

## **Interactive discussion: Response to comments from Anonymous Referee #2**

November 22<sup>nd</sup>, 2024

### **General comment**

In the manuscript, Eslami et al. (2024) have investigated the celerity of aggradations in a controlled flume experiment and explored the relationship between the celerity of aggradation waves with the celerity of propagation of small perturbations. This is a nice work and detailed analysis of the experiments; however, I have a number of questions, partly regarding the theoretical conceptualization and clarifications on several points.

Many thanks for appreciating the work, spending time on it and providing useful comments. We have taken the liberty to group the comments by category (choosing some category titles and changing the comment order), to make our responses more effective.

### **Eigenvalues as the celerities of small disturbances**

The authors seem to accept the celerity parameters are eigenvalues of the coefficient matrix of a system of partial differential equations as proposed previously in the literature. I am skeptical about whether these eigenvalues are indeed representing the celerity of a disturbance (of bed or surface water). Thus, I would recommend adding a theoretical background section to introduce how the eigenvalues become celerity from a mathematical perspective. For example, after linearizing the non-linear partial differential equation and establishing a system of differential equations,  $dU/dt + AdU/dx = 0$ , explain how this concept translates into the celerity of a disturbance happening at a water surface or streambed. Essentially, you can mention that this problem can be treated as Riemann invariant (e.g., Lyn and Altinakar, 2002) under a characteristic path of  $dx/dt = \lambda$ , where  $\lambda$  is characteristic speed or wave speed, which I think is how the term celerity arises. In this path, the disturbance waves move at constant speed over spatial and temporal scales.

We have indeed presented the system of differential equations in eq. (5) of the manuscript (that is, however, a system of non-linear equations). In a revised manuscript, we will expand the explanation at lines 113-116 by adding the concept of Riemann invariant; thanks for the suggestion.

In the introduction, the concept of celerity of propagation of small perturbations and aggradation wave is not clearly defined. I do not understand what are referred to as small perturbations, and in what criteria the authors define a “small” perturbation here. As I read the introduction of the manuscript, I assumed that the celerity of propagation of small perturbations is in this context, water wave celerity and bed wave celerity. It would be nice to introduce/explain this concept in a clearer way.

The “small” or “infinitesimal” disturbances are the terms used in the literature when the surface of bed or water are perturbed locally and the resulting waves are of limited amplitude (e.g., Rosatti et al., 2004). As mentioned, the eigenvalues of the coefficient matrix of a system of partial differential equations are interpreted as the celerity of propagation for these local, small-scale waves. In the revised manuscript we will rephrase for clarity.

Eigenvalues are the solutions of characteristic equations that encompass information of matrix coefficients of a system of differential equations. Hence, it is difficult to say, for example, that  $\lambda_1$  is celerity of water flow, as this manuscript and previous authors suggested, or  $\lambda_2$  and  $\lambda_3$  are celerity of bed perturbations. Rather, it may be a combined effect of both water flow and streambed on these eigenvalues. What are the viewpoints of the authors for these parameters, according to the experiment performed here? Additionally, does this characteristic equation  $\det(A-I) = 0$  always have 3 real roots? Is there any scenario such that the equation yields two negative roots instead of one, as shown in this paper?

Considering the first question, we have mentioned in the manuscript that there is no complete agreement on how the eigenvalues should be associated to perturbations in the water and bed surfaces (lines 129-132 for De Vries, lines 138-140 for Lyn and Altinakar). In our view, the issue can be hardly solved experimentally because, as mentioned in the previous comment of the Reviewer, the “small perturbations” are not well defined and thus can be hardly observed. We will add this consideration to the revised manuscript.

Since the governing system of equations is hyperbolic, it is well-established that it always produces real and distinct eigenvalues (Rosatti et al., 2004; Rosatti & Fraccarollo, 2006). In agreement with this theory, our experimental results consistently yielded three real eigenvalues. A different behavior may be encountered, instead, with approximate determinations of the eigenvalues. While the approximate solution proposed by Goutière et al. (2008), which is valid for any value of the Froude number, always resulted in three real eigenvalues during our experiments, methods such as those by Lyn and Altinakar (2002) or De Vries (1965, 1971, 1973, 1993) sometimes produced non-real roots. These approximated determinations are not valid across the full range of Froude numbers, which led to inconsistencies in certain conditions.

