The authors thank Dr. Lovejoy for the very helpful and constructive comments, which helped improve the accuracy of this article. Aside from minor changes and typo corrections, all changes and comment responses are included below. Editor comments are shown in **bold** and the author responses are indented. Changes to the manuscript are shown in ~~red~~ and blue text. Line numbers correspond to the revised version of the manuscript.

# Editor Report: 'Comment on egusphere-2024-552', Shaun Lovejoy, 26 June 2024

## Overall comments:

**As a preface, I recognize that I am a invested protagonist in the science reported here, so please take these comments as helpful suggestions, not in the spirit of anonymous referee comments. This paper is a welcome update on a key question of atmospheric dynamics: over what ranges are they scaling? The key finding is that observations of cloud radiances over a huge range of horizontal scales are indeed scaling. This vindicates Richardson's wide range scaling hypothesis updated as confirmed by Lovejoy's 1982 area-perimeter analysis (and numerous spectral and other analyses since). Wide range scaling is incompatible with the still prevalent 2D isotropic/3D isotropic paradigm that necessarily involves a "dimensional transition" somewhere in the mesoscale. The question is which symmetry is dominant: the scale symmetry or the rotational symmetry? Richardson believed it was scaling. Following the isotropic 2D Kraichnan 1968 model, and Charney's 1971 quasi-geostrophic variant, the atmospheric community has largely considered isotropy to be the dominant symmetry, thus implying an elusive dimensional transition/scale break somewhere near the mesoscale. This paper contradicts the latter hypothesis but supports the former. It would be worth bringing this out in the introduction, it will enhance the significance of the work.**

> Added to ln 132-137
>
> > Our findings contradict the theories proposing split 2D and 3D isotropic turbulence regimes separated by a scale break that have prevailed over the past decades (Fiedler and Panofsky, 1970; Nastrom et al., 1984), and support the concept of a wide-ranging, scale invariant 2D-3D ~~intermediate~~ anisotropic turbulence regime proposed by Schertzer and Lovejoy (1985), described in detail by Lovejoy and Schertzer (2013)~~, that we show~~. We show that this anisotropic turbulence regime applies to cloud perimeters over a remarkable 10 orders of magnitude ranging from the Kolmogorov microscale $\eta$ to the planetary diameter $2a$.

**My main issue with the paper is that it is monofractal – both in the theoretical model as well as in the data analysis. This aspect with respect to both area-perimeter relations as well as Korcak laws was considered in some detail in several appendices to [Lovejoy and Schertzer, 1991]: http://www.physics.mcgill.ca/ gang/eprints/eprintLovejoy/neweprint/NVAGlovejoy-all.pdf The main conclusion relevant to this paper is that the interpretation of the area-perimeter (A-P) exponent, is quite different when monofractal models (such as fractional Brownian motion), or when multifractal models are used. In the former case (assumed here), the cloud regions exceeding a threshold are assumed to be non fractal (they are assumed to be true (2D) areas, with dimension = 2) so that the usual interpretation of the A-P exponent = D/2 is valid. However if clouds are multifractal, then for any exceedance threshold that define "clouds", the A-P exponent is the ratio D(P)/D(A) of the perimeter dimension D(P) to the fractal dimension of the exceedance set D(A). Since D(P) and D(A) will both decrease with the brightness threshold that defines the sets, the ratio may be quite stable over a range of thresholds, potentially explaining the robustness of the A-P exponent. The authors may want to reflect and comment on this?**

The authors thank Dr. Lovejoy for this helpful explanation, and while we do not address this comment specifically in the article because we do not measure the A-P exponent here, we did add clarification on multifractals based on the reference provided.

Note that Section A.4.ii of Lovejoy and Schertzer 1991 suggests that the fractal dimension of perimeters (to include ensembles) can be obtained through coarsening as long as the *set* rather than the *field* is degraded. Rather than degrading the cloud reflectance value from which the cloud mask is determined, we are only degrading the resolution of the binary cloud mask through averaging. As stated, this method should work whether or not the field is multifratcal since it is converted to a set with a well-defined dimension (2 for binary pixels).

