

Advances in CALIPSO (IIR) cirrus cloud property retrievals – Part 1: Methods and testing

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Abstract. In this study, we describe an improved CALIPSO (Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observation) satellite retrieval which uses the Imaging Infrared Radiometer (IIR) and the CALIPSO lidar for retrievals of ice
10 particle number concentration N_i , effective diameter D_e , and ice water content IWC. By exploiting two IIR channels, this approach is fundamentally different than another satellite retrieval based on cloud radar and lidar that retrieves all three properties. A global retrieval scheme was developed using in situ observations from several field campaigns. The N_i retrieval is formulated in terms of N_i/A_{PSD} ratios, where A_{PSD} is the directly measured area concentration of the ice particle size distribution (PSD), along with the absorption optical depth in two IIR channels and the equivalent cloud thickness seen by IIR.
15 It is sensitive to the shape of the PSD, which is accounted for, and uses a more accurate mass-dimension relationship relative to earlier work. The new retrieval is tested against corresponding cloud properties from the field campaigns used to develop this retrieval, as well as a recent cirrus cloud property climatology based on numerous field campaigns from around the world. In all cases, favourable agreement was found. This analysis indicated that N_i varies as a function of τ . By providing near closure to the ice PSD, the natural atmosphere may be used more like a laboratory for studying key processes responsible for
20 the evolution and life cycle of cirrus clouds and their impact on climate.

1 Introduction

Cirrus clouds contain only ice particles (i.e., no liquid cloud droplets), a condition guaranteed when cloud temperatures (T) are less than $\sim -38^\circ \text{C}$ (Koop et al., 2000). The microphysical and radiative properties of cirrus clouds are subject to very different ice nucleation pathways as well as whether the cirrus clouds are of liquid origin or not (e.g., Krämer et al., 2016), and
25 they also depend on aerosol particles of different sizes in complex ways (Ngo et al., 2024). With the ice particle size distribution (PSD) of cirrus clouds subject to so many factors, factors that may vary with latitude, season, and surface type (e.g., land vs. ocean), there is a need to observe cirrus cloud PSDs from space if cirrus clouds are to be represented accurately in climate models. If PSDs in cirrus clouds are approximated as exponential, they can be characterized through satellite retrievals of the PSD ice water content (IWC), effective diameter (D_e), and ice particle number concentration (N_i) as described

30 in Mitchell et al. (2020). Such satellite retrievals appear to be a necessary but not sufficient condition for understanding aerosol-cloud-climate interactions in cirrus clouds.

Ice crystals in cirrus clouds can form by either of two processes: homogeneous or heterogeneous ice nucleation (henceforth hom and het). The former requires no ice nucleating particles (INP) and can proceed through the freezing of haze and cloud solution droplets when $T \leq 235$ K (-38° C) and the relative humidity with respect to ice (RHi) exceeds some threshold where
35 $RHi > \sim 145\%$ (Koop et al., 2000). This results in generally higher concentrations of ice particles (N_i) relative to het (Barahona and Nenes, 2009; Jensen et al., 2012, 2013a,b; Cziczo et al., 2013). Under weak updraft conditions, N_i resulting from hom may be similar to N_i resulting from het (Krämer et al., 2016), and under atypical conditions (such as high concentrations of mineral dust), N_i resulting from het can exceed 200 L^{-1} which is characteristic of hom (Barahona and Nenes, 2009; Cziczo et al., 2013). In cirrus clouds, het may occur at any $RHi > 100\%$, and in the context of a cloud parcel moving in an updraft, ice
40 is first produced through het, and subsequently through hom if the het-produced ice crystals do not prevent the RHi from reaching the threshold RHi needed for hom to occur (e.g., Haag et al., 2003). Overall, cirrus clouds formed primarily through hom will likely have substantially higher N_i and IWC (due to the higher RHi of ice formation) relative to cirrus formed primarily through het (Krämer et al., 2016). Since the cirrus cloud extinction coefficient for sunlight is proportional to IWC/D_e , these two types of cirrus clouds (i.e., hom and het dominated cirrus) may therefore display considerably different radiative
45 properties.

In addition to extinction effects, relatively high N_i produced through hom can result in smaller ice crystals that fall slower relative to het-formed ice crystals (Krämer et al., 2016). These lower ice fall speeds contribute to higher IWCs and longer cloud lifetimes, and thus greater cloud coverage (Mitchell et al., 2008). In this way hom alters cloud radiative properties through changes in D_e and IWC (that affect cloud extinction and visible optical depth τ) and also cloud coverage. Many
50 modelling studies have demonstrated the important impact that changes in ice fall speed have on climate (e.g., Sanderson et al., 2008; Mitchell et al., 2008; Eidhammer et al., 2017).

To date, there are two methods for retrieving all three cirrus cloud properties (N_i , D_e , IWC) from space: (1) the DARDAR approach based on the CloudSat Cloud Profiling Radar (CPR) and the CALIPSO (Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observation) satellite lidar CALIOP (Cloud and Aerosol Lidar with Orthogonal Polarization) as described in
55 Sourdeval et al. (2018) and Delanoë and Hogan (2010), and (2) a CALIPSO approach combining the Infrared Imaging Radiometer (IIR) with the CALIOP lidar as described in Mitchell et al. (2018; henceforth M2018). These two approaches differ in many respects, with the DARDAR approach sensing optically thicker clouds due to the CPR (i.e., the CALIPSO approach is limited to visible cloud optical depths $\tau < \sim 3$). But 79% of all ice clouds (of which cirrus clouds are a subcategory) have a $\tau < 3$ (Hong and Liu, 2015). Moreover, the DARDAR N_i approach presumes a fixed PSD shape based on Delanoë et
60 al. (2014) whereas the CALIPSO approach does not assume a PSD shape, but rather is based on PSD properties obtained from aircraft measurement probes during cirrus cloud field campaigns. Both methods are sensitive to small ice crystals (that dominate N_i) due to the lidar regarding (1) and due to photon tunnelling (i.e., wave resonance) absorption regarding (2) which is most active when ice crystal lengths are comparable to the wavelength ($\sim 10 \mu\text{m}$ in this case) as described in M2018.

