



Intermittency in fluid and MHD turbulence analyzed through the prism of moment scaling predictions of multifractal models

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Abstract. In the presence of waves due *e.g.* to gravity, rotation or a quasi-uniform magnetic field, energy transfer time-scales, spectra and physical structures within turbulent flows differ from the fully developed fluid case, but some features remain such as intermittency or quasi-parabolic behaviors of normalized moments of relevant fields. After reviewing some of the roles intermittency can play in various geophysical flows, we present results of direct numerical simulations at moderate resolution and run for long times. We show that the power-law scaling relations between kurtosis K and skewness S found in multiple and diverse environments can be recovered using existing multifractal intermittency frameworks. In the specific context of the She-L ev eque model (1994) generalized to MHD and developed as a two-parameter system in Politano and Pouquet (1995), we find that a parabolic $K(S)$ law can be recovered for maximal intermittency involving the most extreme dissipative structures.

1 Introduction

A word-frequency study performed on research papers centered on a variety of atmospheric issues indicated that the most frequent cloud-controlling factor is turbulence (Siebesma et al. (2009); see also Pumir and Wilkinson (2016)), likely because of its ubiquity but also because it could presumably explain a multitude of somewhat puzzling phenomena that occur at small scale, be it only that of order-unity dissipation at high Reynolds number. More recently, fully developed turbulence (FDT) has been associated with the barotropic state of large-scale atmospheric turbulence, with the multiplicative effect due to turbulence in the occurrence of the acceleration of the jet stream, and the rapid intensification of hurricanes (Shepherd, 2020; Emanuel et al., 2023; Shaw and Miyawaki, 2024). A similar study for plasma physics might reveal the same feature, namely that the complexity of nonlinear phenomena is the dominant property impeding the development of wide-encompassing theoretical and modeling techniques of small-scale behavior, thus making the much-needed prediction of disruptions in fusion plasmas difficult, even though it is essential.

Observations of magnetohydrodynamic (MHD) and plasma turbulence in space physics are numerous, with consistent progress in the resolution of satellite instrumentation and with now the exploration of the kinetic regime (Fox et al., 2016;



Muller, D. et al., 2020). The access to small-scale dynamics through newly-launched spacecrafts allows for example for a direct evaluation of the dissipation rate spectrum through the measurement of the current and electric field using the Magnetospheric MultiScale Mission (MMS, He et al. (2019)). It has been known for a long time that vortex sheets observed in the first direct numerical simulations (DNS) of turbulence using pseudo-spectral methods could roll-up into vortex filaments (Patterson and Orszag, 1971; Siggia and Patterson, 1978), whereas in MHD the dynamics leads to complex current and vorticity structures stemming from the sheet destabilization observed in DNS in two dimensions (2D) (Matthaeus and Lamkin, 1986; Pouquet et al., 1986), and leading to various reconnection processes and possible singularities (Friedel et al., 1997; Kerr and Brandenburg, 1999; Cartes et al., 2009), a topic however which will not be covered in this review (see also *e.g.* Bhattacharjee (2004); Mininni et al. (2008); Zweibel and Yamada (2009); Daughton et al. (2011); Zhdankin et al. (2013); Lazarian et al. (2020); Oka et al. (2022) for more details). Reconnection and the intermittency associated with singularities have been related (Osman et al., 2014), including at high cross helicity (Smith et al., 2009), and can lead to plasma heating (Marino et al., 2008). Lastly, analytical and numerical attempts were made to determine the possible development of singular structures in fluids and plasmas in the limit of infinite Reynolds number, but the problem remains open.

Furthermore, new accurate observations of the magnetic field of the Earth have been obtained recently from global ocean circulation measurements, leading potentially to a better understanding of oceanic tides, of ionosphere-magnetosphere interactions and of their variabilities (Hornschild et al., 2022). Thus, one of the marked property of velocity and magnetic fields is that of intermittency (and ensuing anomalous scaling), that is the presence of strong localized structures. These structures can be identified as vortex filaments, as Alfvén vortices which are observed in the solar wind (Wang et al., 2019), or current sheets which undergo instabilities such as Kelvin-Helmholtz (KH) (see Barkley et al. (2015) for a recent review of KH), reconnection and thus dissipation (Matthaeus and Montgomery, 1981; Uzdensky et al., 2010; Faganello and Califano, 2017; Adhikari et al., 2021)). An abundance of observations of our close environment points to a complex suite of systems and structures that include turbulence and nonlinearities in MHD and plasma instabilities, displaying as well anomalous scaling and dissipation (see *e.g.* for recent reviews Matthaeus et al. (2015); Chen (2016); Galtier (2018); Schekochihin (2022); Balasis et al. (2023); Marino and Sorriso-Valvo (2023)). Intermittent dissipation in the MHD range has been shown to lead to beam acceleration in the magnetosphere at ionic scales and below (Sorriso-Valvo et al., 2019)), and particle acceleration has also been observed with MMS in the vicinity of a reconnection X-line, leading also to strong turbulence (Ergun et al., 2020).

There are of course plenty other manifestations of intermittency, *e.g.* through non-Gaussian wings on Probability Distribution Functions (PDFs) for Eulerian and Lagrangian fields. Thus, one way to characterize intermittency in turbulence is through the dual observation of large-scale structures separated by sharp active gradients both for fluids and MHD, particularly noticeable in 2D (Kinney et al., 1995; Meneguzzi et al., 1996; Matthaeus et al., 2015). Another way to quantify the degree of intermittency of a flow is to measure the anomalous exponents of structure functions, *i.e.* measure a departure from self-similarity, as done in the solar wind (Burlaga, 1991) and in DNSs (Politano et al., 1995). MHD intermittency models were built (Grauer et al., 1994; Politano and Pouquet, 1995) to explain the observed behavior, but one difficulty resides in the necessity of having a vast amount of data. In this context, after giving the equations in the next section, we shall analyze in §3 numerical results on the third and fourth-order normalized moments in several systems run at moderate Reynolds numbers for long times, and



give a justification of power-law behavior between moments in the framework of turbulence models in §4. We mention other frameworks for the study of such intermittency in §5, and conclude in the last section.