We have revised several paragraphs of the introduction to address and clarify these points, while also addressing the first minor comment below:

> Fractal geometry is often used as a tool for characterizing the resolution-dependent complexity of shapes. The fractal dimension $D$ was first introduced by Richardson (1961) to characterize the complexity of political borders and was later popularized by Mandelbrot (1967) to describe how the length of a coastline changes depending on the length of the ruler used to measure it. Generally, the perimeter $p$ around an individual fractal object can be related to the measurement resolution $\xi$ through
>
> $$p \propto \xi^{1-D} \tag{1}$$
>
> For the Euclidean case that $p$ is independent of $\xi$ then $D = 1$. At the other extreme, a “space-filling” curve that passes through every resolved point in a unit area has $D = 2$. Lovejoy (1982) first measured $D$ for clouds by relating individual cloud perimeters $p$ to cloud areas $a$ using the expression $p \propto \sqrt{a}^D$. A measured value of $D = 1.35 \pm 0.05 \approx 4/3$ has since been adopted as the canonical value describing individual clouds (Siebesma and Jonker, 2000; Christensen and Driver, 2021)~~. A “monofractal” object has a constant value of~~, although various studies have shown that $D$ ~~, and for the case that its scaling of $p$ with $\xi$ (e.g., by a power-law) is the same at all length scales, it is considered~~ can vary considerably from cloud to cloud. For example, Batista Tomás et al. (2016) found distinct fractal dimension values for cirrus with ragged, tenuous edges of $D = 1.37$, whereas for cumulonimbus with smoother edges, $D = 1.18$. Other analyses of cumulus fields have found $D = 1.28$ (Zhao and Di Girolamo, 20019) and $D = 1.19$ (Mieslinger et al., 2019) using the expression $p \propto \sqrt{a}^D$.
> Generally, we define here a geometric quantity that does not vary with length scale as being “scale invariant~~.~~,” such as the scaling of $p$ with $\xi$ in Eq. (1). For such scale invariance to apply to an atmospheric cloud field, this would require that the physics controlling cloud shapes is unchanged with measurement resolution, at least between the limits of possible cloud sizes. ~~While clouds~~ Clouds have been shown to be broadly scale invariant for the number distributions of cloud ~~perimeters (DeWitt et al., 2024), cloud shapes might better be described as being multifractal, where~~ areas and perimeters (DeWitt et al., 2024) despite previous observations of scale breaks that appeared to separate small and large clouds into different physical regimes. In their study, DeWitt and Garrett (2024) argue that these scale breaks are artifacts that owe to the treatment of clouds that are truncated by the edge of the measurement domain.
>
> Although the initial result of Lovejoy (1982) showed a constant value of $D$ for length scales ranging from 1 to 1,000 km, suggesting a wide-ranging scale invariance of clouds, the value of $D$ ~~is a continuous function of threshold used to define cloud (Lovejoy and Schertzer, 1990, 1991; Marshak et al., 1995; Lovejoy and Schertzer, 2006). Various studies have shown that $D$ can vary considerably from cloud to cloud and even within different regions of the same cloud. For example, Batista Tomás et al. (2016) found distinct fractal dimension values for cirrus with ragged, tenuous edges of $D = 1.37$, whereas for cumulonimbus, with smoother edges, $D = 1.18$. Other analyses of cumulus~~