This study presents a new CALIPSO satellite retrieval that borrows some methodology from M2018 but also develops new methods that greatly increase the sampling range of cirrus clouds and increase the accuracy of the retrievals. It is similar to M2018 in that it retrieves D_e , N_i , and IWC by employing the effective absorption optical depth ratio, β_{eff} [a standard, well characterized CALIPSO IIR retrieval using retrieved absorption optical depths at 12.05 μm and 10.6 μm in this case], but it differs in that new equations are used for calculating N_i , D_e and IWC for greater accuracy and theoretical soundness as described in Sect. 2. As with M2018, empirical $X - \beta_{\text{eff}}$ relationships are developed from cirrus cloud field campaigns as described in Sect. 3, where X is a microphysical property such as N_i/IWC , but IWC is estimated more accurately and the retrieval is based on more field campaigns. Moreover, retrievals (and ice cloud radiative properties) at terrestrial wavelengths can be sensitive to the shape of the PSD as described in Mitchell (2002) and Mitchell et al. (2011). Such a sensitivity was found in the case of tropical tropopause layer (TTL) cirrus clouds, where their PSD shape differed from the anvil cirrus clouds sampled at higher temperatures. Due to this PSD shape difference, TTL and anvil cirrus having the same β_{eff} can have different D_e , which was accounted for in this retrieval scheme. Finally, β_{eff} was obtained with the most recent CALIPSO Version 4.51 Level 2 products. In Sect. 4, the retrievals are tested against corresponding cloud properties from the field campaigns used to develop this method, as well as the cirrus cloud property climatology of Krämer et al. (2020) based on numerous cirrus cloud field campaigns. Conclusions are given in Sect. 5. Scientific discoveries resulting from this CALIPSO retrieval are described in Part 2 of this study (Mitchell and Garnier, 2024).

2 Developing a new CALIPSO IIR retrieval for cirrus cloud properties

2.1 Analytical formulation

The retrieval of M2018 is based on co-located observations from the IIR and the CALIOP lidar aboard the CALIPSO polar orbiting satellite. It retrieves N_i , D_e and IWC as a function of the effective absorption optical depth ratio β_{eff} , where $\beta_{\text{eff}} = \tau_{\text{abs}}(12.05 \mu\text{m})/\tau_{\text{abs}}(10.6 \mu\text{m})$ and $\tau_{\text{abs}}(12.05 \mu\text{m})$ and $\tau_{\text{abs}}(10.6 \mu\text{m})$ are the effective absorption optical depths retrieved in these IIR channels. β_{eff} is considered an effective ratio since the retrieval of β_{eff} from the cloud emissivity at each wavelength includes the effects of scattering. The N_i retrieval depends on three empirical β_{eff} relationships with D_e , the N_i/IWC ratio, and the PSD effective absorption efficiency at 12 μm $Q_{\text{abs,eff}}(12 \mu\text{m})$. These three β_{eff} relationships were derived from in situ measurements during cirrus cloud aircraft field campaigns. The latter is used to derive the visible layer extinction, α_{ext} , from $\tau_{\text{abs}}(12.05 \mu\text{m})$ and the IIR equivalent cloud thickness Δz_{eq} . Layer N_i is derived from the N_i/IWC ratio after retrieving layer IWC from α_{ext} and the empirical $D_e - \beta_{\text{eff}}$ relationship. The uncertainty in N_i can be reduced by eliminating its dependence on the empirical $D_e - \beta_{\text{eff}}$ relationship and by replacing the N_i/IWC ratio with the N_i/A_{PSD} ratio, where A_{PSD} is the PSD projected area per unit volume directly measured by the 2D-S probe (Lawson et al, 2006; Lawson, 2011). That is, the IWCs used to formulate the retrievals in M2018 were calculated from the ice particle projected area (A_p ; directly measured by the 2D-S probe) and the ice particle mass (m) – A_p power law relationship of Baker and Lawson (2006), where considerable uncertainty in IWC enters through this $m - A_p$ power law. This uncertainty can be eliminated by using the relationship between N_i/A_{PSD}

and β_{eff} , which can be determined from PSD in situ measurements during cirrus cloud field campaigns, analogous to the calculation of the N_i/IWC ratio as described in M2018.

To remove these uncertainties, the retrieval of M2018 was reformulated as follows. We begin by equating two different expressions for the effective absorption coefficient α_{abs} for a homogeneous single-layer cirrus cloud:

$$100 \quad \frac{\tau_{\text{abs}}(\lambda)}{\Delta z_{\text{eq}}} = [Q_{\text{abs,eff}}(\lambda)]_{\beta_{\text{eff}}} \left[\frac{A_{\text{PSD}}}{N_i} \right]_{\beta_{\text{eff}}} N_i \quad (1)$$

where $\tau_{\text{abs}}(\lambda)$ is the effective absorption optical depth at a given wavelength (λ), $Q_{\text{abs,eff}}(\lambda)$ is the PSD effective absorption efficiency ($\alpha_{\text{abs}}/A_{\text{PSD}}$) at the given wavelength and both $Q_{\text{abs,eff}}$ and A_{PSD}/N_i are empirical functions of β_{eff} (and are denoted accordingly). Moreover, A_{PSD} is an area concentration, having units of area per unit volume, while N_i is number per unit volume. This gives A_{PSD}/N_i units of area per ice particle (e.g., cm^2). Solving for N_i , and applying to IIR channel at $\lambda = 12.05$

105 μm ,

$$N_i = \left[\frac{N_i}{A_{\text{PSD}}} \right]_{\beta_{\text{eff}}} \frac{\tau_{\text{abs}}(12.05 \mu\text{m})}{[Q_{\text{abs,eff}}(12 \mu\text{m})]_{\beta_{\text{eff}}} \Delta z_{\text{eq}}} \quad (2)$$

Evaluating units, the rhs has units of reciprocal volume. As in M2018, β_{eff} and $Q_{\text{abs,eff}}(12 \mu\text{m})$ are based on in situ PSD measurements and the modified anomalous diffraction approximation (MADA) (Mitchell, 2000, 2002; Mitchell et al., 2006).

Equation (2) is sensitive to the smallest ice crystals (which contribute the most to N_i) due to its dependence on β_{eff} , where β_{eff} is sensitive to photon tunneling (i.e., wave resonance) absorption, and this type of absorption is strongest when ice particle size is comparable to the absorbed wavelength (e.g., M2018). The quantity Δz_{eq} is smaller than Δz , the cloud layer geometrical thickness measured by CALIOP, and accounts for the fact that the IIR instrument does not equally sense all levels of the cloud layer that contribute to thermal emission. This is accounted for through the IIR weighting profile as discussed in M2018 and Garnier et al. (2021) and detailed later in Sect. 2.2.5.

115 The concept and definition of effective diameter D_e is given in Mitchell (2002) as:

$$D_e = \frac{3}{2} \frac{\text{IWC}}{\rho_i A_{\text{PSD}}} \quad (3)$$

where ρ_i is the bulk density of ice. This definition can be expanded to incorporate the β_{eff} relationships pertaining to N_i/A_{PSD} and IWC/N_i (so that the N_i terms cancel):

$$D_e = \frac{3}{2} \frac{1}{\rho_i} \left[\frac{N_i}{A_{\text{PSD}}} \right]_{\beta_{\text{eff}}} \left[\frac{\text{IWC}}{N_i} \right]_{\beta_{\text{eff}}} \quad (4)$$

120 where the subscript β_{eff} indicates that these ratios are retrieved quantities related to β_{eff} . Since the $D_e - \beta_{\text{eff}}$ relationship in M2018 was not as “tight” or precise as the $N_i/\text{IWC} - \beta_{\text{eff}}$ relationship, and the $N_i/A_{\text{PSD}} - \beta_{\text{eff}}$ relationship has a similar shape as the $N_i/\text{IWC} - \beta_{\text{eff}}$ relationship, Eq. (4) is expected to reduce uncertainties in the retrieval of D_e .

