 General

The manuscript "Improve iLOVECLIM (version 1.1) with a multi-layer snow model: surface mass balance evolution during the Last Interglacial" by Thi-Khanh-Dieu Hoang and Co-authors introduces the BErgen Snow Simulator (BESSI) as a potential coupling interface for ice sheets in the Earth system model iLOVECLIM. BESSI is envisioned to replace the previous iLOVECLIM interface, the less complex, empirical insolation-temperature melt equation (ITM). In contrast to ITM, BESSI is physics based and as such might be applicable for a larger range of background climates (e.g. Greenland Ice Sheet during the last interglacial under orbitally changing insolation; or the Antarctic Ice Sheet with only sporadicly occurring melt and precipitation dominating SMB variability) without the need to retune parameters. The authors first evaluate BESSI for the modern Greenland Ice Sheet using 1979-2021 climate forcing from the regional climate model MAR and comparing it to the respective MAR surface mass balance output. Then BESSI is forced with climate output from a iLOVECLIM simulation of the last interglacial period (LIG, 135000-115000 years before present) and compared to a ITM simulation of the surface mass balance using the same climate forcing.

In my opinion, the evaluation could go to greater depth. The manuscript is mostly easy to read but the language is sometimes imprecise (anomaly, bias, difference, gap are genorously used quite exchangeable) or sometimes appears to be non-idiomatic. Figures are mostly of good quality but axes labels are often too small. Nonetheless, the implementation of more physics based schemes is surely a desirable improvement of this Earth System Model of intermediate complexity and will be a valuable innovation and I recommend publication after consideration of the following concerns:

We thank the reviewer for the useful comments. These comments are taken into account in the paper's revised version. Detailed answers to specific comments are provided in the following section.

Here are the main adjustments to the revised manuscript:

- The title of the paper has been changed to make it less technical.
- The description of the BESSI model is shortened in the main text and moved to Appendix A.
- ITM stand-alone replaces the previous version, which runs with the iLOVECLIM time step (4-hourly) and includes the impact of iLOVECLIM temperature biases on the parameter crad in the ITM equation (Quiquet et al., 2024). In the revised version, BESSI is compared to ITM stand-alone, which uses the same input and runs in the same time step. This provides a cleaner comparison and insights into the two models' behavior, as suggested by the reviewer.
- As both reviewers suggested, the bias correction of iLOVECLIM is now included in the main text.
- To put more weight on the Last Interglacial (LIG), as both reviewers suggested, we shortened the part of calibration/validation of BESSI and ITM (stand-alone) with only important results. The climate simulated by iLOVECLIM for the LIG is also evaluated based on existing knowledge.
- All the figures are fixed with the appropriate colorbar, bigger font size, and correct captions.

2 Major concerns and general comments

In the Introduction the authors formulate the aim to "answer the question of whether a physics-based scheme can improve the representation of SMB for paleo timescale", but I don't see that the presented results allow to do so. The BESSI surely is more complex and also provides additional information about SMB components, but the results of the interglacial simulation are not necessarily better (possibly only different) than results of the ITM simulation. As a direct comparison to SMB observations for past climates is not possible, I would recommend to add some in depth analysis of the MAR based simulation with respect to the response to qualitatively different constellation (e.g. showcases for early summer versus late summer melt, bare ice in comparison to high accumulation zone, clear sky vs. overcast conditions...). These constellations are probably quite differently represented by BESSI and ITM and might provide insight into the applicability of the individual models for different background conditions.

For the above analysis it would be nice to add a 1979-2021 MAR-ITM simulation which would also allow to directly compare the skill of the two interfaces for present day climate.

It is true that we cannot validate BESSI and ITM for the paleo period like the LIG. As the reviewer suggested, we compare BESSI to ITM stand-alone in the revised manuscript. In the previous version of the manuscript, we used the ITM embedded in iLOVECLIM, which is not directly comparable to BESSI as it runs in 4-hourly timestep as iLOVECLIM and contains some modifications related to the temperature biases in iLOVECLIM and albedo value as mentioned in Sect 2.1.3.

For the stand-alone version, ITM is forced by a similar input as BESSI and runs in daily timestep. In iLOVECLIM, the albedo of ice grid cells is set to be 0.85. Hence, ITM stand-alone also uses the same value for albedo in the equation. The crad value is obtained by tuning to achieve the best match for total surface mass balance (SMB), taken to be equivalent to the lowest RMSE compared to MAR (Sect. 2.3).