~~fields have found $D = 1.28$ (Zhao and Di Girolamo, 20019) and $D = 1.19$ (Mieslinger et al., 2019) using the expression $p \propto \sqrt{a}^D$.~~ has sometimes been observed to be greater for larger clouds. Cahalan and Joseph (1989) reported $D = 1.27$ for small clouds and $D = 1.56$ for large clouds, supported by Benner and Curry (1998) who found $D = 1.23$ and $D = 1.34$ respectively. Furthermore, after reexamining the data in Lovejoy (1982), Gifford (1989) noted that $D$ increases from 1.35 to 1.77 for the largest clouds with areas $> 2.5 \times 10^4$ km$^2$. ~~Inclusion of holes in the measured cloud areas (Peters et al., 2009) and merged clouds (Cahalan and Joseph, 1989) have been theorized to affect~~ The apparent increase in measured $D$ for larger clouds suggests a violation of scale invariance. However, this is likely another artifact of the data analysis methods. The inclusion of interior cloud holes in area and perimeter measurements has been shown to overestimate calculations of $D$ ~~.~~ using the expression $p \propto \sqrt{a}^D$ (Peters et al., 2009; Brinkhoff et al., 2015). Because interior holes tend to fill when imaged with increasingly coarse resolution, this $\xi$ dependence of $a$ results in an inaccurate value of $D$ – the error of which can be calculated using multifractal analysis (Lovejoy and Schertzer, 1991).

~~The multifractal nature of clouds and their apparent size and type dependence of~~ Clouds have been shown to be multifractal, such that $D$ ~~seem to contradict the argument that cloud geometries are scale invariant. Additionally, a monofractal $D$ does not account for multifractal parameters that account for~~ is a continuous function of threshold used to distinguish clouds from clear skies (Lovejoy and Schertzer, 1990, 1991; Marshak et al., 1995; Lovejoy and Schertzer, 2006). Studies of the multifractal properties of clouds are useful because they can be used to mathematically account for turbulent intermittency (the variability of turbulent fluctuations), notably observed in measurements of water mixing ratio (Tuck, 2022). ~~However, scale invariance might be a reasonable assumption for describing a large ensemble of clouds considered over a sufficiently long period of time and space, especially if turbulent intermittency might be reflected by the geometric intermittency of multiple and varied cloud types in the ensemble. Indeed, the topic of whether or how scale invariance applies to atmospheric structures has~~ We argue that a monofractal assumption is sufficient for the primary conclusions of this study in Section 5.4.

Added Section 5.4 Multifractal considerations to the Discussion:

Because each of the satellite cloud masks considered in Fig. 4 is generated using a single respective cloud definition threshold, the above analysis is implicitly monofractal. Adopting a monofractal analysis of a field that is multifractal for such quantities as cloud brightness can be problematic. Most importantly, if the cloud brightness field were itself coarsened by averaging over adjacent pixels, the threshold applied to define the presence of a cloud would need to be adjusted to account for the inevitable smoothing of very bright regions with dark regions.

Here, this complication is limited because we are averaging adjacent pixels in a binary cloud mask rather than a brightness field, leading to a more accurate measurement of the fractal dimension (Lovejoy and Schertzer, 1991, Sect. A.4.ii), even as it still does not consider how the fractal dimension varies as a function of threshold. To address this question, we applied various threshold parameters $j$ to define cloudy pixels from the modeled cloud field in SAM. The parameter $j$ is the number of cloudy pixels in each vertical column of the 3D volume required to assign a cloudy pixel in the horizontal 2D cloud mask. The vertical resolution of each pixel is 100 m in the vertical portion of the simulation domain that contains the most cloudy pixels. As shown in Fig. 3a, changing the threshold value of $j$ results in a wide range of horizontal cloud fractions spanning $0.15 < \mathcal{A} < 0.60$. The multifractal nature of clouds is evident in Fig. 6: $\mathcal{H}$ and $D_e$ decrease by 0.11 as the threshold parameter $j$ increases from 1 to 15. What remains clear is that, independent of the threshold considered, the central conclusion of this article remains unchanged, which is that measured values for $\mathcal{H}$ are intermediate to those expected for 2D or 3D isotropic turbulence.

## Minor comments (The line numbers are with respect to the second version of the manuscript).:

**Line 55.** ***"The multifractal nature of clouds and their apparent size and type dependence of D seem to contradict the argument that cloud geometries are scale invariant."*** **This is a nonsequitor: by definition, multifractals are scale invariant. What did you mean to say?**

See revisions in the previous comment. This statement is removed and clarification is added regarding multifractals and scale invariance.