Cirrus cloud climatologies such as reported by Krämer et al. (2020) provide the spherical volume radius, R_v , of the mean ice particle mass, IWC/N_i . Unlike D_e , R_v depends only on IWC/N_i as

$$125 \quad R_v = \left(\frac{3}{4 \pi \rho_i} \right)^{1/3} \left[\frac{IWC}{N_i} \right]_{\beta_{eff}}^{1/3}. \quad (5)$$

With unique retrieval equations for N_i and D_e , IWC is determined as

$$IWC = \frac{\rho_i}{3} \alpha_{ext} D_e \quad (6)$$

where α_{ext} is the shortwave or visible extinction coefficient given as

$$\alpha_{ext} = 2 \left[\frac{1}{Q_{abs,eff}(12 \mu m)} \right]_{\beta_{eff}} \frac{\tau_{abs}(12.05 \mu m)}{\Delta z_{eq}}. \quad (7)$$

130 Similarly, the cloud visible optical depth is given as

$$\tau = 2 \left[\frac{1}{Q_{abs,eff}(12 \mu m)} \right]_{\beta_{eff}} \tau_{abs}(12.05 \mu m) \quad (8)$$

and the cloud ice water path, IWP , is given as

$$IWP = \frac{\rho_i}{3} \tau D_e. \quad (9)$$

As noted, the relationships for the quantities N_i/A_{PSD} , N_i/IWC , and $Q_{abs,eff}(\lambda)$ related to β_{eff} were derived from PSD
 135 measurements from cirrus cloud field campaigns. The field campaigns used here and in M2018 are the SPARTICUS (Small Particles in Cirrus) and TC4 (Tropical Composition, Cloud and Climate Coupling) field campaigns; see M2018 for details concerning SPARTICUS and TC4. The current study also uses the PSD measurements from the ATTREX and POSIDON field campaigns conducted in the tropical western Pacific, which are addressed in Sect. 2.3

2.2 CALIPSO processing and sampling improvements

140 The formulations presented above are applied to co-located CALIOP and IIR observations which provide $\tau_{abs}(12.05 \mu m)$, β_{eff} , and Δz_{eq} of cirrus cloud layers for selected scenes.

2.2.1 CALIPSO IIR data

While M2018 was using the version 3 (V3) CALIPSO products, this study uses the most recent version 4.51 (V4.51) products (Vaughan et al., 2024). The IIR Level 2 V4.51 track product reports cloud effective emissivities $\epsilon_{eff}(12.05 \mu m)$ and $\epsilon_{eff}(10.6 \mu m)$ at 12.05 μm and 10.6 μm at 1-km resolution, from which the respective effective absorption optical depths τ_{abs} are derived
 145 as (M2018, Garnier et al., 2021a)

$$\tau_{abs} = -\ln(1 - \epsilon_{eff}). \quad (10)$$

When both ϵ_{eff} (12.05 μm) and ϵ_{eff} (10.6 μm) are strictly between 0 and 1, β_{eff} can be retrieved as

$$\beta_{\text{eff}} = \frac{\tau_{\text{abs}}(12.05 \mu\text{m})}{\tau_{\text{abs}}(10.6 \mu\text{m})} . \quad (11)$$

150 Both in M2018 and in this study, the calibrated and geo-located radiances are from the IIR version 2 Level 1 products (Garnier et al., 2018). The IIR effective emissivity retrievals are informed by CALIOP cloud detection and characterization as reported in the CALIOP V4.51 5-km cloud and aerosols layer products. IIR effective emissivities are similar in this study and in M2018 which was based on improved IIR V3 Level 2 data.

The contribution from the surface that enters in the computation of the effective emissivities was improved in the suite of
 155 version 4 products after the analysis of IIR data in clear sky conditions as determined by CALIOP, following the same rationale as described in M2018. Land and oceans are first identified using International Geosphere and Biosphere Program (IGBP) surface types reported in the CALIPSO products. The presence of snow or sea-ice, which was based solely on a snow/ice index in M2018, is refined in version 4 by using the co-located 532 nm surface depolarization ratio reported by CALIOP. Following Lu et al. (2017), surface depolarization ratios larger than 0.6 are indicative of snow or sea-ice. Water, sea-ice, and snow types
 160 are assigned different sets of static surface emissivities. Over snow-free land, the surface emissivity at 12.05 μm is also static, and the initial surface temperature provided as an input to the algorithm is adjusted to obtain radiative closure in clear air conditions. Surface emissivity at 10.6 μm is from in-house monthly daytime and nighttime maps (resolution: latitude x longitude = $1^\circ \times 2^\circ$) derived by again reconciling simulations and clear air observations.

The determination of the cloud radiative temperature, T_r , for the computation of the blackbody cloud radiance was improved
 165 in version 4 following the rationale described in M2018. It is determined from the temperature at the centroid altitude of the CALIOP 532 nm attenuated backscatter profile and is further corrected using parameterized functions of emissivity and cloud thermal thickness (Garnier et al., 2021a).

In M2018, the atmospheric profiles and surface temperature used for the CALIOP and IIR retrievals were from the Global Modelling and Assimilation Office (GMAO) Goddard Earth Observing System version 5 (GEOS-5) model. In version 4, these
 170 retrievals use the GMAO Modern-Era Retrospective analysis for Research and Applications version 2 (MERRA-2) model (Gelaro et al., 2017).

2.2.2 Cirrus cloud sampling

Because IIR is a passive instrument, we require, as in M2018, the cirrus cloud of interest to be the only cloud layer detected by CALIOP in the atmospheric column seen by the IIR pixel. Only clouds detected with a 5-km and 20-km horizontal averaging
 175 of the CALIOP signal are considered. Importantly, IIR pixels containing clouds detected at the finest single shot (333 m) horizontal resolution are discarded (Garnier et al., 2021a). In addition, atmospheric columns where absorbing dust was detected by CALIOP are discarded. For this study, the identification of cirrus clouds relies on the CALIOP ice/water phase assignment of cloud layers, which was improved in version 4 (Avery et al., 2020). We select those clouds composed of Randomly Oriented Ice with high confidence in the phase assignment. In version 4, CALIOP cloud-aerosol discrimination is performed at any

180 altitude, whereas it was limited to the troposphere in version 3. Because of uncertainties in the determination of the tropopause altitude, upper troposphere tropical cirrus clouds were missed in version 3 but are included in version 4 (Fig. 15 of Avery et al., 2020). In addition, polar stratospheric clouds classified as ice are now sampled.