2 Equations, parameters and numerical set-up

60 The incompressible equations for rotating stratified flows in the Boussinesq incompressible framework are:

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p - N\theta \hat{z} + 2\mathbf{u} \times f_0 \hat{z} + \nu \nabla^2 \mathbf{u} + \mathbf{F}_u, \quad \partial_t \theta + \mathbf{u} \cdot \nabla \theta = Nw + \kappa_0 \nabla^2 \theta + F_\theta, \quad \nabla \cdot \mathbf{u} = 0, \quad (1)$$

with \mathbf{u}, θ the velocity and the temperature fluctuations (in velocity units here), w the velocity in the direction of imposed gravity and/or rotation (here, the vertical z direction), p the pressure, N and $f_0/2$ the Brunt-Väisälä and rotation frequencies, and ν, κ_0 the viscosity and thermal diffusivity, taken equal (unit Prandtl number). \mathbf{F}_u, F_θ are forcing terms. For $N = 0, f_0 = 0$, one
 65 recovers the Navier-Stokes (NS) equations with a passive scalar. We also write the magnetohydrodynamic (MHD) equations:

$$[\partial_t + \mathbf{u} \cdot \nabla] \mathbf{u} \equiv D_t \mathbf{u} = -\nabla P + \mathbf{b} \cdot \nabla \mathbf{b} + \nu \Delta \mathbf{u} + \mathbf{F}_u, \quad [\partial_t + \mathbf{u} \cdot \nabla] \mathbf{b} \equiv D_t \mathbf{b} = \mathbf{b} \cdot \nabla \mathbf{u} + \eta \Delta \mathbf{b}, \quad \nabla \cdot \mathbf{b} = 0, \quad P_M = \nu/\eta; \quad (2)$$

\mathbf{b} is the induction in Alfvén velocity units, $P = p + |\mathbf{u}|^2/2$ the total pressure, η the diffusivity, and $P_M = \nu/\eta$ the magnetic Prandtl number. The results described herein have been obtained integrating numerically these equations with pseudo-spectral accuracy using the GHOST (Rosenberg et al., 2020) or CUBBY (Ponty et al., 2005) codes. In the absence of dissipation
 70 ($\nu = 0, \eta = 0, \kappa_0 = 0$), the total energy is conserved as well as cross-helicity and magnetic helicity in MHD, and potential vorticity in the stratified case (see §3.2 for definitions of the helicities which are pseudo-scalars).

Given a typical large scale taken as the integral scale L_0 , and a characteristic *r.m.s.* velocity at that scale, u_0 , one defines the kinetic and magnetic Reynolds numbers and the Froude and Rossby numbers, Fr, Ro in a standard way, namely:

$$R_V = \frac{u_0 L_0}{\nu}, \quad R_M = \frac{u_0 L_0}{\eta}, \quad Fr = \frac{u_0}{L_0 N}, \quad Ro = \frac{u_0}{L_0 f}; \quad R_B = Ro Fr^2, \quad R_\lambda = \frac{\lambda}{L_0} R_V, \quad Ri_g = N(N - \partial_z \theta) / [\partial_z u_\perp]^2. \quad (3)$$

75 Fr, Ro measure the wave period *vs.* the turn-over time $\tau_{NL} = L_0/u_0$, and R_B the intensity of the waves. Are also defined the Taylor Reynolds number R_λ based on the Taylor scale $\lambda = \sqrt{\langle u^2 \rangle / \langle \omega^2 \rangle}$, with $\omega = \nabla \times \mathbf{u}$ the vorticity, and the gradient Richardson number Ri_g . The kinetic and magnetic energies are $E_V = \langle \mathbf{u}^2 \rangle / 2, E_M = \langle \mathbf{b}^2 \rangle / 2$, and $\mathbf{u} \cdot \mathbf{F}_V$ is the kinetic energy input. The point-wise dissipation rates of kinetic and magnetic energy are $\epsilon_v(\mathbf{x}) = \mathbf{u} \cdot \partial_t \mathbf{u}, \epsilon_m(\mathbf{x}) = \mathbf{b} \cdot \partial_t \mathbf{b}$. They can be expressed in terms of the symmetric part of the velocity gradient tensor, S_{ij} , and of j^2 , with $\mathbf{j} = \nabla \times \mathbf{b}$ the current density:

$$80 \quad S_{ij}(\mathbf{x}) = \frac{\partial_j u_i(\mathbf{x}) + \partial_i u_j(\mathbf{x})}{2}, \quad \epsilon_v(\mathbf{x}) = \Sigma_{ij} S_{ij}(\mathbf{x}) S_{ij}(\mathbf{x}), \quad \epsilon_m(\mathbf{x}) = j^2(\mathbf{x}). \quad (4)$$

Finally, the skewness and excess kurtosis are written below for a scalar field f , with $S_f = 0, K_f = 0$ for a Gaussian distribution:

$$S_f = \langle f^3 \rangle / \langle f^2 \rangle^{3/2}, \quad K_f = \langle f^4 \rangle / \langle f^2 \rangle^2 - 3, \quad K_f(S_f) \sim S_f^{\alpha_f}.$$

In the following sections, variations of α_f with parameters will be succinctly analyzed for several turbulence fields and settings.

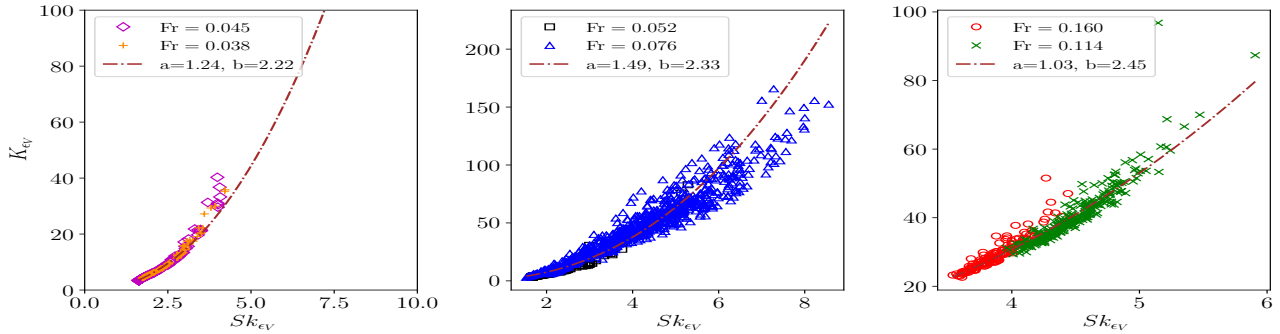


Figure 1. Stratified turbulence: Kurtosis-skewness plots for ϵ_v for several Froude numbers (see insets in which are also given the fit parameters assuming $K \sim a * S^b$). Note the different scales for S and K , and in particular the high K values for the run with $Fr \approx 0.076$ (middle).

