Addundum to the Response to Referee 1 on "A fast and unified subglacial hydrological model applied to Thwaites Glacier, Antarctica" by Kazmierczak, Gregov, Coulon & Pattyn.

Dear Referee,

Following the comments of all the Referees, we have made changes and improvements to several sections of our manuscript. In this addendum to our previous response, we would like to describe in more detail the additional changes that are relevant to your comments, as we believe they may be valuable. This addendum is divided into four parts:

A. Assumptions of the hydrological model

B. Clarification of the expression of the melt rate

C. Form of the Darcy-like flow equation

D. Justification and impact of the drainage density

The modifications and additions to the initial manuscript are written in blue.

Best regards,

On behalf of the authors,
Thomas Gregov

A. Assumptions of the hydrological model

In the new structure of the model description, we have tried to improve the description of the hydrological model by stating the main assumptions prior to the derivation of the model. This has resulted in an additional subsection to the Model section, which is now structured as follows:

1. Ice-flow model
2. Hydrological model
    - 2.1. Simplifying assumptions
    - 2.2. Subglacial water routing
    - 2.3. Subglacial effective pressure
    - 2.4. Bed rheology

Specifically, the following discussion has been added to the new 'Simplifying assumptions' subsection:

"*The key simplifying assumptions are given by the following:*

1. *There is limited temporal melt variability so that the hydrological system is in a quasi-static equilibrium with respect to the ice-sheet geometry. Therefore, changes in ice geometry will be the main driver for changes in subglacial water variability (both spatial and temporal).*

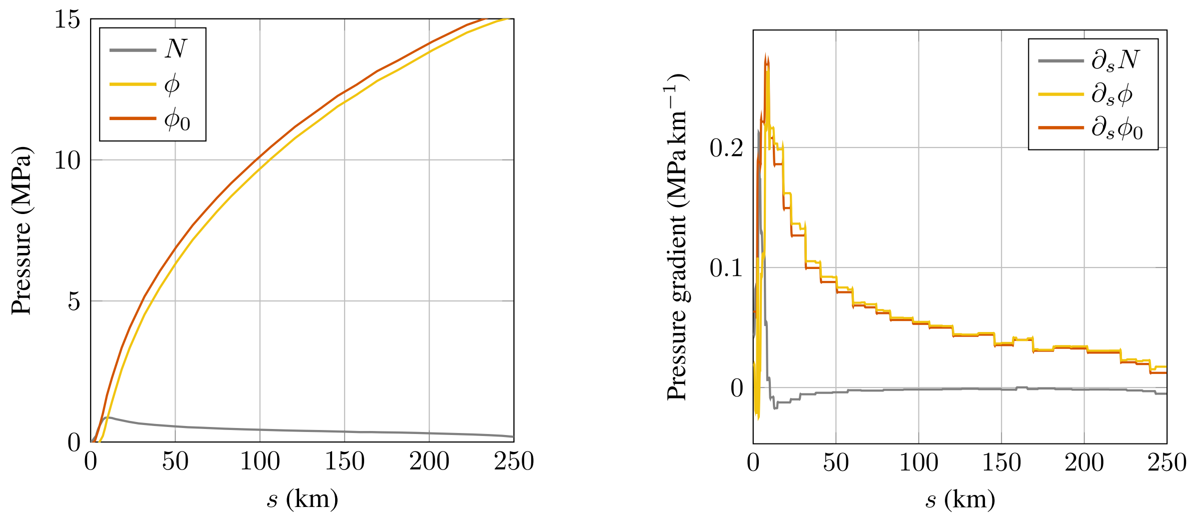
2. *A few kilometers upstream of the grounding line, the hydraulic gradient is approximated by the geometric gradient.*

3. *The drainage density is uniform and the effective pressure is not calculated at a sub-grid level.*

The first assumption is based on several studies of subglacial hydrology in Antarctica (Le Brocq et al., 2009; Pattyn, 2010; Kazmierczak et al., 2022), among others, that demonstrate that — contrary to the Greenland ice sheet— there is limited surface meltwater infiltration. Hence, *changes in hydrology are primarily due to changes in ice geometry. Since the time scales associated with water flow are much smaller than those associated with ice flow, subglacial hydrology automatically adapts to any change in ice geometry and reaches the associated equilibrium. The second assumption is motivated by a scaling analysis through an estimation of the dimensionless ratio $\eta := [\nabla N]/[\nabla\phi_0]$, where $[\nabla N]$ is the scale of the spatial gradients for the effective pressure and $[\nabla\phi_0]$ is the characteristic scale for the geometric potential gradient. For the former we take $[\nabla N] = [N]/[x]$, with $[N] = 1\,MPa$ and $[x] = 10^3\,km$. For the latter we take $[\nabla\phi_0] = 5 \times 10^{-2}\,MPa\,km^{-1}$, which is a plausible value for ice sheets (Hewitt, 2011). This results in $\eta = 2 \times 10^{-2} \ll 1$, suggesting that $\|\nabla N\| \ll \|\nabla\phi_0\|$ and $\nabla\phi \approx \nabla\phi_0$. We further note that profiles obtained with a high-resolution subglacial hydrology model suggest that $\nabla\phi$ and $\nabla\phi_0$ have a correlation of at least $\sim 80\%$ for a region that is several kilometers upstream of the grounding line (see Supplementary Material S1). Finally, the third assumption follows from our modeling approach, where we do not describe the effective pressure at the sub-grid scale and where we assume the same number of conduits in each grid cell, similar to Gowan et al. (2023).”*