We further require that the cirrus clouds are fully sampled by CALIOP to ensure that their true base is detected. These semi-transparent clouds that do not fully attenuate the CALIOP signal have an IIR effective emissivity at 12.05 μm smaller than
185 about 0.8 or visible optical depth smaller than about 3 (Fig. 2 of Garnier et al., 2021b).

The radiative temperature is deemed representative of the IIR layer retrievals and for this study, we require T_r to be colder than 235 K. Unlike in M2018, cirrus clouds with base altitude warmer than 235 K are included because of their CALIOP classification as ice is with high confidence.

2.2.3 Absorption optical depth uncertainties

190 Uncertainties in ϵ_{eff} (12.05 μm) and ϵ_{eff} (10.6 μm) induce uncertainties in τ_{abs} (12.05 μm) and τ_{abs} (10.6 μm) and subsequently in β_{eff} . In semi-transparent clouds, the main sources of error are from the measured radiances and from the surface contribution estimates, and errors increase as effective emissivity and optical depth decrease (M2018, Garnier et al. (2021a)). Errors in the surface contribution estimates are larger over land than over oceans due to the larger variability of surface emissivity and surface temperature over land. To evaluate IIR τ_{abs} (12.05 μm), we use CALIOP 532 nm layer integrated attenuated backscatter
195 (IAB), which is an independent and measured quantity related to visible optical depth. Even though the relationship between CALIOP IAB and IIR τ_{abs} (12.05 μm) depends on $Q_{\text{abs,eff}}$ (12 μm), the extinction-to-backscatter lidar ratio, and the contribution of multiple scattering to the lidar backscatter (Garnier et al., 2015), CALIOP IAB is a reliable reference to assess IIR retrieval errors as optical depth and IAB tend to 0. Furthermore, CALIOP IAB uncertainties are not sensitive to land-oceans differences. Figures 1 and 2 show median IIR τ_{abs} (12.05 μm) and percentiles vs. CALIOP IAB in six latitude bands over land and oceans,
200 respectively, during December-January-February (DJF) and June-July-August (JJA) of 2008, 2010, 2012, and 2013. The statistics are built using IAB bins of $5 \times 10^{-4} \text{ sr}^{-1}$ up to $\text{IAB} = 0.02 \text{ sr}^{-1}$ including non-physical τ_{abs} (12.05 μm) negative values resulting from retrieval errors. Because most of the samples have $\text{IAB} > 5 \times 10^{-4} \text{ sr}^{-1}$, the lowest bin is from $5 \times 10^{-4} \text{ sr}^{-1}$ to 10^{-3} sr^{-1} where median IAB is $\sim 7.6 \times 10^{-4} \text{ sr}^{-1}$. As CALIOP IAB tends to 0, median τ_{abs} (12.05 μm) tends to zero as expected, both over land and over oceans. To $\text{IAB} = 7.6 \times 10^{-4} \text{ sr}^{-1}$ corresponds a visible optical depth (τ) ~ 0.016 - 0.026 assuming an extinction-
205 to-backscatter lidar ratio between 21 and 35 sr^{-1} (Young et al., 2018), that is median τ_{abs} (12.05 μm) ~ 0.0058 - 0.013 assuming $Q_{\text{abs,eff}}$ between 0.72 and 0.96 (see Eq. (8) and Sect. 3). The median τ_{abs} (12.05 μm) values in the lowest bin reported in Table 1 are $\sim 0.0091 \pm 0.0054$ over land and $\sim 0.0065 \pm 0.0013$ over oceans. This is consistent with expectations and therefore shows no evidence of bias in the retrievals. Over land, the largest discrepancy is at 82° S - 60° S in DJF where median τ_{abs} (12.05 μm) might be too large by ~ 0.01 . Otherwise, the discrepancies are smaller than ± 0.003 . Over oceans, the largest discrepancy is at
210 60° N - 82° N in DJF where median τ_{abs} (12.05 μm) might be too small by ~ 0.002 . The spread of τ_{abs} (12.05 μm) values at a given IAB is clearly larger over land in Fig. 1 than over oceans in Fig. 2 in the smaller range of IABs and up to $\text{IAB} = 0.02 \text{ sr}^{-1}$ in winter in the polar regions, which is due to the larger IIR uncertainties over land resulting from the variability of surface

conditions. The τ_{abs} dispersions over land at high latitudes are about twice during winter relative to summer, which might be related to larger uncertainties in surface and atmospheric parameters and smaller radiative contrast between the surface and the cloud temperature.