3 Numerical data on $K(S) \sim S^\alpha$ behavior for a few turbulent flows

3.1 The fluid case with or without stratification and possibly also rotation

We briefly give numerical results showing the ubiquity of $K(S) \sim \kappa S^\alpha$ scaling in turbulence, stressing the following examples:

85 Navier-Stokes fluids; stratified flows without or with rotation, and MHD in the fast dynamo regime. In Sreenivasan and Antonia (1997), one finds a compilation of skewness and flatness up to Taylor Reynolds number in excess of 3×10^4 , for a variety of flows, experimental, numerical and in the atmospheric boundary layer (see their Figures 5 and 6). By digitalizing the data, making log-log fits and selecting points with $R_\lambda \geq 660$, one finds a fit $K \approx S^{2.34}$ (see also (Sattin et al., 2009)). It will be of interest to redo this compilation with more recent experiments, but this already tells us that a pseudo-parabolic scaling between

90 K and S is present for fluid turbulence, as shown as well in numerical simulations of the Navier-Stokes equations with a passive scalar (Pouquet et al. (2023))¹, Table 1. Note that the analysis in PRM2 was done rather in terms of the variation with governing parameters, say the Froude number Fr , of the coefficient assuming a parabolic fit, *viz.* $K \sim a(Fr)S^2$, whereas here we do not assume *a priori* the power-law scaling between K and S , and instead search for α . In that context, we observe that the first figure of PRM2 gives $K(S)$ for the vertical buoyancy flux $\langle w\theta \rangle$; a quasi-parabola emerges, and with kurtosis up to

95 ≈ 12 . Moreover, $K(S)$ statistics of local square vorticity and local dissipation differ somewhat, in particular at moderate R_V values as shown in PRM2, but such statistics are found in Donzis et al. (2008) to be quite similar for the most extreme events, defined as having 10^4 times the mean dissipation at high R_λ . It will thus be of interest to extend this study to higher R_V .

In the presence of stable stratification, as is found in the atmosphere and the ocean, we plot in Fig. 1 above for several Froude numbers (see insets), the power-law fits to $K(S) \sim S^{\alpha_\epsilon}$ for the kinetic energy dissipation ϵ_v , a good indicator of

100 clear-air turbulence (Storer et al., 2019); this leads to an exponent α_ϵ that increases continuously with Fr , from ≈ 2.22 to 2.45 (parameters for the runs are given in Table 1 of PRM2). The highest values of both S and K are reached for the run with

¹This paper, by Pouquet, Rosenberg, Marino & Mininni (2023), is denoted hereafter PRM2.

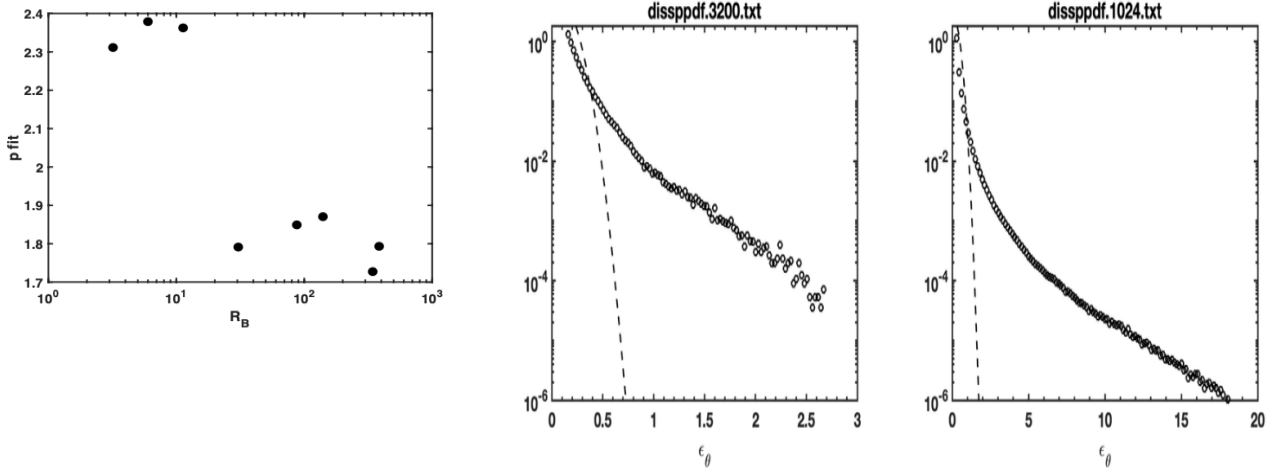


Figure 2. Quasi-geostrophic (QG) turbulence: *Left:* Variation with R_B of exponent for $K(S)$ scaling for kinetic energy dissipation for QG runs. Note the sharp transition which occurs at $R_B \approx 25$ corresponding to $\langle Ri_g \rangle \approx 1.5$. *Center and right:* PDFs of potential energy dissipation for runs with $R_B = 3$ (center) and $R_B = 385$ (right; Q1 and Q8 in PRM2, see Table 1), with dashed lines for equivalent Gaussians.

$Fr \approx 0.076$ corresponding to the strongest intermittency of the vertical velocity in particular (Feraco et al., 2018; Marino et al., 2022), strong local dissipation and associated localized shear layers.

When now combining rotation and stratification of comparable magnitude as found in the ocean ($N/f_0 \approx 5$), we observe in
 105 Fig. 2 (left) for quasi-geostrophic (QG) runs (see Table 1 of PRM2 for run specifications) a sharp transition in the exponent
 of the $K(S) \sim S^\alpha$ fits for the buoyancy flux around $R_B \approx 25$, corresponding to an average gradient Richardson number
 $\langle Ri_g \rangle \approx 1.5$, close to that for a KH transition to instability. This points to the importance of the occurrence of turbulence at
 small scale once the Ozmidov scale is larger than the Kolmogorov scale, $\ell_{Oz} \geq \eta_K$ with $\ell_{Oz}^2 = \epsilon_V / N^3$, $\eta_K^4 = [\epsilon_V / \nu^3]^{-1}$. We
 also give in Fig. 2 the PDFs of the potential energy dissipation $\epsilon_\theta = \kappa_0 \langle |\nabla\theta|^2 \rangle$ for QG runs Q1 (center) and Q8 (right) with
 110 buoyancy Reynolds numbers R_B of 3.2 and 385. The respective fits ($0.1 \exp^{-2.7\epsilon_\theta}$ and $0.0008 \exp^{-0.37\epsilon_\theta}$) are in agreement
 with the expected increase in small-scale structures and dissipation as R_B grows and a fully turbulent regime is reached.

