

Reviewer #1:

The two-layer model (modeling boundary layer and free atmosphere), developed by the authors, can express the effect and feedback from the boundary layer to the free atmosphere. The effect and feedback cannot be expressed by the previous one-way configuration. The model has the potential to understand essential processes of the TC vortex development. The model is also suitable for the scaling law study. A scaling law is a power law that asserts a proportional relationship between relevant quantities. In general, the finding of the scaling law for a phenomenon in fluid systems can be largely helpful for understanding the phenomenon. It will be very fascinating to apply the two-way setting model to the scaling law studies in many different typhoon cases. Overall, the paper is well-written and can be published.

Thank you for reviewing the manuscript and providing constructive and helpful comments. We have made edits to the manuscript incorporated with your suggestions. Reviewers' comments are shown in black, our response to each comment is shown in blue, and changes to the manuscript are shown in red.

Major

My major concern with the paper is a potential gap between the authors' model and the real observations or full-physics model simulations. Specifically, the momentum transport in the two-layer model may need to be considered on some occasions. As the authors also pointed out, even in the two-way configuration, the updraft from the SBL does not provide momentum to the SWM (L332-340). That is, the vortex or tangential wind in the free atmosphere in the two-way configuration may be enhanced through gradient wind adjustment by the change in the pressure distribution of the free atmosphere associated with the updraft from the SBL. If so, there may be some differences in the intensifying process of the vortex between the authors' model and the full physics model. In the full physics model, the momentum transport associated with the updraft from the boundary layer to the free atmosphere greatly affects the development of the vortex in the free atmosphere (e.g., Fig. 6 of Wang et al. 2016). This suggests that the TC vortex can develop by a process that cannot be described by the authors' model. To what extent the evolution of the vortex represented by the authors' model is valid for the evolution of real TC vortices does not seem to be discussed in the current manuscript. If the authors can quantify or estimate the validity of their model, it may be useful to add a discussion in the text.

Reference: Wang, H., C. Wu, and Y. Wang, 2016: Secondary Eyewall Formation in an Idealized Tropical Cyclone Simulation: Balanced and Unbalanced Dynamics. *JAS*, 73, 3911-3930, <https://doi.org/10.1175/JAS-D-15-0146.1>

Thank you for this important and insightful point, and we agree with you that there are indeed caveats and limitations to this idealized model. We are fully aware of the importance of the vertical advection of momentum transport out of the boundary layer in the intensification process. Our intention with this simplified modeling framework is to distill the impacts on the vortex intensification process from diabatic heating, which is parameterized by a mass sink in this study. Our results do not preclude the possibility of other

mechanisms being important or even perhaps essential in the real atmosphere, and we have added new text to the manuscript to emphasize this important point. As we outlined in the introduction, while using a full-physics model simulation could aid in studying the kinematic and thermodynamic processes, the complexity involved makes it challenging to isolate and examine our proposed mechanism amidst the intricate interactions among various processes. We think that selective addition of various processes, such as vertical advection of momentum, could be very enlightening and a fruitful avenue for future work. In light of these considerations, we have incorporated an additional discussion in the manuscript to underscore the limitations of our model at L399:

We reiterate that the only forcing in the SWM is through the mass sink term, which is driven by the boundary layer updraft. The location of the boundary layer updraft in turn is controlled primarily by the pressure gradient force aloft, which is associated with the structure of the PV field. Our simplified modeling framework is specifically designed to distill the impacts on the vortex intensification process from diabatic heating forced by boundary layer convergence. However, such a simplification neglects other processes such as vertical advection of momentum out of the boundary layer, entrainment, vortex alignment and vertical growth, and other factors which impact the intensification process in the real atmosphere. We have identified one pathway to rapid intensification of a vortex in a highly simplified framework, but our results do not preclude the possibility of other mechanisms being important or even essential in the real atmosphere. For example, the advection of the boundary layer momentum and the adjustment of the supergradient winds in the boundary layer to gradient flow aloft is essential to prevent an eventual shock and frontal collapse in the shallow water layer. There is not a sufficiently strong supergradient flow to prevent the inward contraction of the RMW in the absence of that forcing, such that in the current framework, we must simply end the simulation before such effects become too strong to ignore. The slab boundary layer is in itself a simplification and likely overestimates the strength of the updraft in some areas, so additional momentum effects would need to be considered and modeled carefully. The current modeling framework provides an avenue for such additions of selective processes to design experiments with other intermediate levels of complexity. Such additions and experiments would enable a better assessment of which factors are indeed critical for intensification in the real atmosphere, and are recommended for future work.

Minor

There are some typographical errors in the model equations. Please correct them. For example, it may be necessary to revise the sign of the pressure gradient force in Eq. (5), u_b (correctly) in the last term of the right side of Eq. (5), and the specific expression of the Laplacian in Eqs. (4) - (5).

Thank you for catching the errors. We have corrected the terms and provided the expression of the Laplacian.

$$\nabla^2 = \partial^2/\partial r^2 + (1/r)(\partial/\partial r) + (1/r^2)(\partial^2/\partial \lambda^2)$$