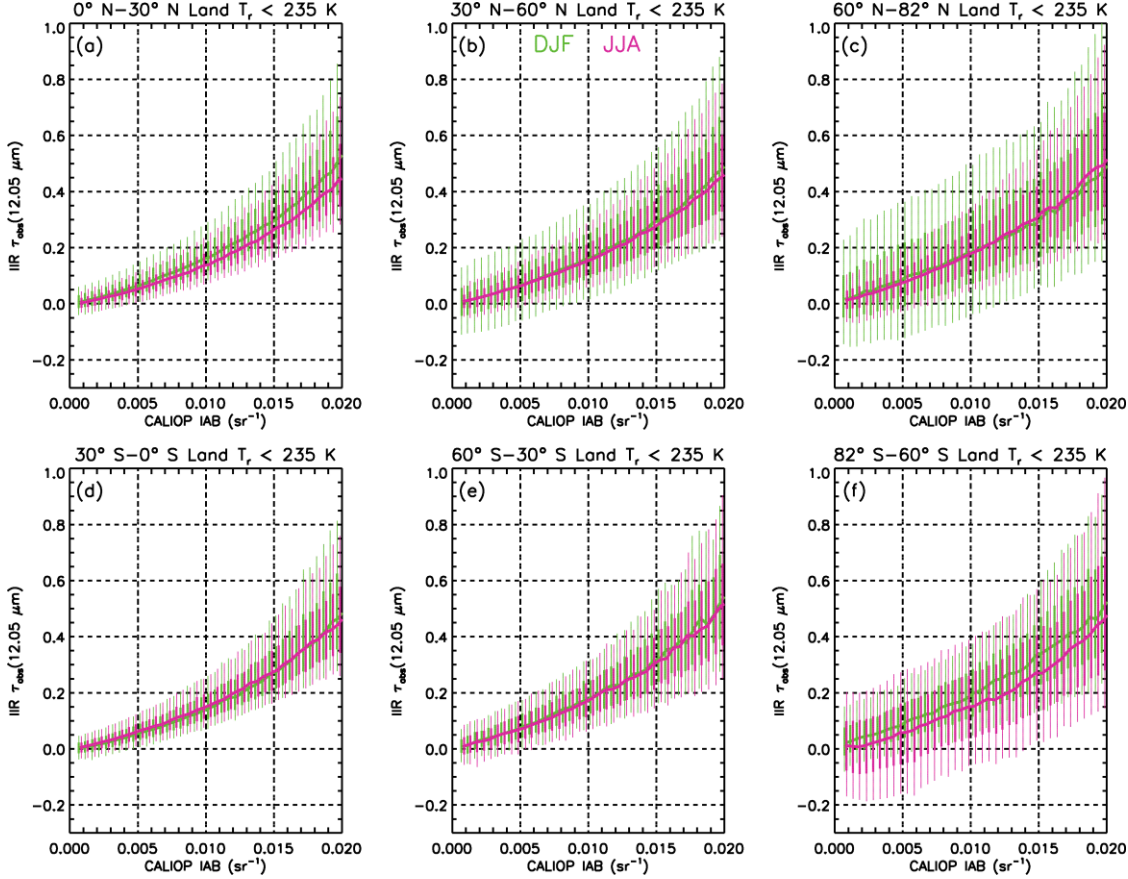
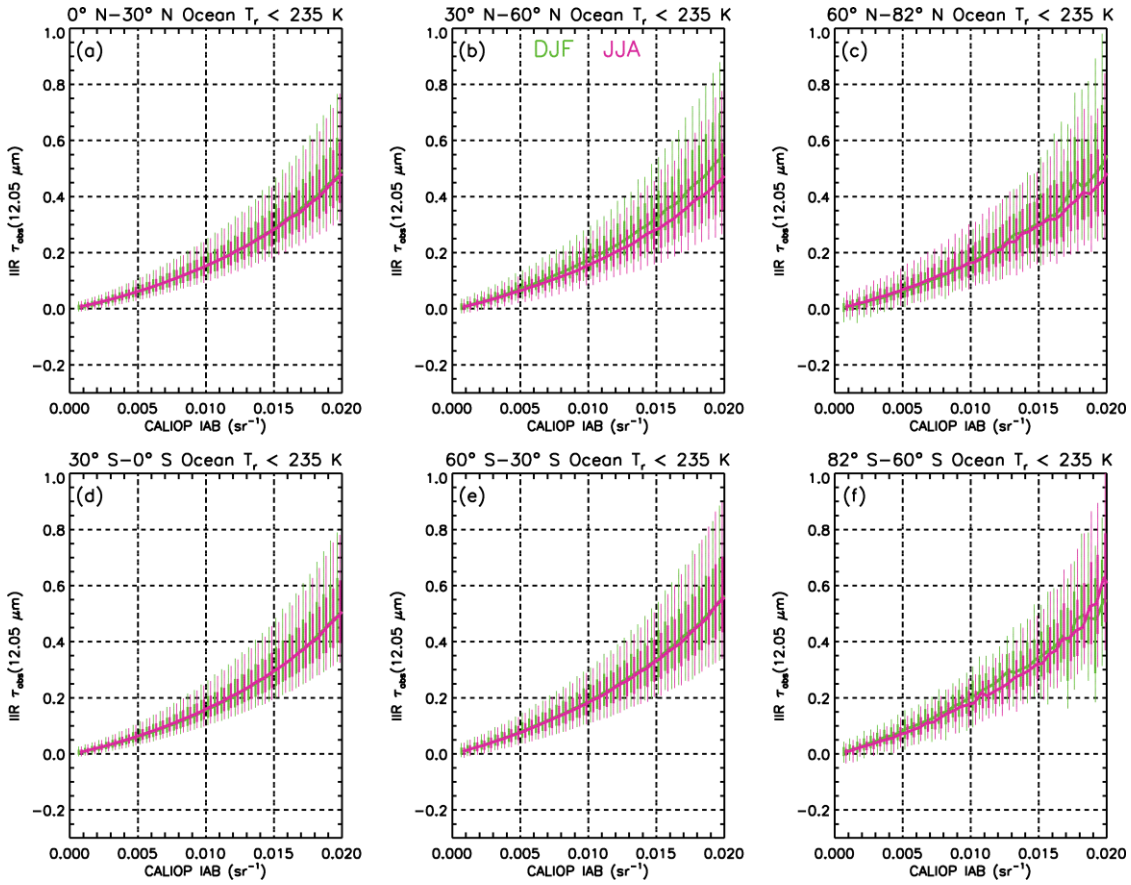


Figure 1. IIR $\tau_{\text{abs}}(12.05 \mu\text{m})$ vs. CALIOP IAB over land in December-January-February (DJF, green) and in June-July-August (JJA, magenta). The solid curves show medians. The thin vertical lines are between the 10th and 90th percentiles and the superimposed thick lines are between the 25th and 75th percentiles. Each row features the tropics (0–30°, panels a and d), midlatitudes (30–60°, panels b and e), and high latitudes (60–82°, panels c and f) in the northern (panels a–c) and in the southern (panels d–f) hemisphere during 2008, 2010, 2012 and 2013.



225 **Figure 2. Same as Fig. 1 but over oceans.**

Table 1: Median IIR $\tau_{\text{abs}}(12.05 \mu\text{m})$ in the lowest bin at CALIOP IAB $\sim 7.6 \times 10^{-4} \text{ sr}^{-1}$ using all retrievals (cf Figs. 1 and 2).

Latitude	Land		Oceans	
	DJF	JJA	DJF	JJA
60°-82° N	0.0161	0.0116	0.0043	0.0088
30°-60° N	0.0064	0.0076	0.0063	0.0062
0°-30° N	0.0064	0.0032	0.0054	0.0071
30°-0° S	0.0032	0.0047	0.0056	0.0059
60°-30° S	0.0089	0.0081	0.0083	0.0081
82°-60° S	0.0212	0.0119	0.0067	0.0056

2.2.4 Impact of optical depth uncertainties in β_{eff} and measurement thresholds

230 Uncertainties in β_{eff} are driven by optical depth uncertainties at 12.05 and 10.6 μm . In addition to the random noise, inter-channel biases of the retrievals could yield systematic biases in β_{eff} , which need to be assessed. A first approach is to evaluate the median $\tau_{\text{abs}}(12.05 \mu\text{m}) - \tau_{\text{abs}}(10.6 \mu\text{m})$ differences (hereafter $\tau_{\text{abs}12-10}$) when IAB tends to 0, i.e. in the lowest bin at CALIOP IAB $\sim 7.6 \times 10^{-4} \text{ sr}^{-1}$ and using all retrievals. Since the imaginary index of refraction (a measure of absorption efficiency) at 10.6 μm is lower than at 12.05 μm , we expect positive differences. The results in Appendix A show that the
235 median differences are overall consistent with expectations, thereby showing no evidence of detectable biases.