3.2 Coupling to a magnetic field in MHD: fast dynamos in the ABC, Roberts and Taylor-Green flows

The dynamo problem is that of the growth of magnetic fields due to either, at small scale, chaotic streamlines of the velocity or,
 at large scales, the kinetic helicity content of the flow, where $H_V = \langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle$ (Steenbeck et al., 1966; Moffatt, 1969; Zel'dovich
 115 et al., 1983; Brandenburg and Subramanian, 2005), and it plays an essential role in the solar context in the presence of convec-
 tion (see *e.g.* Ponty et al. (2001)). Both the cross-helicity $H_C = \langle \mathbf{u} \cdot \mathbf{b} \rangle$ (Pouquet et al., 1986; Yokoi, 2013), and the magnetic
 helicity $H_M = \langle \mathbf{A} \cdot \mathbf{b} \rangle$, with $\mathbf{b} = \nabla \times \mathbf{A}$, also play a role, the latter in the nonlinear saturation of the dynamo associated with an
 inverse cascade of H_M (Pouquet et al., 1976). In fact, with sufficient large-scale separation, a dynamo can occur with $H_V \equiv 0$
 overall but with sufficient local fluctuations (Gilbert et al., 1988). The dynamo can also be sub-critical because the growing
 120 magnetic seed will alter the flow and reduce the turbulence (Ponty et al., 2007; Mannix et al., 2022). The resulting 3D turbulent



flow is made-up of current and vorticity sheets, rolling-up around the local mean magnetic field and with a strong twist of \mathbf{b} across the sheet (Mininni et al., 2006; Ponty and Plunian, 2011; Homann et al., 2014); see also Uzdensky et al. (2010); Lazarian et al. (2020); Oka et al. (2022).

Parabolic $K(S)$ laws in MHD have been found both in laboratory plasmas and in the cosmos (Labit et al. (2007); Krommes
125 (2008); Osmane et al. (2015)). Recently, variations of α with parameters have been discussed briefly in the context of the
classical She-L  v  que (SL) model as found in the fast dynamo context (Ponty et al., 2024), and we are expanding on these
results presently for generalized SL models (see equ. (7) analyzed in §4.1), as well as for higher order moment ratios.

Figure 3 first shows $Q_5(S)$ with Q_5 the normalized fifth-order moment (defined in equ. (10) below); the data is given for
the vertical velocity and magnetic field (v_z, b_z), as well as the kinetic and magnetic dissipation (ϵ_v, j^2). In the $Q_5(S)$ data,
130 power-laws emerge in the tails of the kinetic variables and b_z , and throughout for j^2 . Variation of the power-law index α with
a threshold in skewness is given in the middle-right plots for Q_5 for the two TG flows (top), and for the normalized sixth-order
moment H_6 (bottom) for the two ABC flows (see Table 1 in Ponty et al. (2024) for details on these four runs, and see equ. (10),
with $\sigma = [53/22]$ for Q_5 and $[63/22]$ for H_6). The variation of the fit constant κ is shown as a function of a given threshold
in skewness, $S \rangle$ (top right), and we give (bottom right) the number of data points involved at a $S \rangle$ level (see Table 1 in
135 Ponty et al. (2024) for more information on the runs). Note also that these results are linked to the Lagrangian statistics of
the flow, as discussed in Homann et al. (2014). High S are reached for H_6 for the ABC flows. As discussed in §4.3 below,
the power-law indices are noticeably lower than the predicted values using the standard SL models developed in Grauer et al.
(1994); Politano and Pouquet (1995), and also, for Q_5 , the high-intermittency limit of $\alpha_Q \rightarrow 3$ appears plausible. For H_6 ,
the discrepancy is higher and might indicate an insufficiently high Reynolds number leading to too few extreme events, or an
140 insufficient complexity in the models themselves.

New dynamo computations with the G.O. Roberts flow with a helical forcing give the following scaling results (third row of
Fig. 3). The overall diagnostics for these runs stem from runs using grids of $64^2 \times 128$ and $128^2 \times 256$ (runs GOR1 and GOR2),
with respective Reynolds numbers of 147 and 445 (and R_λ of 38 and 66). These flows are well resolved (the dissipation scale
is more than twice the numerical cut-off according to the Kaneda criteria), and they are run for long times (15000 and 2000
145 τ_{NL} resp.); however, the energy spectra (not shown) are not yet sufficiently developed (see also Ponty and Plunian (2011) for
different runs using the G.O. Roberts dynamo configuration), although the skewness of j^2 reaches high values above 18.

Preliminary results indicate the following. We reach here a ratio $r_E = E_M/E_V$ approaching equipartition (Fig. 3, bottom
row, left), and the scatter plot for j^2 for run GOR2 indicate a clear power law, the blue line following the parabola $K(S) =$
 $3/2[S^2 - 1]$. The scaling exponent α and constant κ given here for the TG, ABC and GOR runs show variations with forcing
150 function, with Reynolds number and with the threshold in S used for the plot, as well as possibly with the equipartition ratio
 r_E . As is the case for the TG and ABC flows, the Reynolds number leads to a difference in scaling for the fit parameters. Thus,
runs at higher Reynolds numbers with all three configurations will have to be performed in order to study the scaling of the
 α, κ parameters for relevant variables, but in the next section we begin an approach that can elucidate these scalings laws in the
framework of three multifractal intermittency models.