**Line 60:** ***"Indeed, the topic of whether or how scale invariance applies to atmospheric structures has been the topic of decades of debate (Lovejoy and Schertzer, 2018)."*** **In the turbulence community, scale invariance itself is a mainstay for all the theories, the question is the type and range(s) of the scaling: the standard 2D isotropic / 3D isotropic turbulence model with dimensional transition somewhere in the meso-scale versus a single wide range but anisotropic scaling regime (the 23/9D) model. The debate is about the type of scaling: anisotropic or isotropic, the limits of the scaling regime(s) and the values of the scaling exponents.**
**Note: there is no Lovejoy and Scherzter 2018, you seem to be referring to Lovejoy and Schertzer 2013; please change this throughout the text.**

Corrected the year in the reference throughout and revised ln 67-73:

> While the fractal dimension and scale invariance are intrinsically linked, their relationship to turbulent structures in the atmosphere is less clear. Two paradigms of turbulence scaling in the atmosphere have been the topic of decades of debate~~(Lovejoy and Schertzer, 2013). The~~ : split 2D and 3D isotropic scaling regimes for large and small scales (Fiedler and Panofsky, 1970; Nastrom et al., 1984), and wide-ranging anisotropic scaling (Lovejoy, 2023) Both theories originated from the pioneering work of ~~Richardson (1926)~~ Richardson (1926), who showed that the turbulent eddy diffusivity $K$, measured using the relative motion of pairs of particles separated by distance $\ell$, followed a power-law with a 4/3 exponent from the millimeter scale for molecular diffusion to the length scale of atmospheric cyclones ($\ell \sim 10^3$ km), $K \propto \ell^{4/3}$, termed the Richardson "4/3 law" of atmospheric diffusion.

**Line 72: Eq. 2 needs an absolute value sign around the difference. In addition, H is only the usual Hurst exponent in the nonintermittent (Gaussian) case. In equation, the H is inspired by Hurst, but is not the same. Also, if fluctuations are defined by other wavelets (i.e. not the differences as indicated), then H can in principle take any real value, the range $-1 < H < 0$ being particularly important in the macroweather regime.**

Added absolute value signs in Eq (2) and specified further about $\mathcal{H}$ in the footnote (ln 80):

> For turbulent scalars, the function tends to be a power-law given by
>
> $$S(\ell) = |\Delta\Theta(\ell)| = \left\langle |\Theta(x+\ell) - \Theta(x)| \right\rangle \propto \ell^{\mathcal{H}} \tag{2}$$
>
> where brackets indicate averaging over many iterations of the experiment, and $\mathcal{H}$ is the Hurst exponent[1] ~~bounded by~~ with bounds $0 < \mathcal{H} < 1$ (Schertzer and Lovejoy, 1984; Hentschel and Procaccia, 1984; Lovejoy and Schertzer, 2012).

[1] The Hurst exponent has various mathematical applications~~in other fields~~, but here we employ its ~~common~~ usage in the field of fractal geometry (for the non-intermittent case) to relate the scaling of turbulent fluctuations with respsect to separation distance $\ell$.

**Line 83: The law eq. 3 ignores intermittency, it is at best an average law. Statistics of other orders will presumably define a hierarchy of (multifractal) exponents. Your mention of the dimensionality is in fact a reference to the 2D isotropic/ 3D isotropic versus 23/9D debate.**

Revised ln 87-92:

> The dimensional approximation that $K \sim \ell v$ (Tennekes and Lumley, 1972) results in $K \sim \varepsilon^{1/3}\ell^{4/3}$, reproducing Richardson's 4/3 power-law, and implying that the relationship between diffusivity and the Hurst exponent $\mathcal{H}$ (again ignoring intermittency) follows
>
> $$K \sim \ell^{1+\mathcal{H}} \tag{3}$$
>
> As Sect. 5 elaborates, the value of $\mathcal{H}$ ~~depends~~ differs based on the dimensionality of the turbulence ~~.~~ (e.g., the case of 2D isotropic turbulence).