The Supplementary Material S1 refers to the assessment of the assumption that $\nabla\phi \approx \nabla\phi_0$ outside the vicinity of the grounding line based on data. Here is the content of this addition to the supplementary materials:

“Here, we provide additional data to underpin the validity of the assumption that $\nabla\phi \approx \nabla\phi_0$ outside the range of influence of the grounding line, which is a few kilometers from it. Since there are no direct observations of the effective-pressure field in Antarctica, we have to rely on high-resolution models. A first test case comes from Lu and Kingslake (2023) who uses a high-resolution model that couples ice-sheet dynamics and subglacial hydrology for hard beds. Potential limitations of that study is that it considers a flow line and a smooth bedrock. The assumption that $\nabla\phi \approx \nabla\phi_0$ a few kilometers upstream of the grounding line is confirmed numerically (Figure R1).

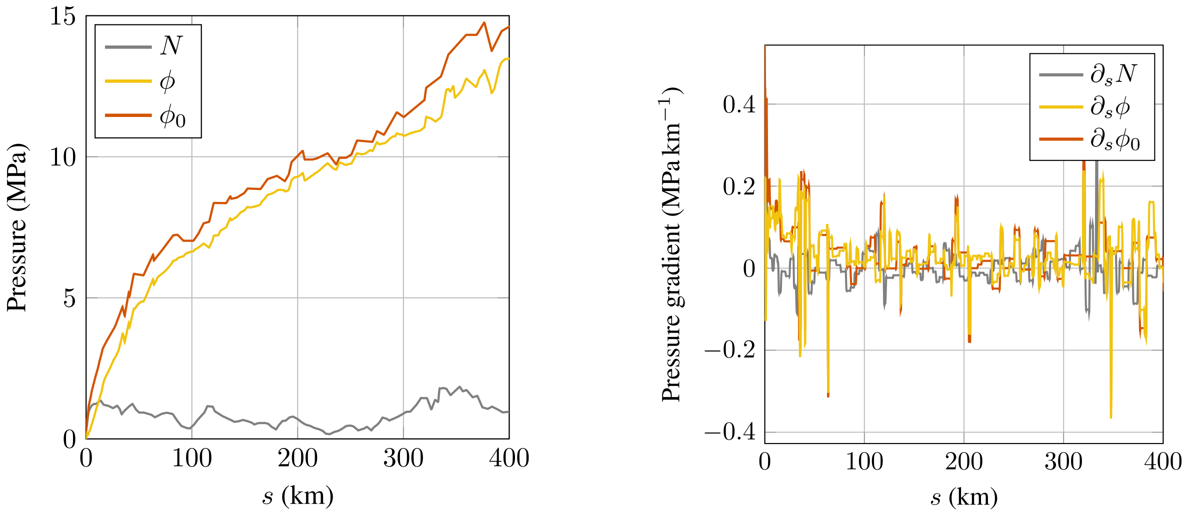


(a) Numerical results.

(b) Computed gradients.

Figure R1: Data derived from Figure 4 of Lu and Kingslake (2023).

A second test case comes from Hager et al. (2022) who applied the high-resolution model MALI (Hoffman et al., 2018) to Thwaites Glacier. They also consider a hard-bed hydrology. The computed effective pressures along a center-line transect are shown in Figure R2. Note that the signals are much more noisier compared to the first test case. This noise can be attributed to the model resolution, but also to the presence of localized hydrological features that cross the center-line transect at which the effective pressures are evaluated, therefore resulting in very localized variations. However, we observe a good correlation between $\partial_s\phi$ and $\partial_s\phi_0$ out of the *vicinity of the grounding line (Figure R2): $\sim 80\%$ over the range $[10, 400]$ km, suggesting that the assumption that $\nabla\phi \approx \nabla\phi_0$ is valid in this region."*



(a) Numerical results.

(b) Computed gradients.

Figure R2: Data derived from Figure 8 of Hager et al. (2022).

B. Clarification of the expression of the melt rate

The expression for $\dot{m}_\mathrm{w}$ was missing from the manuscript. We have modified the introduction of the melt rate as follows:

"The latter is computed from the energy balance within the ice sheet and includes effects of geothermal heat flux, frictional heating due to the motion of both ice and subglacial water, and thermal conduction, i.e.,

$$\dot{m} = \frac{G + \boldsymbol{\tau}_\mathrm{b} \cdot \boldsymbol{v}_\mathrm{b} - q_T}{L_\mathrm{w}} + \dot{m}_\mathrm{w}\,, \tag{R1}$$

where G is the geothermal heat flux, q_T is the thermal conduction flux, L_w is the latent heat for ice, and $\dot{m}_\mathrm{w} = |\boldsymbol{q}_\mathrm{w} \cdot \nabla\phi|/L_\mathrm{w}$ is the water melt rate due to the dissipated energy from the subglacial water conduits. However, we do not include this last term in our simulations as it was found to be negligible compared to the other terms."