However, β_{eff} can be computed only when $\tau_{\text{abs}}(12.05 \mu\text{m})$ and $\tau_{\text{abs}}(10.6 \mu\text{m})$ have physical positive values. Therefore, because of retrieval random errors, especially over land, the $\tau_{\text{abs}}(12.05 \mu\text{m})$ and $\tau_{\text{abs}}(10.6 \mu\text{m})$ distributions are truncated when computing β_{eff} , because the scatter around median τ_{abs} at low IAB values will tend to be more negative at 10.6 μm relative to 12.05 μm . To illustrate the impact of these truncations, Fig. 3 shows the median $\tau_{\text{abs}}(12.05 \mu\text{m}) - \tau_{\text{abs}}(10.6 \mu\text{m})$ differences vs.
240 IAB over land for all retrievals (solid lines) and for samples having only positive values for both $\tau_{\text{abs}}(12.05 \mu\text{m})$ and $\tau_{\text{abs}}(10.6 \mu\text{m})$ (dashed line) for which β_{eff} can be retrieved. These β_{eff} values are also shown (diamonds) and are given by the right axis values. Figure 4 is the same as Fig. 3 but for retrievals over oceans.

The comparison of the solid and dashed lines in Fig. 3 over land shows that discarding non-physical absorption optical depths yields underestimated optical depth differences, and therefore underestimated β_{eff} values. These systematic low biases
245 result largely from the greater percentage of negative values in the dispersion around $\tau_{\text{abs}}(10.6 \mu\text{m})$ at low IAB, so that using only the positive values yields a smaller (or negative) difference for $\tau_{\text{abs}}(12.05 \mu\text{m})$ minus $\tau_{\text{abs}}(10.6 \mu\text{m})$.

The greater separation between the solid and dashed curves in Fig. 3 during polar winter may relate to a lower contrast between the surface and cloud radiances and/or overall weaker radiances. But more generally, the divergence between these curves relates to the truncation bias noted above. Moreover, the decrease in β_{eff} with decreasing IAB for IAB $< 0.01 \text{ sr}^{-1}$ in
250 Fig. 3 in the Polar Regions, and IAB $< 0.005 \text{ sr}^{-1}$ elsewhere, tends to roughly correspond with the divergence between the solid and dashed curves. These two trends are largely absent over oceans (cf Fig. 4) where surface emissivity and temperature are well characterized, thus greatly reducing the amount of scatter around the median values of $\tau_{\text{abs}}(10.6 \mu\text{m})$ and $\tau_{\text{abs}}(12.05 \mu\text{m})$. The only exception is at high latitude in the northern hemisphere (Fig. 4c) for both seasons where β_{eff} at IAB smaller than 0.003 sr^{-1} appears to be underestimated.

255 From this analysis, we chose a threshold IAB $> 0.01 \text{ sr}^{-1}$ over land to ensure that the distributions are not or only slightly truncated. Nevertheless, for high latitudes in winter, median β_{eff} is likely underestimated for IAB up to 0.02 sr^{-1} . The chosen IAB $> 0.01 \text{ sr}^{-1}$ threshold corresponds to median $\tau_{\text{abs}}(12.05 \mu\text{m}) \sim 0.15$ (Figs. 1 and 2), that is $\tau > \sim 0.24-0.3$ on average. In M2018, an IAB threshold of 0.01 sr^{-1} was applied to all retrievals, both over land and oceans. However, Fig. 4 shows that this condition can be relaxed over oceans. We refined the analysis over oceans by inspecting the fraction of negative $\tau_{\text{abs}}(10.6 \mu\text{m})$ values as $\tau_{\text{abs}}(12.05 \mu\text{m})$ increases from 0 with increments of 0.001. We estimate that the $\tau_{\text{abs}}(10.6 \mu\text{m})$ distribution is not
260 significantly truncated when more than 90 % of the $\tau_{\text{abs}}(10.6 \mu\text{m})$ values are positive, yielding a lower threshold of 0.006 for

$\tau_{\text{abs}}(12.05 \mu\text{m})$ or $\tau > \sim 0.01$. The asterisks in Fig. 4 show that applying this threshold slightly increases median β_{eff} at $\text{IAB} \leq 0.002 \text{ sr}^{-1}$, most notably in the tropics and at mid-latitude. Nevertheless, the width of the β_{eff} distributions increases rapidly as IAB and optical depth approach zero, which is due in large part to increasing random uncertainties (Garnier et al., 2021b; M2018). This is illustrated in Table 2 for JJA at $0^\circ\text{--}30^\circ\text{N}$, where the difference between the 75th and 25th β_{eff} percentiles is ~ 0.49 for median $\tau_{\text{abs}}(12.05 \mu\text{m}) = 0.02$ but only ~ 0.07 for median $\tau_{\text{abs}}(12.05 \mu\text{m}) = 0.49$. The difference between the 90th and 10th percentiles is about twice these values.