As per the last question, we have never encountered cases with two negative and one positive eigenvalues.

### **Definition of an aggradation celerity and its relationship with the previous one**

As mentioned in the manuscript, the celerity of the aggradation is easily quantified in low Froude number flow since the translational migration of bedload can be captured visually. The authors noted that in the high Froude number flow, sediment and water are moving more dispersive; thus, the celerity of aggradation needs to be more precisely defined before diving into the analysis of the partial differential equations in chapter 2, and later quantified by equation 20.

An aggradation wave represents a larger-scale process compared to “small perturbations” and thus requires a different definition for its celerity of propagation. We will rephrase the Introduction of the revised manuscript to improve clarity.

Line 165, equation 19: I don't understand why setting  $dX/dt = 0$  in Equation 19. Does that imply that  $C = -(dX/dt)/(dX/dx) = 0$ ? Later, the authors used bed elevation for their

calculations, but  $dz_b/dt$  is not 0, which is a contradiction. Also, in lines 163 and 164, if I understand correctly, celerity is a scalar quantity, while velocity is a vector quantity; they are not the same. On line 67, you mentioned “the celerity of propagation of small perturbations is not the celerity of propagation of the aggradation wave.” Then, in equation 19, you defined the local and instantaneous  $C_x = dx/dt$ , which is the same as the celerity of a disturbance. This is again another contradiction.

The short reply is that there are no contradictions but, evidently, in the manuscript we have not been effective enough in our explanations. We here reply to all the questions of the Reviewer and declare an intent to expand the explanation in the revised manuscript.

Regarding the first question: as mentioned in the manuscript (line 164) the celerity of propagation of a certain quantity  $X$  is equal to the velocity at which an observer needs to move to see a constant value for  $X$ . “Constant value for  $X$ ” then translates into  $dX = 0$  and then  $dX/dt = 0$ . Here we have a total derivation, “ $d$ ”. On the other hand, in eq. (19) we have partial derivatives, “ $\partial$ ”. Therefore, not at all a condition that  $dX/dt = 0$  will imply that  $C = 0$ .

We impose  $dz_b/dt = 0$  to find the celerity at which any value of the bed elevation migrates, and this is fully appropriate even if the bed elevation varies in both space and time. In this respect, we present the following depiction, that we produced reworking Fig. 2(a) of the manuscript. Here, we see that any  $z_b$  value in a profile for a certain time may be found in a profile for larger time, at a downstream location. This indeed demonstrates that a  $z_b$  value migrates downstream, at a certain celerity whose determination requires looking for the same value at another space and another time, so exploiting  $dz_b/dt = 0$ . In the plot, we show values of  $z_b$  in a range from 2 cm to 9.5 cm, every 0.5 cm. Migration is depicted by a horizontal displacement from a red circle to a black star (note that a black star may have a red circle around if it becomes the starting point for another displacement).