**Line 94: The correct reference for the spurious nature of the scale breaks in aircraft data is [Lovejoy et al., 2009]**

Corrected this reference.

**Line 100: The expression *"intermediate turbulence regime"* is unfortunate since readers will likely think this is a regime intermediate in spatial scales whereas I understood (only later in the text) that you meant intermediate in the value of the dimension (i.e. $3 > 23/9 > 2$). The key point to make here is that rather than 2 isotropic regimes separated somewhere in the meso-scale, a single (much wider scale range) anisotropic regime was proposed.**

Revised ln 108-112:

> Specifically, Lovejoy et al. (2007) (hereafter L07), and more comprehensively Lovejoy and Schertzer (2013), provided evidence that, rather than two separate isotropic turbulence regimes, the atmosphere is best characterized by a ~~consistent intermediate turbulence regime at all scales following the~~ single anisotropic turbulence regime spanning all scales in the atmosphere. Following the framework of generalized scale invariance (GSI), which accounts for stratification~~in~~, the "23/9D" elliptical model of turbulence in the atmosphere is characterized by a dimension intermediate to 2D and 3D (Schertzer and Lovejoy, 1985).

and ln 129-137:

> Section 4 presents the values of the ensemble fractal dimension obtained using several satellite and numerical model datasets. Section 5 interprets the significance of the results by comparing them to the expected values of $D_e$ and $\mathcal{H}$ for 2D and 3D isotropic turbulence, as well as for an ~~intermediate~~ anisotropic turbulence regime that ~~combines the two~~ is intermediate to 2D and 3D at all scales. Our findings contradict the theories of split 2D and 3D isotropic turbulence regimes separated by a scale break, which have prevailed over the past decades (Fiedler and Panofsky, 1970; Nastrom et al., 1984), and support the concept of a wide-ranging, scale invariant 2D-3D ~~intermediate~~ anisotropic turbulence regime proposed by Schertzer and Lovejoy (1985), described in detail by Lovejoy and Schertzer (2013)~~, that we show~~. We show that this anisotropic turbulence regime applies to cloud perimeters over a remarkable 10 orders of magnitude ranging from the Kolmogorov microscale $\eta$ to the planetary diameter $2a$.

**Line 104: The 23/9D model proposes that the volume of NONfractal structures scales as L23/9. 23/9 is an upper bound on the dimension of the (sparse) fractal structures (i.e. rather than the usual upper bound of D =3). In the 23/9D model, only structures with $D < 23/9$ are fractal. Also, the exponent is Hz, not H so that it is NOT a Hurst exponent. In the equation "D = 2.55 = 2+H" , H is in fact a RATIO of exponents Hz =Hhor/Hvertical. I'm puzzled**

**because later in the paper, this fact is acknowledged. In terms of the spectral exponents B, the relationship is Hz = (Bvertical-1)/(Bhorizontal-1) (this is true for both monofractal and multifractal variants of the 23/9D model). I could also note that the relationship B = 1+2H is only valid for the Gaussian (nonintermittent, nonmultifractal) case (this should be stated), otherwise the are intermittency corrections that are (inconsistently) invoked later (line 111).**

Revised ln 112-118:

> Power spectra of radar reflectivity, cloud radiance, wind speed, and temperature all revealed length-scaling exponents that lie between purely 2D and 3D turbulence cases, consistent with an ~~intermediate~~ anisotropic turbulence regime predicted to have a ~~(fractal)~~ volume dimension of ~~$D = 2.55 = 2 + \mathcal{H}$ where $\mathcal{H} \approx 0.55$~~ $D = 2.55 = 2 + \mathcal{H}_z$ where $\mathcal{H}_z \approx 0.55$ is the ratio of horizontal and vertical values of $\mathcal{H}$ (discussed further in Sect. 5) (Schertzer and Lovejoy, 1985; Lovejoy and Schertzer, 1985; Lovejoy et al., 1993; Lovejoy, 2021).. ~~In these cases,~~ For the Gaussian case, which does not include intermittency or multifractal aspects, $\mathcal{H}$ is calculated from the power spectrum of the observed phenomenon, $E(k) \sim k^{-B}$, where $B = 2\mathcal{H} + 1$. In the 23/9D theory, which incorporates the vertical and horizontal aspects of separation, $\mathcal{H}_z = (B_V - 1)/(B_H - 1)$.