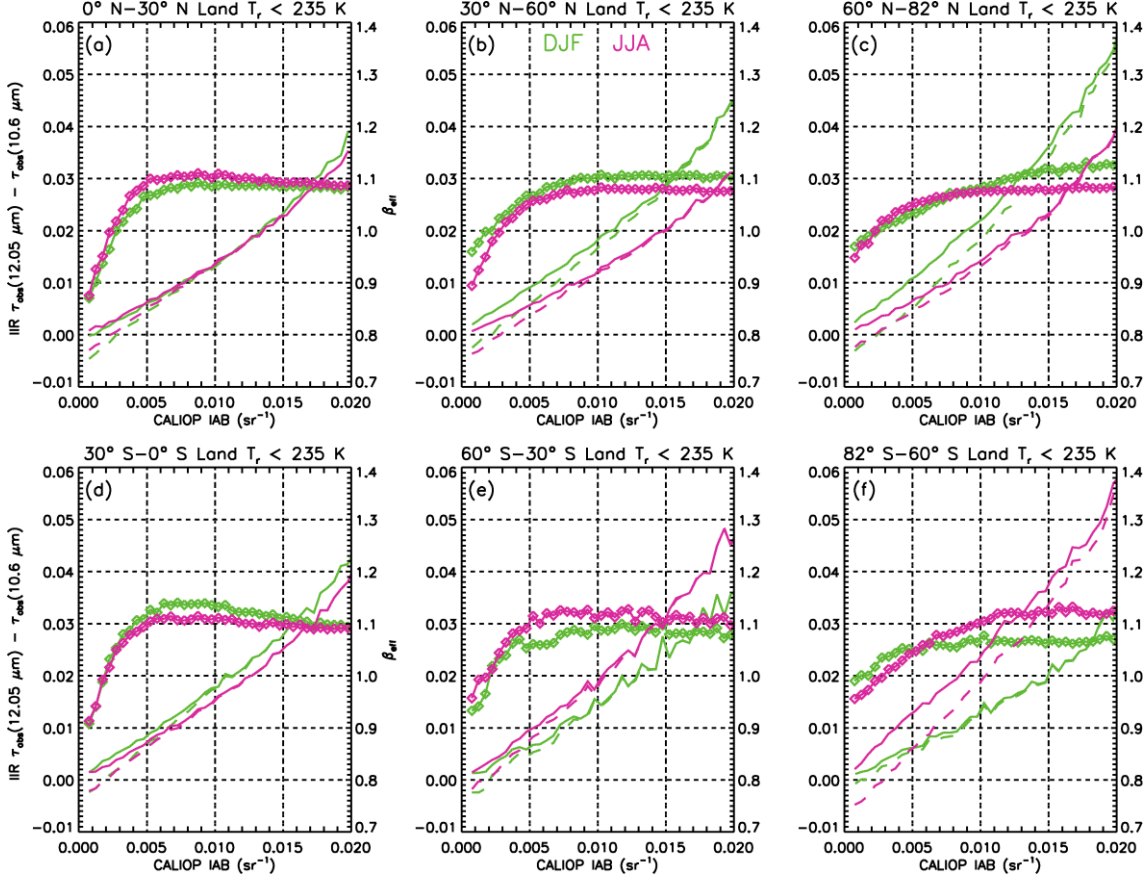


Figure 3. Median $\tau_{\text{abs}}(12.05 \mu\text{m}) - \tau_{\text{abs}}(10.6 \mu\text{m})$ difference vs. CALIOP IAB over land for all samples (solid), and for samples with positive absorption optical depths (dashed) for which β_{eff} (diamonds, right vertical axis) can be retrieved. Each row features the tropics ($0\text{--}30^\circ$, panels a and d), midlatitudes ($30\text{--}60^\circ$, panels b and e), and high latitudes ($60\text{--}82^\circ$, panels c and f) in the northern (panels a-c) and in the southern (panels d-f) hemisphere during December-January-February (DJF, green) and June-July-August (JJA, magenta) of 2008, 2010, 2012 and 2013.

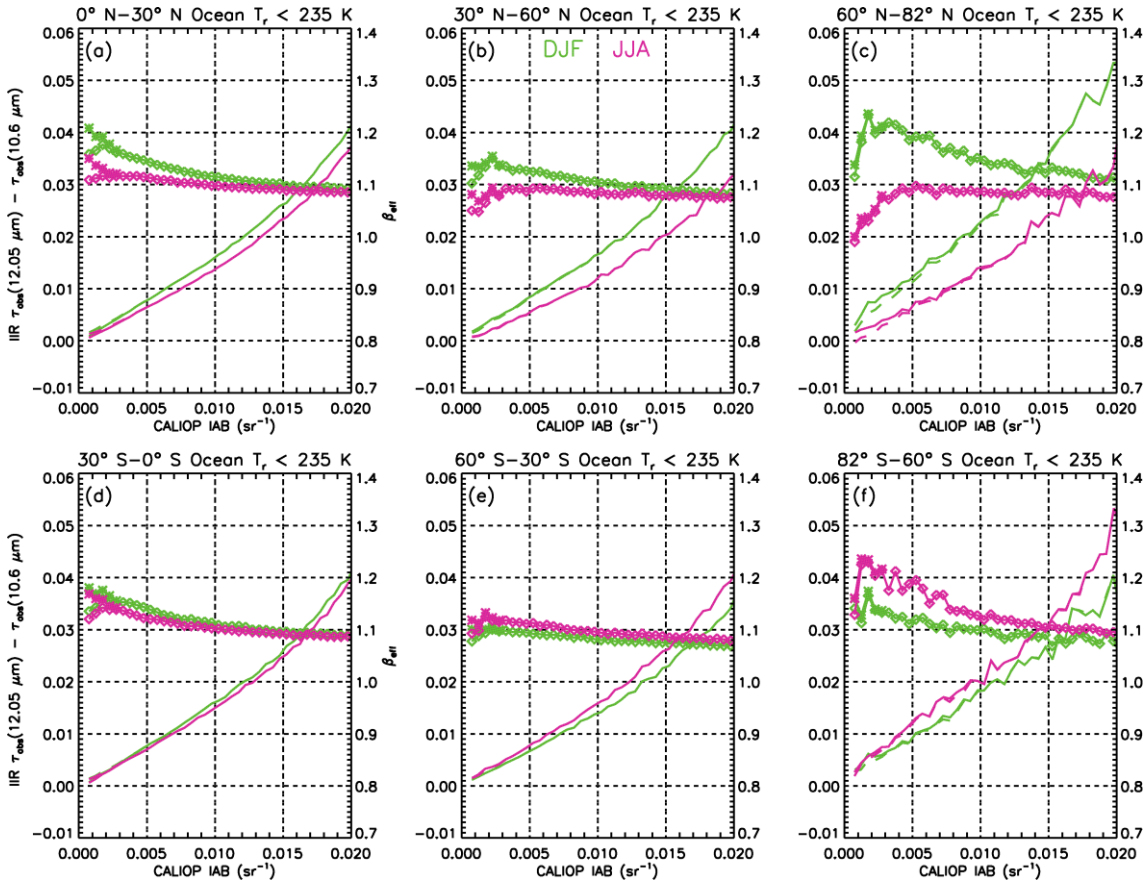


Figure 4. Same as Fig. 3 but over oceans. In addition, the differences between the asterisks and the diamonds show that requiring $\tau_{\text{abs}}(12.05 \mu\text{m}) \geq 0.006$ or visible optical depth $> \sim 0.01$ increases median β_{eff} at $\text{IAB} \leq 0.002 \text{ sr}^{-1}$.

280 Table 2: β_{eff} percentiles, as well as 75th-25th and 90th-10th percentiles differences for three CALIOP IAB bins over oceans where $\tau_{\text{abs}}(12.05 \mu\text{m}) > 0.006$ (or $\tau > \sim 0.01$).

IAB (sr ⁻¹)	Median τ_{abs} (12.05 μm)	β_{eff}						
		10 th	25 th	median	75 th	90 th	75 th -25 th	90 th -10 th
0.0008	0.02	0.765	0.943	1.150	1.431	1.946	0.488	1.181
0.0103	0.16	1.003	1.048	1.098	1.154	1.226	0.106	0.223
0.0202	0.49	1.025	1.054	1.086	1.122	1.164	0.068	0.139

2.2.5 IIR equivalent layer thickness, Δz_{eq} , and radiative temperature

Even though the IIR is a passive instrument that retrieves layer integrated quantities such as cloud optical depth, the cloud boundaries information provided by CALIOP allows one to retrieve vertically resolved layer properties such as the layer extinction coefficient. However, the high sensitivity of CALIOP to cloud detection and the expected variability of extinction within the layer are such that only a portion of the cloud layer detected by CALIOP is “seen” by IIR. Thus, relevant for our retrievals is the IIR equivalent layer thickness, Δz_{eq} , which is estimated using the IIR in-cloud weighting function derived from the in-cloud 532 nm CALIOP extinction profile of vertical resolution, δz (Garnier et al., 2021a). For this analysis, we choose to use IIR channel centered at 12.05 μm .