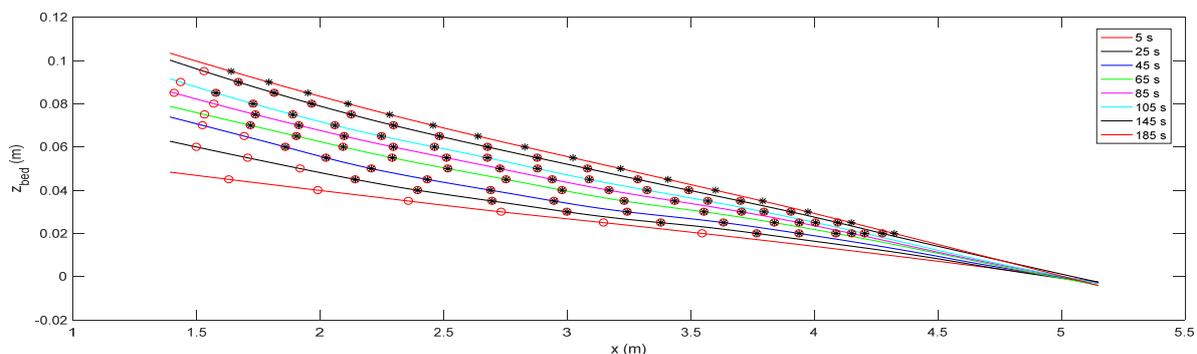


Figure R1. Migration of  $z_b$  values along the channel during the experiment (symbols explained in the text just above).

Third issue: velocity is indeed a vector, but here we are exploring the propagation in 1D conditions. Therefore, the direction of this vector would be confined to the channel's longitudinal direction, effectively reducing it to a scalar in the 1D framework.

Last issue is a contradiction possibly arising due to using  $dx/dt$  for every celerity. Actually, indeed a celerity is always a  $dx/dt$ , since it is a velocity. But different objects may have different celerities (like, for example, a person walking on a

moving train and the train itself). In the manuscript, we wrote (line 383) that “the smaller, the faster”. Morphological processes, indeed, are frequently characterized by multiscale propagation. Let us mention a couple of examples from our recent experience. Zanchi and Radice (2021) noticed that, in their aggradation processes under subcritical flow, dunes were sometimes superimposed on depositional forms. The propagation celerity of the dunes may be different from that of the aggradation front. Radice (2021) investigated the propagation of bed-load dunes along a channel at equilibrium, and spotted a multi-celerity propagation considering the dunes, the sediment particles, and the sediment gusts triggered by turbulent flow events. In the present study, an aggradation wave (large object) moves at a different celerity compared to the “small perturbations”.

Line 255, equation 20: Is equation 20 derived from equation 19? It seems to me that celerity can be simply defined as  $C = dx/dt$  (as in de Vries, 1993; if  $x$  is difficult to quantify, you can then use  $dx/dt = (dX/dt)/(dX/dx)$  where  $X$  is easily to quantify (e.g., bed elevation in the paper). I am not sure about the minus sign in this equation, as it contradicts the standard definition from de Vries or from Morris and Williams.

We first rewrite the Reviewer’s equation with other symbols, because again it is important to see partial derivatives correctly. The Reviewer writes:

$$\frac{dx}{dt} = \frac{\partial X / \partial t}{\partial X / \partial x}$$

In a standard case with aggradation, depicted below (fig. R2), this quantity is negative if applied to the bed elevation, because the numerator is positive while the denominator is negative. Since we intend to use these derivatives to depict the downstream propagation of an aggradation wave, the change of sign is appropriate.

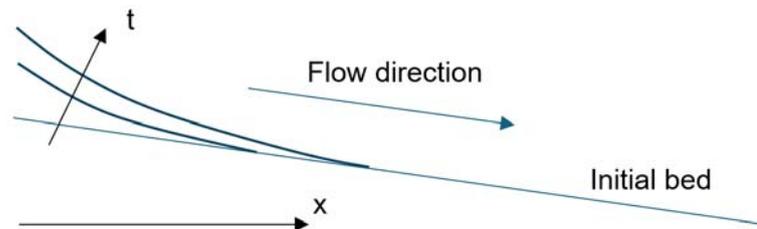


Figure R2. Sketch of the aggradation process.

Line 67-68: Why is the celerity of propagation of small perturbations not the celerity of propagation of the aggradation wave? For example, if the aggradation wave is moving downstream and generating disturbances, can these disturbances generate perturbations of water and bed?