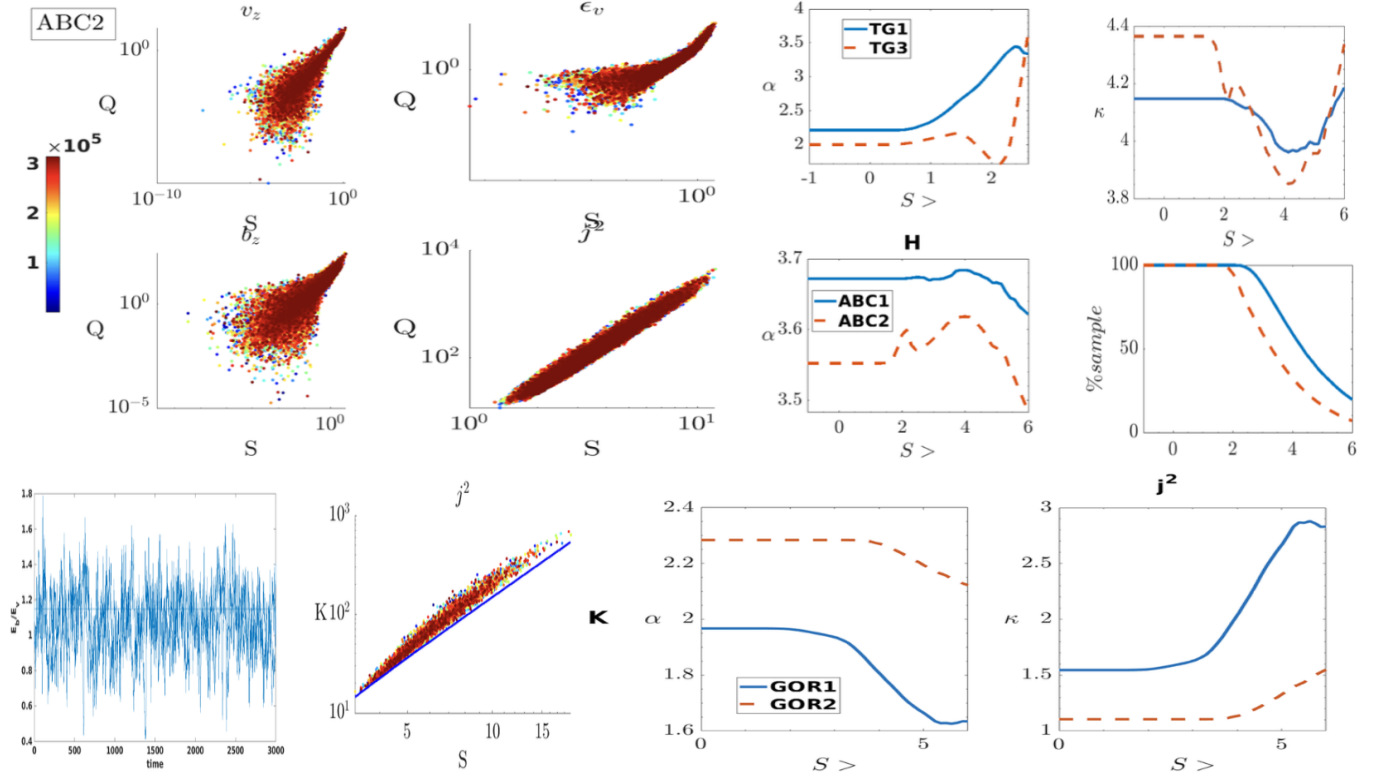


Figure 3. Fast dynamics in MHD. Top and middle rows, left: Variation with skewness of $Q_5 \sim \kappa_Q S^{\alpha_Q}$, in log-log, ABC2 run for kinetic (top) and magnetic (middle) fields for v_z, b_z (left), and ϵ_v, j^2 (center left); the vertical color bar far left gives the timestamp, from early (blue) to late (red), with $\approx 3 \times 10^5 \tau_{NL}$ altogether. Center right: Variation of the power-law α_Q for the TG_{1,2} runs for ϵ_v for Q_5 (top row), and α_H for H_6 for j^2 for the ABC_{1,2} runs (middle row), as a function of increasing threshold in skewness. Rightmost: for j^2 , ABC_{1,2} runs, constant κ_Q (top), and percentage of points at a given threshold in S (bottom). Third row, Run GOR2, $R_V \approx 445$; leftmost: $E_M/E_V(t)$, and (center left), $K(S)$ for j^2 . Runs GOR1 and GOR2 for j^2 : power-law $\alpha(S >)$ (center right) and constant $\kappa(S >)$ (right).

155 4 Theoretical moment scaling using several multi-fractal intermittency models

4.1 Expression for the kurtosis-skewness scaling exponent, $K \sim S^\alpha$, for both fluids and MHD in the SL framework

We now give a path towards a theoretical formulation for $K(S)$ scaling using a classical intermittency model and several of its extensions. Assuming a power-law scaling for velocity and magnetic field structure functions $\delta u(r), \delta b(r)$ defined as

$$\langle [u(x+r) - u(x)]^p \rangle \equiv \langle \delta u(r)^p \rangle \sim r^{\zeta_p^{(f)}}, \quad \langle [b(x+r) - b(x)]^p \rangle \equiv \langle \delta b(r)^p \rangle \sim r^{\zeta_p^{(m)}}, \quad (5)$$



160 and assuming as well a power-law scaling between kurtosis and skewness for both fluids (f) and MHD (m), we easily obtain:

$$K_f(S_f) \sim S_f^{\alpha_f}, \quad K_m(S_m) \sim S_m^{\alpha_m}, \quad \alpha_f = \frac{\zeta_4^{(f)} - 2\zeta_2^{(f)}}{\zeta_3^{(f)} - 3\zeta_2^{(f)}/2}, \quad \alpha_m = \frac{\zeta_4^{(m)} - 2\zeta_2^{(m)}}{\zeta_3^{(m)} - 3\zeta_2^{(m)}/2}, \quad (6)$$

with the functions $\zeta_p^{(f,m)}$ depending on the (fluid or MHD) intermittency dynamics or on an explicit model.

We now recall the scaling laws derived in the context of the She-Lévêque formulation for fluids (1994) and for MHD as generalized in Politano et al. (1995); these GSL models, named *gslf* and *gslm* respectively depend on two open parameters,
 165 $0 < x < 1$ and $0 < \beta < 1$, with $x = 0, \beta = 0$ in the non-intermittent case; they are respectively for fluids and MHD²:

$$\zeta_p^{gslf} = \frac{p(1-x)}{3} + \frac{x(1-\beta^{p/3})}{1-\beta}; \quad \zeta_p^{gslm} = \frac{p(1-x)}{4} + \frac{x(1-\beta^{p/4})}{1-\beta}. \quad (7)$$

We note that x is related to the co-dimension of the most dissipative structures in the nonlinear system, and that β is a measure of the efficiency of energy transfer and dissipation among intermittent structures as the moment order varies. This formulation leads to log-Poisson statistics (see *e.g.* She and Lévêque (1994); Dubrulle (1994); Frick et al. (1995)). A further assumption
 170 of the models concerns the scaling of nonlinear transfer in terms of characteristic times of the problem, namely the nonlinear eddy turn-over time, the wave period (in MHD, the Alfvén wave) and the transfer time of energy to small scales.

The multi-fractal framework (see Frisch (1995)) allows for a multiplicity of dissipative structures of diverse physical (co)-dimensions: vortex and current sheets, flux tubes, current filaments or bubbles, resulting in a non-integer effective β parameter. An extension of the multifractal framework to vectors (velocity field) as opposed to scalars (velocity amplitude), can be found
 175 in Schertzer and Tchiguirinskaia (2020). The SL formulation for MHD has been used for example in modeling intermittent nano-flares in connection with solar wind data (Veltri et al., 2005). In the numerical context, it is stressed in Servidio et al. (2011) that a high resolution is needed to quantify properly the properties of local reconnection and current sheets; moreover, reconnection events and the ensuing dissipation are highly local and very varied in amplitude, somewhat reminiscent of the multifractality property reviewed in detail in Lovejoy and Schertzer (2012); Benzi and Toschi (2023).