C. Form of the Darcy-like flow equation

Regarding of the use of the Darcy–Weisbach equation equation for the relation between Q_w, S, and $\nabla\phi$, we now mention that we follow the approach taken in Schoof (2010), although we acknowledge the possibility of considering other parametrizations:

"Following Schoof (2010), we assume a turbulent flow, with $\alpha = 5/4$, $\beta = 3/2$, and $K = (2/\pi)^{1/4}\sqrt{(\pi+2)/(\rho_\mathrm{w} f)}$, where f is a friction coefficient (e.g., Clarke, 1996). Other choices have been considered for subglacial hydrology in the literature; we refer to Hewitt (2011) and Werder et al. (2013) for laminar parametrizations, and to Hill et al. (2023) for a discussion of the transition between laminar and turbulent flows and their range of validity."

D. Justification and impact of the drainage density

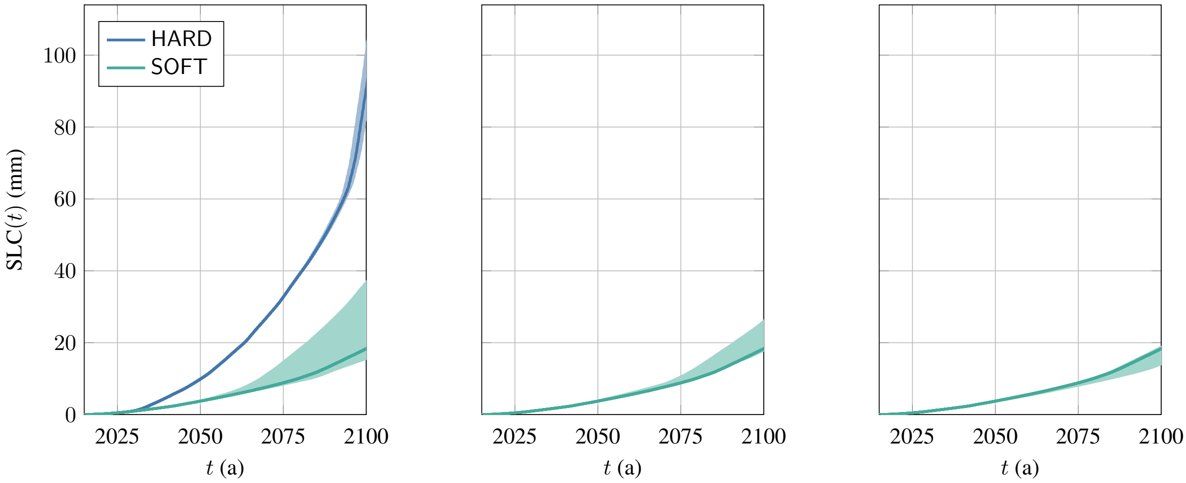
We have made several changes to the manuscript with respect to the uniform drainage density. It is now explicitly mentioned in the additional 'Simplifying assumptions' subsection (see comment in part A of this addendum).

Moreover, we have added the following after sentence the introduction of the parameter l_c:

"*We take $l_c = 10$ km, which is similar to the value considered in Gowan et al. (2023) based on observations of distances between eskers formed under the Laurentide Ice Sheet (Storrar et al., 2014).*"

Finally, we have added an appendix with a sensitivity analysis with respect to l_c, and also with respect to the other unconstrained parameters, namely, Q_c and F_{till}:

"*We performed a sensitivity analysis of the least constrained parameters of our model, i.e., l_c, Q_c, and F_{till} (Figure R3). It can be observed that l_c has only a limited effect for hard beds, while it has a more pronounced impact for soft beds. From equation (4), a change in l_c results in a change in the water flux Q_w, which will be important if water flow transitions from an efficient to an inefficient flow (or the reverse). However, for hard beds, the entirely efficient or inefficient cases yield similar results (Figure 9b). On the contrary, for soft beds, the difference between the entirely efficient or inefficient cases is more pronounced (Figure 9b), and it follows that there is a stronger dependence with respect to l_c. For Q_c and F_{till}, the impact is limited. Finally, it can be noted the spread in the results increase over a time.*"



(a) Effect of l_c. (b) Effect of Q_c. (c) Effect of F_{till}.

Figure R3: Sensitivity analysis of the results with respect to the parameters l_c, Q_c, and F_{till}. The set-up is the same as the one described in the forcing experiments over Thwaites (subsection 4.2, Subglacial hydrology on homogeneous beds), except that different values of these parameters are chosen. The shaded areas correspond to the ranges $l_c \in [5, 15]$ km, $Q_c \in [0.5, 1.5]\, m^3/s$, and $F_{till} \in [1, 2]$, and the lines correspond to the nominal values considered in the original experiment.

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