The replies above should have clarified the issue that different objects may move at different velocities. Here we add that the points depicted in Fig. R1 can be used to estimate the celerity of the aggradation wave, since any couple of points (a red circle and a black star) corresponds to a displacement within a certain time and thus to a celerity. The celerity values computed for those couples of points are depicted in Fig. R3 (color bar on the right is for the celerity values). Taking

0.015 m/s as a reference value, it corresponds to  $C/U = 0.02$  for  $U = 0.705$  m/s that is the nominal flow velocity for the experiment. These values are, as expected, in agreement with those of Fig. 5 of the manuscript, obtained from eq. (20); by contrast, they are largely different from those of the eigenvalues (see again Fig. 10 of the manuscript).

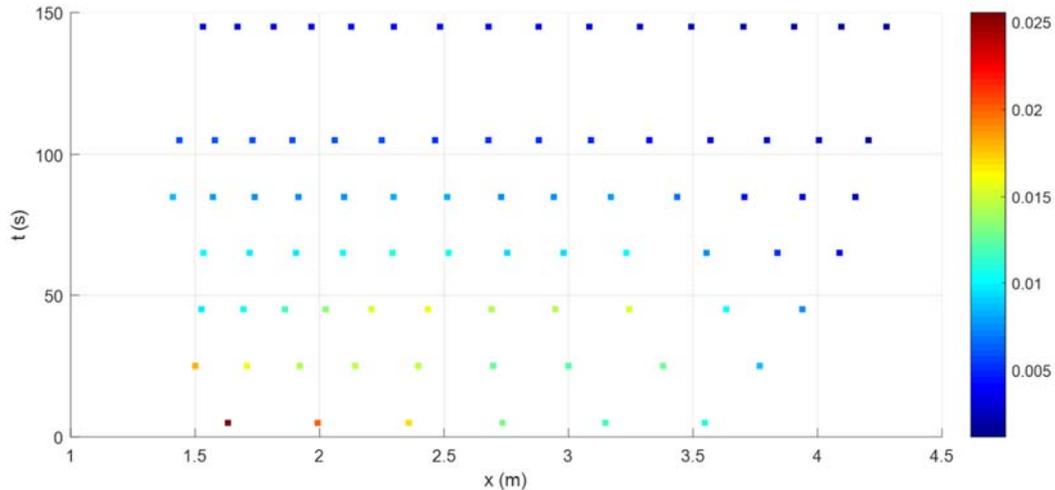


Figure R3. Celerities of migrating bed elevations computed from the point couples of Fig. R1.

### Research questions and their answers

The goal of the present manuscript is not clear according to the results presented there. The authors aim to answer 3 questions. (1) How can one quantify the scales of propagation in an aggradation process? (2) What is the relationship between the aggradation celerity and the celerity of small perturbations? And finally, (3) Which is the impact of considering or not the sediment concentration on the previous point? However, results and discussions were not focused on answering them; instead, the authors presented some graphical results between the aggradation celerity and the celerity of small perturbations, and no specific formula or relationships between have been clearly determined. What are some findings for questions 1 and 2, I did not see it? How one can predict the spatial and temporal scales of an aggradation wave, given the information to solve the system of differential equations here? Having read the title and introduction, I hope the present study can show when and where the aggradations are likely to happen based on the celerity of propagation of aggradations.

Regarding the first question: as mentioned, under high Froude number conditions, the aggradation induced by sediment overloading is dispersive and, as a result, front tracing methods cannot be used to estimate the celerity of the aggradation wave. We thus employed the standard definition of the celerity of a specific quantity, as outlined by Chow et al. (1988) and Jain (2001), which is presented in section 3 of the manuscript. Indeed, by introducing the bed elevation as the specific quantity, we successfully determined the celerity of propagation of the aggradation wave. Figure 5 shows this celerity of propagation in dimensional and dimensionless form. Therefore, the short answer to question 1 is: by applying eq.

(19-20) to the bed elevation. We might rephrase slightly question 1, pointing at supercritical/dispersive aggradation conditions.