180 From equations (7), we can compute the general scaling exponents of kurtosis *vs.* skewness using equation (6). We obtain:

$$\alpha_{gslf} = \frac{2(1-2\beta^{2/3}+\beta^{4/3})}{1+2\beta-3\beta^{2/3}}; \quad \alpha_{gslm} = \frac{2(1-2\beta^{1/2}+\beta)}{1+2\beta^{3/4}-3\beta^{1/2}}, \quad \beta \neq 1. \quad (8)$$

Note that, interestingly, both the α_{gslf} and α_{gslm} exponents are independent of x , the fractal co-dimension of the most dissipative structures. It is also straightforward to see, again both for fluids and for MHD, that the limit, for $\beta \rightarrow 0$, is $\alpha \rightarrow 2$. In other words, a parabolic law is reached when the most dissipative structure dominates the small-scale dynamics, irrespective of
 185 its geometrical (co)-dimension, likely at high R_V, R_M as well as $\langle Ri_g \rangle \approx 1$ (see equations (3)). However we note that, in the shell models examined in Frick et al. (1995), β never reaches this low limit. Another point concerns the fact that the variation of α could reflect the dependence on the form of the second invariant in the shell models, that is akin to helicity (Frick et al., 1995; Kadanoff et al., 1995). This may point to a limitation of such models when restricted to nearest-neighbor interactions or with different sets of invariants, restrictions that cannot encompass by construction the highly non-local (in scale) interactions
 190 leading to anomalous dissipation. Thus, this point will need further investigations.

²The standard SL-MHD model is generalized in Merrifield et al. (2005) to include extended self-similarity (see also Merrifield et al. (2007) for 2.5D).



4.2 The standard choice of parameters for the SL models for fluids and MHD

The standard case for the classical fluid SL model is obtained for $x = 2/3$, $\beta = 2/3$ associated with vortex filaments, whereas in the MHD case with wave-vortex interactions and current sheets, the standard parameters become $x = 1/2$, $\beta = 1/2$ (Grauer et al., 1994; Politano and Pouquet, 1995). This yields respectively for the $K(S) \sim S^\alpha$ for fluids (*ssl**f*) and MHD (*sslm*):

195 $\zeta_p^{sslf} = \frac{p}{9} + 2 \left[1 - \left(\frac{2}{3} \right)^{p/3} \right]$, $\alpha_{sslf} \approx 2.56$; $\zeta_p^{sslm} = \frac{p}{8} + 1 - \frac{1}{2^{p/4}}$, $\alpha_{sslm} \approx 2.53$; these α values for the standard SL models are also given in Ponty et al. (2024). Note that such values are sensitive to the number of decimals taken; in the fluid case using strictly 2 decimals throughout, one finds $\alpha_{sslf} \approx 2.00$.

All α s are close except for extreme cases (β at its limits), in part because the values of the anomalous exponents for structure functions for fluids are anchored at $\zeta_3 \equiv 1$. For θ, \mathbf{v} and \mathbf{b} , there are more complex constraints since they involve

200 cross-correlations between fields at third order (Yaglom, 1949; Antonia et al., 1997; Politano and Pouquet, 1998), and also because the analytical expressions for α lead to small fractional power of β , and we are at relatively low orders of the structure functions. In fact, an extension of the SL theory to the intermittency of the passive scalar θ in the fluid case can be found in L ev eque et al. (1999). We can then derive the expression $K_{F_\theta} \sim S_{F_\theta}^{\alpha_\theta}$ in the framework of that model. Here, the scalar flux F_θ is defined as $F_\theta(r)^{(p)} := \langle |\delta u(r) \delta \theta(r)^2|^{p/3} \rangle \sim \langle |\delta u(r) \delta \theta(r)^2|^{\zeta_p} \rangle$, an expression using the flux arising from the aforementioned

205 exact law for the conservation of scalar energy derived in Yaglom (1949). With the numerical values given in L ev eque et al. (1999), we find $\alpha_\theta^E \approx 2.61$ using anomalous exponents stemming from experiments, $\alpha_\theta^D \approx 2.38$ for DNSs, and $\alpha_\theta^T \approx 2.44$ using the theory developed in that paper. This shows again the sensitivity of these power-laws to the accuracy of the data.

4.3 Generalized scaling for higher-order normalized structure functions in the framework of the She-L ev eque models

Let us now rewrite the generalized SL models for fluid and MHD slightly differently, with as before $0 < \beta < 1$ and $0 < x < 1$:

210 $3(1 - \beta)\zeta_p^{gslf} = x[3(1 - \beta^{p/3}) + p(\beta - 1)] + p(1 - \beta)$, $4(1 - \beta)\zeta_p^{gslm} = x[4(1 - \beta^{p/4}) + p(\beta - 1)] + p(1 - \beta)$. (9)

We now compute the scaling of a generalized adimensionalized structure function vs. another one, provided they exist, writing:

$$K_{pq} = \frac{\langle \delta u^p \rangle}{\langle \delta u^q \rangle^{p/q}}, \quad K_{rs} = \frac{\langle \delta u^r \rangle}{\langle \delta u^s \rangle^{r/s}}, \quad K_{pq} = f(K_{rs}) = K_{rs}^{\alpha_\sigma}, \quad \sigma = [pr/qs], \quad \alpha_\sigma = \frac{\zeta_p - [p/q]\zeta_q}{\zeta_r - [r/s]\zeta_s}, \quad (10)$$

with $\sigma \in \mathbb{N}^+$, $p > \max[q, r]$, $r > s$. In §4.1, we considered the case $K = S^\alpha$, or in the present notation, $K_{42} = K_{32}^{\alpha_{43/22}}$, with $p = 4, q = 2 = s, r = 3$. After a slightly cumbersome but straightforward computation, one obtains that again α_σ is independent

215 of x , the co-dimension of dissipative structures, for all values of the indices encapsulated in σ ; one finds specifically:

$$\alpha_\sigma^{(gslf)} = \frac{s}{q} \left[\frac{q(1 - \beta^{p/3}) - p(1 - \beta^{q/3})}{s(1 - \beta^{r/3}) - r(1 - \beta^{s/3})} \right]; \quad \alpha_\sigma^{(gslm)} = \frac{s}{q} \left[\frac{q(1 - \beta^{p/4}) - p(1 - \beta^{q/4})}{s(1 - \beta^{r/4}) - r(1 - \beta^{s/4})} \right]. \quad (11)$$

In the case of extreme intermittency with $\beta \rightarrow 0$, we also have, for both fluids and MHD, and with $s \neq r$ as stated before:

$$\beta \rightarrow 0, \quad \alpha_{pr/qs}^{(gslf),(gslm)} \rightarrow \frac{s(p - q)}{q(r - s)}. \quad (12)$$

This formula simplifies, for $q = s$ (same normalization of moments) into $[p - q]/[r - q]$, and gives a parabolic scaling for

220 $p + q = 2r$. Thus, when choosing for the normalisation the second-order energy moment ($q = s = 2$), we have a parabolic



scaling for $2r = p + 2$. Similarly, for a normalisation by the skewness, $q = s = 3$, we obtain again a parabola for $2r = p + 3$. These parabolic solutions, for $\beta \rightarrow 0$, are directly linked to the algebraic, hierarchical formulations of the SL models.