To investigate the behaviors of BESSI and ITM for the present-day condition with MAR as forcing, we select 2 points in the accumulation zone and 2 points in the ablation zone. Point 1 is in an accumulation zone with a cold and dry climate. In a similar zone, point 2 has mild and wet conditions. Points 3 and 4 are similar to points 1 and 2 but located in the ablation zone. By selecting points from different climate conditions (humidity and temperature) for bare ice and accumulation zones, we compare the behaviors of MAR, BESSI and ITM. The location of the points is provided in Fig. RA1a.

For each point, we plot surface mass balance - SMB, runoff - RU and sublimation - SU of MAR, BESSI-MAR, and ITM-MAR climatological mean daily variation. Following the suggestion of reviewer 1, melt computed by ITM is considered as runoff. In general, compared to MAR and BESSI, ITM tends to simulate runoff earlier due to the sensitivity to the temperature while missing the explicit refreezing calculation.

Differences in each process of the three models are provided in annual mean maps in the Supplement (Fig. S2 and S3 for Greenland and S6 for Antarctica).

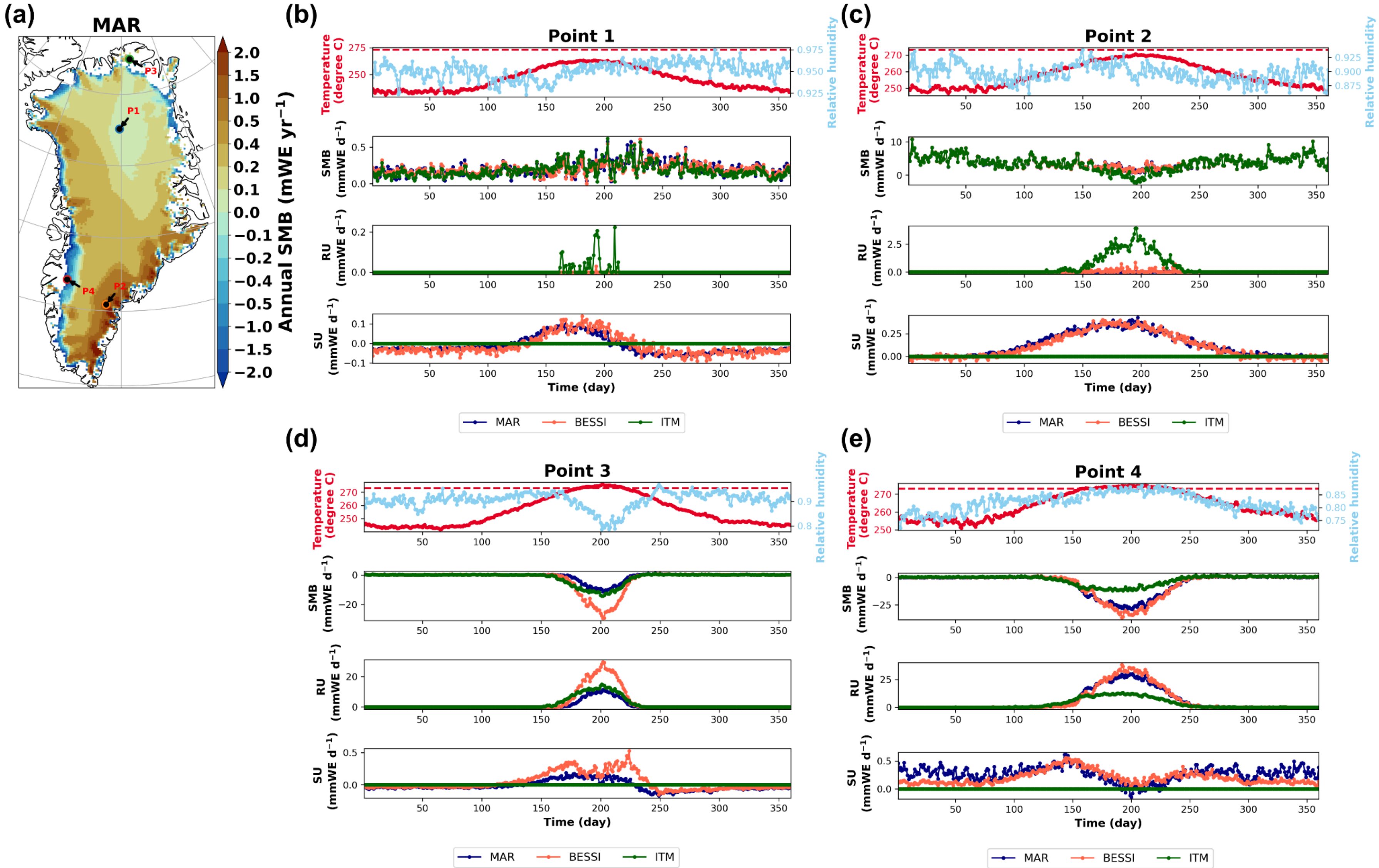

Figure RA1. (a) The location of four selected points on the Greenland Ice Sheet. The climatological variation of inputs (temperature and humidity), SMB, RU and SU for **(b)** point 1, **(c)** point 2, **(d)** point 3 and **(e)** point 4.

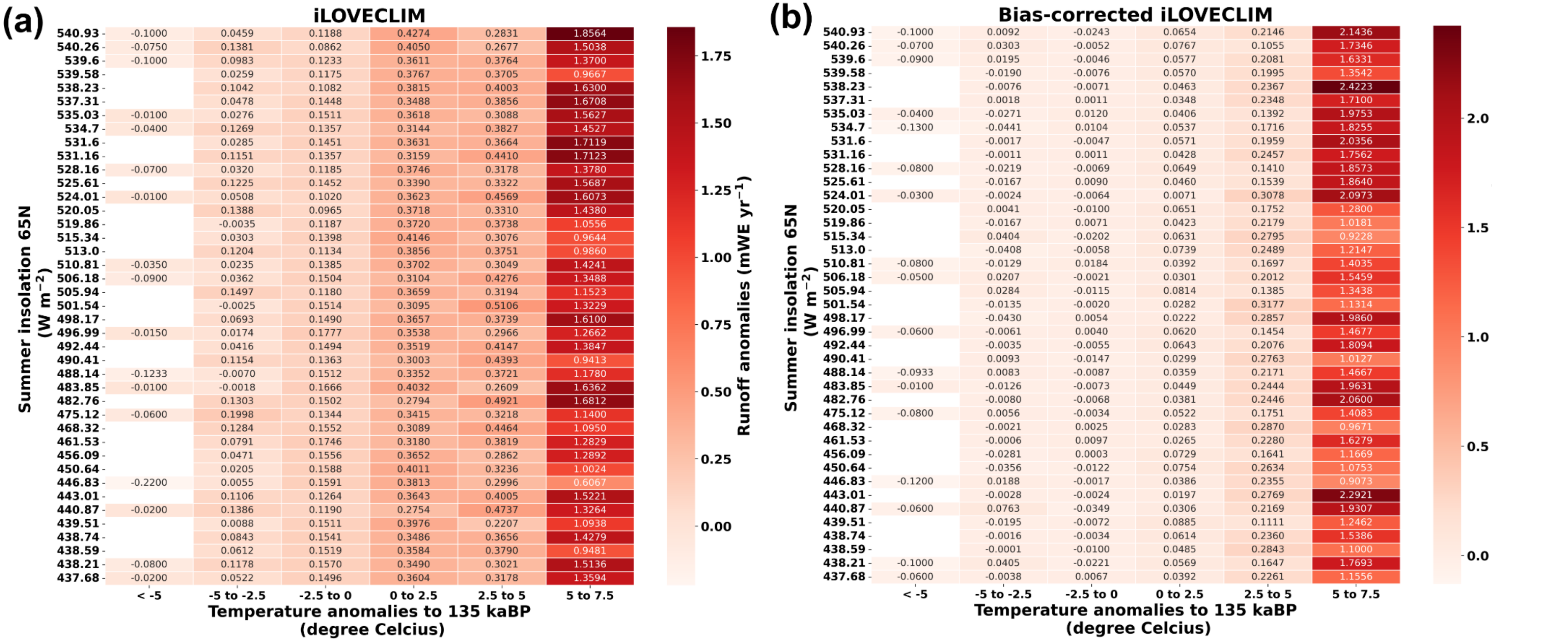

Figure RA2. Mean runoff differences between BESSI and ITM forced by iLOVECLIM of Greenland Ice Sheet during the LIG respect to the mean temperature anomalies (compared to 135 kaBP) and the summer insolation at 65N **(a)** before and **(b)** after the bias-correction.