Regarding the second question: Figure 10 illustrates the correlation between the dimensionless eigenvalues of the system (representing the celerity of propagation of small perturbations) and the dimensionless celerity of the aggradation wave (determined in section 3). The aim of this work is not to derive a formula explicitly relating the two celerity scales, but rather to demonstrate how these two scales correlate to each other. Based on this, it might be more appropriate to revise the second question to: “How does the aggradation celerity correlate with the celerity of small perturbations?”. Furthermore, in response to one of the first reviewer’s suggestions, we have computed the Pearson correlation coefficients to quantify the strength of the relationships between  $C/u$  and  $Fr$ , as well as  $C/u$  and  $\lambda_i/u$  (Figures 8 and 10). The values in the following table range between 0.5 and 0.6, indicating a moderate correlation between the variables. We can add this table and the resulting interpretation to our manuscript to strengthen this aspect of the analysis, and thus the answer to question 2.

	<b>Goutière et al. (2008)</b>				<b>Morris and Williams (1996)</b>		
	<b><math>Fr</math></b> (Fig. 8)	<b><math>\lambda_1/u</math></b> (Fig. 10)	<b><math>\lambda_2/u</math></b> (Fig. 10)	<b><math>\lambda_3/u</math></b> (Fig. 10)	<b><math>\lambda_1/u</math></b> (Fig. 10)	<b><math>\lambda_2/u</math></b> (Fig. 10)	<b><math>\lambda_3/u</math></b> (Fig. 10)
<b><math>C/u</math></b>	-0.53	0.54	-0.60	-0.54	0.55	-0.59	-0.55

Regarding, finally, the third question: in different parts of the manuscript, we have addressed the impact of considering or disregarding sediment concentration on the correlation between the celerity of small perturbations and the celerity of the aggradation wave. For example, in discussion part, line 374 it has been mentioned that: “In the context of a relationship between the  $\lambda$  and  $C$ , the possibility or not to discard the sediment concentration while expressing the eigenvalues loses its merit. Obviously, formulating the eigenvalues with or without concentration changes their values (even by 50 – 100% for  $\lambda_2$ , that is the eigenvalue most affected by the sediment concentration), but it will be always possible to find a trend linking the celerities of small perturbations and the celerity of the aggradation wave”. Similarly, the conclusion chapter briefly summarizes these findings: “From a mathematical point of view, using a model that considers or not the solid concentration may obviously lead to different results for the eigenvalues of the system (while  $\lambda_1$  is almost the same for the two approaches and a slight difference exists in  $\lambda_3$ , a major difference is observed for  $\lambda_2$  that has a higher absolute value when the concentration is discarded). However, considering or not solid concentration in the governing equations does not affect the qualitative relationships between the different celerities”.

### **Interpretation/discussion**

In the discussion section, there is little interpretation of values of  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$ , as well as  $c$ , in the context of the performed experiment. What exactly do these lambdas represent? The authors just presented results with minimal intuition and without

comparison with previous work (i.e., Zanchi and Radice, 2021) to distinguish between subcritical and supercritical cases. I expected to see more on the comparison of how changing lambdas can impact the bed aggradation and bed elevations.

Based on the explanations provided above and in the manuscript, we believe we have addressed the first part of the question regarding the interpretation of eigenvalues and  $C$ . We will ensure this is made clearer in the manuscript, particularly by elaborating on the distinction between small-scale waves and large-scale waves, as well as their respective celerities.

Regarding the second part of the comment, it is important to note that the work of Zanchi and Radice (2021) primarily focused on a bulk approach to the issue rather than investigation of the correlation between the two types of celerity of propagation. However, in the reply to a following issue (implications for field conditions) we present the  $c/U$  values of the experiments of Zanchi and Radice (2021). It is worth mentioning that the experiment presented in this manuscript is one of several conducted in an extended experimental campaign, the global results of which will be presented in future papers. Basically, in a bulk analysis, we have extracted bulk values of celerity for each experiment and derived a formula predicting the celerity of propagation of the aggradation wave based on the control parameters. However, this is beyond the scope of the present manuscript.

Line 345: I don't quite understand how the correlation trends are obviously consistent. Which one is consistent with the other?