Finally, let us take two specific examples: $Q_5(S)$ with $p = 5, q = s = 2, r = 3$, and $H_6(S)$ with $p = 6, q = s = 2, r = 3$; the first example for $Q_5(S)$ is also discussed in Sardeshmukh and Sura (2009). We find in the standard case ($\beta = 2/3$ for fluids and $\beta = 1/2$ for MHD) the scaling $\alpha_{53/22}^{sslf} \approx 4.6$, $\alpha_{53/22}^{sslm} \approx 4.5$, whereas the numerical estimate for the ABC runs gives a maximum of $Q_5 \approx 3.5$. When $\beta \rightarrow 0$, $\alpha_{53/22}^{\beta \rightarrow 0} \rightarrow 3$, a value advocated in Sardeshmukh and Sura (2009) for this $Q_5(S)$ scaling for both vorticity and potential height using a linear stochastically forced Langevin equation model for climate dynamics with correlated additive and multiplicative noise (see also §5.1 below). To give a second and final example, for H_6 in the standard case again, we have $\alpha_{63/22}^{sslf} \approx 7.1$, $\alpha_{63/22}^{sslm} \approx 6.8$, and when $\beta \rightarrow 0$, $\alpha_{63/22}^{\beta \rightarrow 0} \rightarrow 4$, whereas the numerical value we find for the ABC runs is close to 3.7. The discrepancy with the numerical data given in Fig. 3 is thus large; in this context, a study in terms of variation with Reynolds number will be informative, but one may have to investigate the MHD turbulence case in 2D, or so-called “2.5D” (two space variations, three components of the fields) to reach substantially higher R_V, R_M .

4.4 $K \sim S^\alpha$ scaling for the Yakhot intermittency model

One can use other models of structure function scaling in turbulent flows. For example, a model of intermittency in fluid turbulence due to Yakhot (2006) (herewith model Y) yields the scaling:

$$\zeta_{2p}^{(Y)} = \frac{2(1 + 3\beta_y)p}{3(1 + 2p\beta_y)}, \quad \zeta_3^{(Y)} = 1 \quad \forall \beta_y; \quad \text{with } q = 2p, \quad \zeta_q^{(Y)} = \frac{q(1 + 3\beta_y)}{3(1 + q\beta_y)}. \quad (13)$$

The model comes from evaluating perturbatively the corrections to two-dimensional turbulence when close to a critical dimension at which the energy cascade reverses its direction to the small scales. One can verify that $\beta_y = 0$ gives a $\zeta_p = p/3$ standard scaling. We immediately get $\alpha_y \approx 2.56$ for the relationship $K \sim S^{\alpha_y}$ when choosing for the open parameter the value $\beta_y \approx 0.05$ close to that given by experiments (see also Nickelsen (2017); Friedrich and Grauer (2020) for recent analyses of this and other models³). The anomalous ζ_p exponents themselves (see Fig. 1 in Friedrich and Grauer (2020)) do not differ by much from model to model, specially at relatively low order. But in view of the sensitivity of α to the evaluation of the anomalous exponents, α -scaling in an empirical $K(S)$ law may prove a valuable tool in order to discern between different intermittency modeling and small-scale parametrisation in general, somewhat better than with the ζ_p themselves, given sufficiently resolved data leading to precise fits to the quasi-parabolic power-law behavior for long runs in terms of turn-over times.

In conclusion, if the change of $K(S)$ scaling with Reynolds number is not known, and is difficult to evaluate experimentally or numerically, the data is sufficient to assess that such scalings will be observed at high R_V ; indeed, it can be expected in the framework of random multiplicative systems (see Benzi and Toschi (2023) for a recent introduction). We also recall here that a parabolic law can be justified on several grounds. First, one can write a Taylor expansion for $K(S)$ for a PDF close to a Gaussian, and note that $K \geq 0$; this was performed by Longuet-Higgins (1963) in the context of sea-surface elevations. Another reason for observing a $K(S)$ parabolic law relies on the existence of Cauchy-Schwarz relationships (and their generalisations)

³In the Markov process (M) interpretation of the SL model, the independence of α on the co-dimension of dissipative structures is an independence of the jump distribution on the associated stochastic process due to M , and only the amplitude of the velocity jumps (leading to dissipation intermittency) matters.



between the third- and fourth-order moments of a stochastic variable f , namely $S_f^2 \leq K_f + 3$, with a tightening of the inequality for a unimodal PDF, namely $S_f^2 \leq K_f + 186/125$ for a finite fourth-order moment (Klaassen et al., 2000). We also note that *Beta* distributions are advocated in Labit et al. (2007) for the intermittency of density fluctuations in drift-exchange turbulence in plasmas, in particular because they admit both positive and negative skewness as observed in many instances such as the fast dynamo (Ponty et al., 2024). These $K(S)$ relations also provide useful bounds for the data.

5 Other approaches for quasi-parabolic scaling beyond the GSL and Y models

5.1 Linear and non-linear Langevin models

Langevin equations have long been written in the context of turbulent flows, for example in order to take into account the nonlocality of mode interactions leading to intermittency, modeling as such the separation of spatial and temporal scales (Nazarenko et al., 2000; Laval et al., 2003). Indeed, dissipative structures such as shear layers or current sheets are multi-scale, spanning a range from the integral scale characteristic of their length to the dissipative scale defined by viscosity or resistivity, *e.g.* the Kolmogorov scale η_K for NS, although we note that, in the multi-fractal framework such as in the SL models, there is a range of dissipative scales corresponding as well to a range of spectral indices. This provides a justification for the application of a Langevin framework, where the original nonlinearities of the primitive equations are modeled through fast-evolving additive and multiplicative stochastic noise. It is shown in Wan et al. (2012) that the kurtosis of the magnetic field filtered at the dissipation scale and smaller increases sharply and significantly both in high-resolution 2D DNS and in ACE and Cluster solar wind data. Recent observations in the heliosphere analyzing data from the Parker Solar Probe confirm the importance of such non-local interactions in the case of so-called imbalanced MHD turbulence with $\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{b}$ of unequal amplitudes (Yang et al., 2023), an imbalance enhanced by the quasi-absence of collisions (Miloshevich et al., 2021).