**Line 110: Eq. 4 applies to the fractal dimension of the geometric set of points on the graph (x,B(x)) where x is the position in a 1-D cloud transect), B is the brightness of 1-D transects through monofractal cloud such as a fractional Brownian motion (fBm) cloud with structure function exponent H. In this case, the fractal dimension of the set of "zero-crossing points" (the intersection of the line B=T = constant with the cloud brightness B(x) is D = 1-H for any threshold T. That is why fBm is a monofractal function. If this fBm model is extended to two dimensional space B(x,y) then the codimension is still H, so that the dimension of the zero-crossing sets (the perimeter set) is independent of the brightness threshold.**

Revised ln 123-125

> Equation (4) ~~has also been related directly to cloud perimeter fractal dimension as adjusted for intermittency $\mu$ through $D_\mu = 2 - \mathcal{H}$ (Hentschel and Procaccia, 1984).~~ is the 2D analog of the fractal dimension of a geometric set of points. For example, given $(x, \Theta(x))$ where $x$ is the position in a 1D transect and $\Theta$ is the measured cloud brightness, the 1D case $D = 1 - \mathcal{H}$ extends to the 2D cloud perimeter $(\Theta(x, y)$ as $D = 2 - \mathcal{H}$ (Hentschel and Procaccia, 1984).

**Line 150: The nondimensionalization is not only a question of convenience. If the process is multifractal, the key scale is the outer scale and the dissipation plays the role of small cut-off. At any intermediate scale (between the smallest dissipation scale and the outer scale, only the outer scale intervenes, not the inner scale.**

Removed from ln 23 because we clarify in more detail later

> (defined as either the pixel side length in a satellite image or the grid spacing in a model~~following Garrett et al. (2018)).~~ ).

Removed the following (ln 165)

> ~~Note that $\xi$ is normalized here by $\eta$ rather than its common normalization by an outer scale $L$ (Lovejoy, 2023). We choose this normalization to more conveniently relate $\mathcal{P}_\xi$ to $K_\xi$ and $K_\eta$.~~

and added the following as a footnote:

Note that $\xi$ is normalized here by $\eta$ rather than the more common normalization by outer scale $L$, the largest eddy of the turbulent flow, from which energy is transferred to smaller eddies of observation scale $\ell = \xi$ in the energy cascade. Because the choice of normalization length scale does not affect calculations of the value of $\mathcal{H}$ or $D_e$, we choose $\eta$ to relate $\mathcal{P}_\xi$ to $K_\xi$ and $K_\eta$. This is consistent with the approach taken by Krueger et al. (1997); Garrett et al. (2018) who focused on the relationship between cloud measurements at scale $\xi$ and turbulent processes at the Kolmogorov microscale $\eta$.

**Line 184: The intermittency correction arises because turbulence is multifractal, not monofractal.**

Revised ln 195-200:

However, while $D = 4/3$ is consistent with values seen for individual clouds, a larger value is required for cloud ensembles, in which case the inequality $D < D_e$ predicted by ~~Mandelbrot (1977); DeWitt et al. (2024) applies. To allow for an~~ Mandelbrot (1977); DeWitt et al. (2024) applies. In a similar adjustment to the individual fractal dimension, Hentschel and Procaccia (1984) related the perimeter fractal dimension of clouds to $\mathcal{H}$ through the expression $D = 2 - \mathcal{H}$ (Eq. 4), ~~but required~~ with a correction for ~~intermittency in turbulence~~ turbulent intermittency ($\mu$, where $D_\mu = (4+\mu)/3 \approx 5/3$ (described below). We obtain, from Eqs. (11) and (4), an adjustment to $D$ for an ensemble of clouds:

$$D_e = 3 - D \tag{4}$$

**Line 188: The quantity 3-D is the fractal codimension of a fractal set embedded in a three dimensional space. In (multifractal) turbulence, the codimension is in fact a function (not a unique value) that depends on the threshold used to define the fractal set. At best this equation is useable for a Gaussian model.**