What we intended is the following. Let one consider three quantities,  $A$ ,  $B$  and  $C$ . If  $A$  increases with  $B$  and decreases with  $C$ , then  $B$  needs to decrease with  $C$ . If one finds the latter the trends are consistent; if one instead finds that  $B$  increases with  $C$ , then there is something wrong. In the present study we found consistent trends; in the revised manuscript we will rephrase to make the statement better understandable.

Line 347: The authors stated: "The second correlation ( $Fr - C/u$ ) has returned the dimensionless celerity as a decreasing function of the Froude number". However, there are some clusters (e.g., 3 lines on top) where  $C/u$  increases with respect to Froude number (Figure 8).

As mentioned in line 378, these points correspond to the initial stages of the experiment, where the water discharge had not yet been adjusted to its nominal value. Specifically, the text states: "The points before the water discharge achieves its nominal value show opposite trends to the others. The approaches introduced above are for unsteady flows, so the different trend cannot be attributed to a limitation of the mathematical depiction. The different trend thus needs to be attributed to the swift change of the flow rate at the very beginning of the experiment."

Line 360: If  $\lambda_2$  and  $\lambda_3$  are attributed to the bed perturbations, can you explain more on the negative values of  $\lambda_2$ ?

In numerical studies, the eigenvalues are considered to propagate the effect of boundary conditions into the domain (e.g., Fasolato et al., 2009). A negative value

of  $\lambda_2$  would thus indicate that this eigenvalue propagates into the domain the boundary condition imposed at the downstream end. An equilibrium condition is frequently imposed at the downstream end of a simulated channel (i.e., no variation in the bed elevation), which is also what we did experimentally with a downstream sill (see profiles in Fig. R1). In this context, a higher  $|\lambda_2|$  would tend to transport faster a condition of no aggradation and thus increase the local channel slope and in turn the aggradation celerity. The first version of the manuscript did not contain these arguments, that we could instead add to a revised one.

Line 378: The authors claimed that it is always possible to find a trend linking celerity of small perturbations and celerity of aggradation wave. This is a bold statement that needs to be tested not only in experimental studies but also at field scale, where things are much more complicated. And I am highly skeptical about this.

The statement was related to Fig. 10, where the plots on the left and those on the right have the same shape, even if the eigenvalues may change. So, what we intended to say is that, since considering concentration or not does not change the shape of the point scatter, a correlation would exist in both cases. On the other hand, we do not know if this would hold for any concentration value, thus the Reviewer is right in issuing a warning. In the revised manuscript, we will reduce the boldness of the statement adding the above considerations.

### **Reproducibility**

In the result section, I assumed the author performed many experiments and listed the average results or best results, but I only see a table summary of the experiment. Is this experiment (and results) reproducible? Updated: I see now in the results section, the authors mentioned that "This result, shown here for a single experiment, was confirmed by the others run in the current experimental campaign". If the authors run other experiments to confirm the findings here, they should be presented in the manuscript, or at least should be in supplemental information.

Line 215, table 1: It seems that only one experiment was performed (e.g., Table 1). I'm worried about the reproducibility of the results presented in this manuscript.

The key trends found in this study apply also to the other runs of the campaign, even though in this manuscript we preferred to show one as a proof of concept. This will be made explicit in the discussion.

### **Implications for field conditions / Engineering relevance**

It is very nice that the authors found the ratio  $C/u < 0.04$  based on the experiment. If I interpret it correctly, doesn't this suggest that during high Froude number flows, sediment transport is more dispersive, making aggradation much harder to occur? I wonder what the threshold for the ratio  $C/u$  would be under field conditions. If the results presented here are valid, I am curious whether this has implications for understanding aggradation

at field conditions. For example, how long would it take for sediment transport under high Froude number flow to alter riverbed and channel morphology?

The last question of the Reviewer is exactly the initial thrust that motivated an extensive campaign on propagation of aggradation.