One can write a stochastic Langevin equation for a fluctuating field \tilde{c} , *viz.* $D_t \tilde{c} = -(\bar{\lambda}_k + \lambda'_k) \tilde{c} + \tilde{\zeta}_k$, where $[\bar{\lambda}_k, \lambda'_k]$ represent large-scale and fluctuating small-scale velocity stretching the magnetic field lines in the kinematic phase, and $\tilde{\zeta}_k$ is an additive noise due to (plausible) rapid small-scale fluctuations. The essential features in the development of Sura and Sardeshmukh (2008) for climate can thus be reproduced in the MHD case; this will likely lead to the same conclusion of a parabolic behavior. The large-scale velocity and induction are constrained by divergence-free conditions, by Galilean invariance for the velocity, and perhaps even more importantly by existing so-called exact laws⁴. Such laws involve third-order cross-correlations of \mathbf{u} and \mathbf{b} (see Marino and Sorriso-Valvo (2023) for a recent review), whereas the fourth-order moments do not have such constraints for quadratically nonlinear equations. A non-zero energy dissipation rate (a plausible conjecture) thus implies non-Gaussianity ($S \neq 0, K \neq 0$). A Langevin equation developed in the kinematic dynamo regime can be amended to model the back reaction of the Lorenz force, as discussed briefly in Ponty et al. (2024). We finally note that, starting from well-resolved data, one can reconstruct a Langevin equation model of the observed stochastic process (Friedrich et al., 2011; Rinn et al., 2016). This may prove instructive, in particular if different models were to emerge for different regimes or dynamo types.

⁴These exact laws have been extended to fluid and MHD turbulence as apply to the heliosphere, see *e.g.* Ferrand et al. (2021); David and Galtier (2022).



5.2 Self-organized criticality and $1/f$ law as another possible framework for intermittent quasi-parabolic scaling

Self-Organized Criticality (SOC) has been introduced in the context of sandpile systems and their avalanching properties to
285 model solar flares (Lu and Hamilton (1991); see also Bramwell et al. (2000); Chapman and Watkins (2001); Osman et al.
(2014); Watkins et al. (2016); Balasis et al. (2023) for recent discussions). In the context of DNS in three-dimensional (3D)
MHD, Uritsky et al. (2010) identified SOC in the dissipative range of decaying runs (so with a local critical Reynolds number
of order unity). However, SOC was not found in the inertial range, a fact that was interpreted as SOC properties propagating
from the dissipative to the inertial range, with merging of current structures. The critical state is that in which the source (the
290 energy cascade at a fixed rate) and the sink (the dissipation at a fixed rate through *e.g.* eddy viscosity) balance, as they do
on average. Note that Smyth et al. (2019) identified SOC in rotating stratified flows with the Richardson number, governing
shear instabilities such as KH, being the critical parameter (see also Fig. 2). The nonlinear interactions in the inertial range
are conservative, and dissipation sets in through nonlocal interactions between energy-containing eddies and dissipative ones,
lending these interactions to be described by SOC together with $1/f$ noise (Vespignani and Zapperi, 1998). As shown in
295 Dmitruk and Matthaeus (2007), this leads to an emphasis on the dynamics of the largest modes, and on their interactions with
the early dissipative range where intermittency is strongest (Kraichnan, 1967; Chen et al., 1993). Also, the sharp variations of
the flow and field due to the nonlinearities of the primitive equations can be treated as a stochastic force using renormalization
group techniques, reminiscent again of a Langevin approach (Materassi and Consolini, 2008). In all these studies, nonlinear
shear instabilities appear central to the inter-related small-scale and large-scale behavior of the stochastic turbulent flows.

300 6 Conclusion and perspectives

We have analyzed in this paper the relative behavior of normalized moments of the velocity and magnetic fields in a variety
of contexts, and have given a rationale to cast these results in the mold of classical intermittency models for fluid and MHD
turbulence, models which provide a natural framework for such relative scalings. The variability of the scaling is linked to the
details of the dissipative structures and their relative intensities. The ubiquity of a quasi-parabolic $K(S) \sim S^\alpha$ law could be
305 interpreted as it having no specific physical meaning; on the other hand, it may be pointing to a universality of intermittency in
turbulent flows. We also note that the power-law exponent α is independent of the (co)-dimension of the dissipative structures.
The abrupt transition in α -scaling for the rotating-stratified case when shear instabilities arise (see Fig. 2) is indicative of
an underlying dynamics where the development of turbulence, as measured by the Ozmidov scale becoming larger than the
dissipative scale in that case, plays a dynamical role (Pouquet et al., 2023). In MHD, one issue absent from the present analysis
310 is to incorporate the potential effect of helical structures (with non-zero kinetic, magnetic and/or cross helicity) on the $K(S)$
scaling. It is known from multiple studies that helicity plays a central role in large-scale dynamos (see Brandenburg and
Subramanian (2005)), and that its incorporation in closures of turbulence leads to better modeling of these flows (Yokoi, 2013).

In order to pursue the investigation of $K(S)$ laws in turbulence at higher Taylor Reynolds number, one can implement
hyper-viscosity algorithms, or else use models which, because they are significantly less costly numerically, will allow for
315 longer statistics at substantially higher R_V . Such approaches are numerous. One can think of shell models retaining only



one mode per field per wavenumber shell and only nearest-neighbor interactions as developed for MHD in Gloaguen et al. (1985) (see Plunian et al. (2013) for review). One can also simplify the dynamics by lowering the space dimension, as for the 1D, 2D and 2.5D cases (see *e.g.* Thomas (1970); Hada (1993); Laveder et al. (2013); Merrifield et al. (2007); Servidio et al. (2011)). Numerical adaptation, preferably spectral when dealing with L_∞ norms as for extreme intermittent events (see
320 Ng et al. (2008)), as well as various large-eddy simulations (Sagaut and Cambon (2008)), or the so-called α -model (Holm et al., 1998) used for example in the framework of oceanic dynamics (Pietarila Graham and Ringler, 2013) or analyzed as well in MHD (Montgomery and Pouquet, 2002) will be similarly useful. These methods will allow for disentangling between Reynolds number and intermittency effects, the consequences of the presence or not of helicity linked to vortex filaments and to the dynamo, as well as equipartition or not of kinetic, potential or magnetic energy.

325 One further important issue will concern incorporating the role of anisotropy which can affect scaling properties and interpretations of the intermittency, as shown in the context of the atmosphere in Lovejoy et al. (2001), or in Schekochihin (2022) for MHD. Finally, it was shown in Yeung et al. (2015) that for extreme events, defined as having their local dissipation being more than 10^4 above the mean, the strongly intermittent vorticity structure is a sub-part of the usual vortex filament and appears more as an (isotropic) blob; thus, these intense structures are not force-free (which would require, for the NS case, quasi-parallel ve-
330 locity and vorticity as in a filament) and are therefore dissipative. These authors also noted that the grid resolution and machine precision both affect the estimate of the overall enstrophy (see Fig. S1 of that paper). This type of analysis is not performed here for lack of sufficiently large Reynolds number, and the ensuing lack of sufficient intense localized dissipation, but a study of intermittent structures in MHD at substantially higher Reynolds number is planned for the future.

Code and data availability. Codes and data are available upon reasonable request.

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