At several places the authors claim that ITM is more sensitive to temperature than BESSI - it would be helpful to illustrate this with for example a scatter plot of MELT(BESSI)-MELT(ITM) against temperature and insolation.

In the revised version, the paper includes more results and a discussion on the sensitivity of ITM to the input data (temperature and short-wave radiation). Since the melt in ITM is treated as Runoff to compare to Runoff of BESSI, as Reviewer 1 suggested, the heat map of Runoff(BESSI)-Runoff(ITM) against local temperature anomalies (compared to 135 kaBP) and summer insolation of Greenland Ice Sheet during the LIG are presented in Fig. RA2. We show a heatmap instead of a scatter plot since we only have discontinuous values for the insolation (41 time slices). Fig. RA2 displays the annual mean runoff anomalies between BESSI and ITM spatially averaged for a given temperature range and a given summer insolation. In general, the figure indicates that the role of temperature is more significant than insolation since the horizontal gradients (impact of temperature change) are larger than the vertical gradients (effect of insolation change). However, it is hard to draw any firm conclusion from this figure since the temperature change integrates all the other changes, including insolation change.

The comparison of the simulated runoff rate by BESSI and ITM is also shown in Fig. 12 for Greenland and Fig. 14 for Antarctica in the revised manuscript. From these figures, the change of runoff in ITM due to the insolation and temperature is more significant. In addition, these figures indicate that ITM is sensitive to its inputs and needs to be retuned when the climate conditions change.

It is remarkable that ITM, with c = $-25 Wm^{-2}$ exhibits a similar sensitivity to interglacial climate change as BESSI with bias correction. With three tuneable parameters it will probably be possible to find ITM parameters which would be in general agreement with BESSI both for preindustrial and last interglacial climate and difference in the behaviour of the two schemes maybe depending on parameter choices in ITM. Also the choice of c = $-25 Wm^{-2}$ might represent a first order bias correction of the iLOVECLIM climate. Please discuss.

ITM inside iLOVECLIM is partly bias corrected by changing the crad with respect to the bias in the temperature (Quiquet et al., 2024). For example, when the temperature difference between iLOVECLIM and ERA-Interim is +10°C, the crad value is double, which means less runoff to account for this warm bias.

In the revised version, we use ITM stand-alone with one crad value for the whole ice sheet. In the new results, we can see that even when both BESSI and ITM are calibrated to behave similarly to MAR for Greenland, their behaviors start to be different when the climate conditions or the forcings change. For Antarctica, unlike BESSI, without retuning, ITM simulates unrealistic high melt with MAR as the forcing.

I recommend to evaluate the melt, refreezing and sublimation separately in the figures, as biases are compensating and might be masked by strong precipitation contribution to the signal and e.g. in Fig. S1 the color scale does not resolve differences in sublimation.

Plots of the differences in simulated albedo, melt, refreezing, runoff, and sublimation of BESSI-MAR, ITM-MAR, and MAR for Greenland and Antarctica are now in the Supplement.

Finally I am wondering if this paper would be better placed in a more method-focused journal such as Geoscientific Model Development.

We agree that the title and the description of the BESSI model make the paper seem technical. To fix this, we have reshaped the manuscript to focus more on the differences between BESSI and ITM as well as the effects of using BESSI for the paleo studies. We have also changed the paper's title to "Using a multi-layer snow model for transient paleo studies: surface mass balance evolution during the Last Interglacial." The BESSI model's description has now been moved to the Appendix to make the paper less technical.

3 Some specific comments

We have fixed all the errors the reviewer mentioned. However, as the paper's content has changed a lot, some of the sentences have been removed.

title: "Improve iLOVECLIM"... => "Improved iLOVECLIM"?

We have changed the title.

l.42: rephrase, maybe: the albedo feedback being absent in the simulation.

Fixed

l.43: maybe better: The first option is to use dedicated snow pack models coupled to RCMs...

Thanks for the suggestion. We have fixed it now in the revised version.

l.61: maybe: the model's performance => the model's behaviour

Fixed

10 l.77: please reformulate more carefully as LIG SMB is not known.

Done, the word "reproducing" is replaced with "simulating".

l.78: also here: I don't see how the advantage can be evaluated. Maybe "evaluate the effect..."