Revised ln 201-202:

The quantity $3-D$ has been ~~described as the fractal intermittency of turbulence~~ defined as the intermittency exponent by Hentschel and Procaccia (1984) and the multifractal codimension[2] (Schertzer and Lovejoy, 1987)within a 3D ~~volume (Mandelbrot, 1977; Hentschel and Procaccia, 1984).~~ space.

**Line 354: *"Because stratification is only observable in vertical velocity perturbations"*. I don't understand: the role of vertical velocity is not clear, and the data on vertical velocities is inadequate. However, the fact of scale dependent stratification and the key Hz parameter ( the ratio of horizontal and vertical scaling exponents) has been estimated in several fields: Temperature, potential temperature, humidity, horizontal velocity, lidar reflectivity (aerosols), radar reflectivity (clouds). This is reviewed and summarized in ch. 6 of [Lovejoy and Schertzer, 2013], see in particular, table 6.5.**

Removed this statement. The other points are addressed through minor clarifications in other comments.

**Eq. 15: The Richardson law is nearly equivalent to the Kolmogorov law. In the 23/9D model, the standard Kolmogorov law holds in the horizontal (but not vertical), and therefore, we expect the Richardson 4/3 law to hold in the horizontal (but not vertical). Using your eq. 8, we expect the vertical exponent to be 1+3/5 = 8/5 rather than the horizontal value 4/3 (the value 14/9 is not justified). Here you imply the existence of an isotropic Richardson law that would certainly contradict the highly anisotropic 23/9D model.**

[2] The difference between the spatial dimension of the domain and the fractal dimension

Revised ln 374-381:

> To account for this anisotropy in the vertical, ~~$\mathcal{H}$~~ $\mathcal{H}_z$ for the combined turbulence case was derived from the ratio of the horizontal and vertical Hurst exponents $\mathcal{H}_H = 1/3$ and $\mathcal{H}_V = 3/5$, resulting in ~~$\mathcal{H}_H/\mathcal{H}_V = 5/9 \sim 0.56$~~ $\mathcal{H}_z = \mathcal{H}_H/\mathcal{H}_V = 5/9 \sim 0.56$. From Eq. (11), the elliptical dimension becomes $D_{el} = 14/9 = 1.56$ (for the volume, $23/9 = 14/9 + 1$. See Lovejoy (2023) for a review.) From Eq. (3), the turbulent diffusivity for this intermediate 23/9D regime then scales as
>
> $$K_{\xi,int} \sim \xi^{14/9} \tag{5}$$
>
> Note that Eqs. (13) and (14) correspond to the isotropic cases of 2D and 3D turbulence, while Eq. (15) combines the vertical and horizontal components of $\mathcal{H}$ to arrive at an anisotropic case of turbulent diffusivity that applies at all scales.

**Line 353: maybe stress that these exponents correspond to the horizontal velocity component with the subscript only indicating the direction of the separation.**

Revised ln 367-371:

> This continuous scaling accounts for the horizontal-vertical anisotropy of the atmosphere due to stratification and is determined by comparing velocity fluctuations $\Delta v_H$ and $\Delta v_V$ ~~in the horizontal and vertical directions. Because stratification is only observable in vertical velocity perturbations, it is~~ corresponding to the horizontal velocity component with subscripts $H$ and $V$ indicating the horizontal or vertical separation between measurements. Horizontal velocity fluctuations have been widely observed to follow a 3D scaling $\Delta v_H \sim \varepsilon^{1/3}\ell^{1/3}$ ~~up~~ where $\ell$ ranges from order $\sim$1 m to the planetary scale (Lovejoy and Schertzer, 2013).