Let us tell the full story: Radice et al. (2013) discussed how morphologic processes may impact hydraulic hazard assessment and management in upland environments. They thus performed numerical simulations for a mountain river and found that the aggradation in the downstream portion of the modelled reach (that was an in-town portion, thus a key spot to be considered for flood hazard) was independent on a sediment feeding used as an upstream boundary condition. Then the question was, indeed: how long will it take for the upstream feeding condition to get to the town? Radice and Rosatti (2012), for the same river of the other study, compared the bed profiles obtained for a numerical simulation with a certain upstream sediment yield and for another one with zero yield, and tracked the point at which the two solutions coincided; this point moved at around 50 m/h, which is obviously not a general result and also requires trusting the numerical models, but corresponds to a small percentage of the typical flow velocity in a mountain stream.

Finally, once a suitable laboratory facility was available, experiments have been performed, first in subcritical conditions (Zanchi and Radice, 2021 and other companion works), then in supercritical conditions (the present study and companion works).

By the way, the plots below (Fig. R4) present a manipulation of Zanchi and Radice's data, showing a front celerity (that is actually a bulk celerity values, since in that case a front could be indeed tracked) that, with one exception, was below 0.05 times the flow velocity.

We would not put all this story in the revised manuscript; even if it is the path that took us here, it is probably too long and irrelevant for a reader. Furthermore, we think that the value of 0.04 may be left unchanged in the manuscript to stick to the present experimental conditions, possibly adding a warning that the results should not be generalized at this stage. As mentioned in a previous reply, we are presently working on another manuscript that will aim at being a counterpart of Zanchi and Radice (2021) for supercritical flows, to provide a bulk celerity predictor valid for these conditions.

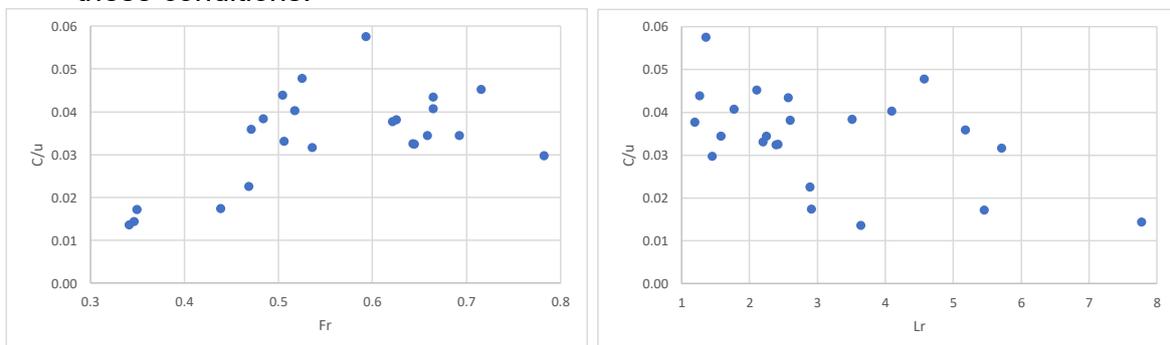


Figure R4. Dimensionless front celerities for the experiments of Zanchi and Radice (2021).

## Other

Line 69: What is “something different” here?

We think that it is actually the definition of eq. (1), then becoming (20). We will rephrase for clarity.

Line 73: Missing parenthesis.

Will be fixed.

Line 129, equation 10: Lack of definition for Froude number.

Will be defined.

Line 221: The authors mentioned that they did not set the camera to capture the profile before 140 cm, which piques my curiosity. Was there any aggradation or erosion in this section during the experiment?

In all the experiments conducted during our campaign, the overloading ratio was consistently greater than 1. As a result, aggradation was observed along the entire length of the channel, thus also for  $x < 140$  cm.

Line 320, 335: Were results shown in Figures 8 and 10 for the entire profile of the riverbed in this experiment (e.g., 140 to 520 cm) or only for some selected locations along this section?

They are for the entire profile of the riverbed (140 to 520 cm).

Line 383: What are other processes in this context?

We were referring, as examples, to the processes investigated by the referenced studies (long waves for Lanzoni et al., 2006, and bed-load dunes for Radice, 2021). In the revised manuscript, we will revise for clarity.

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