Corrected

sect. 2.1.1: Please highlight changes with respect to the earlier published model version.

In the revised version, we provide the differences compared to Zolles et al., 2021 in Section 2.1.1.

sect. 2.3: Maybe include a table of all experiments with some climte characteristics (mean JJA temperature and insolation range) and a figure of the topographies used (15km, 40km, T21).

For Section 2.3, a new table 1 with the characteristics of each experiment is included. Figures of topography for MAR, iLOVECLIM NH40, and SH40 as well as the native T21 grid, are also included (Fig. 2 and 3).

Fig 2: Maybe include sublimation,refeezing and melt in the pannels in the middle. Increase font size.

As we want to put more weight on the LIG part, the plots of differences in albedo, melt, refreezing, runoff, and sublimation between BESSI-MAR, ITM-MAR and MAR are in the Supplement.

l.290ff: a bit clumsy, please rephrase.

This sentence is removed as the comparison of MAR and iLOVECLIM in terms of climate forcing is put in the Supplement to support the results of Sect. 3.1 and 3.2.

table 2: Typo: "Greeland", maybe also include bias corrected BESSI.

The table is now replaced by Fig. 8.

The bias correction part is now included in the main text with the results of SMB of BESSI and ITM stand-alone with iLOVECLIM forcing before and after bias correction for both ice sheets (Sect 3.2 and 3.3.2).

l. 305: the "negtive SMB zone" is somewhat quite comparable to the more extensive ablation zone in MAR. Please discuss.

This is due to the bias correction impact on the crad included in ITM when running coupled to iLOVECLIM (Quiquet et al., 2024). In the new results of ITM stand-alone, we can see that the ablation zone in the southwest of Greenland is linked to temperature and short-wave radiation.

l. 324: "North of AIS" is no good orientation here, maybe Eastern Weddell Sea sector.

Fixed.

l. 355: avoid "overestimation" because this would imply that BESSI serves as the reference here- rather use something like "more sensitive than"

We have improved the word choices in the revised version.

Figure 9, upper panel: maybe include mean summer temperature for the two ice sheets.

Instead of the mean summer temperature for the two ice sheets, we now included a figure of temperature simulated on the ice sheet compared to the proxy (NGRIP and EPICA Dome).

l. 365: albedo feedback should be discussed in greater detail.

The albedo feedback is discussed in the Discussion as follows: "The albedo in ITM is fixed at 0.85, which is the value of ice grid points in iLOVECLIM, to give a clean comparison to BESSI. This can also be the reason behind the low runoff simulation in ITM-iLOVECLIM during the LIG. A lower albedo value, which means more solar radiation is considered, can increase the simulated runoff rate of ITM (Supplement Fig. S12b). However, using only one albedo value for the whole ice sheet is not realistic. ITM with a range of albedo for different altitudes and locations can provide satisfied results as in Quiquet et al. (2021)."

l. 374-380: a bit confusing, please rephrase.

The writing of this part is polished in the revised version.

l. 400: correct: south-western part of the GrIS

Done

l. 406: "cheaper cost" specify the computational cost for orientation (e.g. wall clock time/100 model years) of BESSI-iLOVECLIM and ITM-iLOVECLIM, maybe here, maybe somewhere else...

We are sorry for the missing information. The information is now mentioned in the Discussion.

l. 415: more processes do not always increase reliability- additional, poorly constrained feedbacks might actually increase uncertainty...

We agree with the reviewer. BESSI might not be reliable as its model structures are simplified. The simulation of various processes and feedback in BESSI does have some limitations. However, they are in the acceptable range as the model exhibits good results compared to MAR in the calibration/validation. Also, compared to ITM, which needs to be tuned for different climate conditions and tends to overestimate the runoff rate, BESSI is still more favorable to simulate SMB for transient paleo studies while maintaining a low computational cost.

30 l. 420: insolation is a common forcing for both BESSI and ITM.

Yes, sorry for the mistake.

l. 446-447: check grammar.

The sentence is rewritten as "The current SMB scheme of iLOVECLIM needs to be retuned for different climate forcings and study periods, which is not ideal for application in paleo studies."

Fig. B2: Caption does not seem to belong here.

We are sorry for the misplacing of the Appendix figures. It is fixed in the revised version.

Fig 1,2,4,5,6,7,8...: Increase font size.

All the figures are increased in font size in the revised version.

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