The effective emissivity ε_{eff} of a cloud composed of n vertical bins, i , from $i = 1$ at the cloud base to $i = n$ at cloud top can be seen as the vertically integrated in-cloud IIR effective emissivity attenuated profile $\varepsilon_{att}(i)$ written as:

$$\varepsilon_{att}(i) = \varepsilon(i) \prod_{j=i+1}^{j=n} (1 - \varepsilon(j)). \quad (12)$$

In Eq. (12), $\varepsilon(i)$ is the emissivity of bin i , and the second term represents the transmittance through the overlying cloudy bins. The $\varepsilon(i)$ term is derived from the CALIOP cloud extinction coefficient of bin i , $\alpha_{part}(i)$, as

$$\varepsilon(i) = 1 - \exp\left(-\alpha_{part}(i) \delta z / r\right) \quad (13)$$

where r is the scaling ratio between the CALIOP layer optical depth, τ_{CAL} , and the cloud effective absorption depth τ_{abs} . The IIR weighting function, $WF_{IIR}(i)$, is obtained from Eq. (12) after normalization by ε_{eff} as

$$WF_{IIR}(i) = \frac{\varepsilon_{att}(i)}{\varepsilon_{eff}} \quad (14)$$

so that

$$\sum_i WF_{IIR}(i) = 1. \quad (15)$$

Then, we compute the IIR-weighted layer extinction coefficient, $\alpha_{CAL-IIR}$, as

$$\alpha_{CAL-IIR} = \sum_{i=1}^{i=n} \alpha_{part}(i) WF_{IIR}(i). \quad (16)$$

This IIR-weighted layer extinction coefficient is larger than or equal to the mean layer extinction, noted $\text{mean}(\alpha_{CAL})$. Finally, the IIR equivalent layer thickness, Δz_{eq} , is related to the geometric thickness Δz as

$$\Delta z_{eq} = \Delta z \frac{\text{mean}(\alpha_{CAL})}{\alpha_{CAL-IIR}}. \quad (17)$$

The ratio r in Eq. (13) is estimated using CALIOP visible optical depth, and might thus differ from $2/Q_{abs,eff}$ (12 μm) used in this study to derive visible optical depth, τ (Eq. (8)). Importantly, Δz_{eq} does not depend on this ratio. Had we used τ instead of τ_{CAL} , both α_{part} and r in Eq. (13) would have been multiplied by τ/τ_{CAL} and $\varepsilon(i)$ and the subsequent $WF_{IIR}(i)$ would have been

310 unchanged. Likewise, both $\text{mean}(\alpha_{\text{CAL}})$ and $\alpha_{\text{CAL-IIR}}$ in Eq. (17) would have been multiplied by τ/τ_{CAL} , leaving Δz_{eq} unchanged. We note, however, that without vertically resolved information, $2/Q_{\text{abs,eff}}(12 \mu\text{m})$ (or r) is supposed constant within the layer.

Two cirrus examples are shown in Fig. 5 where the CALIOP extinction coefficient (α_{part}) profile is in black and the IIR weighting function (WF_{IIR}) profile is in red. The vertical resolution is $\delta z = 0.06 \text{ km}$. The first example in panel a is a TTL cirrus between 15.13 and 16.5 km observed in June 2010. Retrieved ε_{eff} is equal to 0.06 and the black and red curves have an

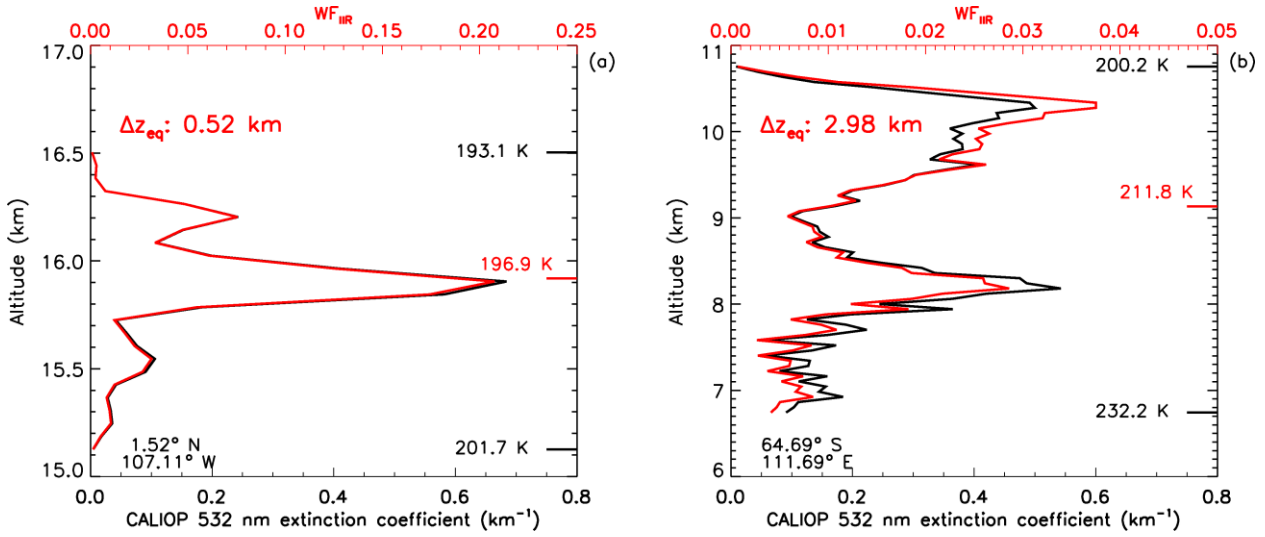
315 almost identical shape because the attenuation term in Eq. (12) is close to 1. We find $\Delta z_{\text{eq}} = 0.52 \text{ km}$, which corresponds roughly to the main marked peak and to the secondary maximum. In panel b, the cirrus is between 6.74 and 10.74 km in the Southern Ocean in August 2008. Here, retrieved ε_{eff} is equal to 0.44 and relative to the black curve, the lower part of the cloud contributes less to the cloud emissivity than the upper part. The equivalent thickness $\Delta z_{\text{eq}} = 2.98 \text{ km}$ can be seen as the portion of the cloud above 7.8 km where WF_{IIR} exceeds about 0.008. In these examples, $\Delta z_{\text{eq}}/\Delta z$ is equal to 0.38 (a) and 0.74 (b).

320 Figure 6 shows Δz_{eq} vs. Δz for all the sampled cirrus over oceans, showing that $\Delta z_{\text{eq}}/\Delta z$ is globally mostly between 0.5 and 0.9.

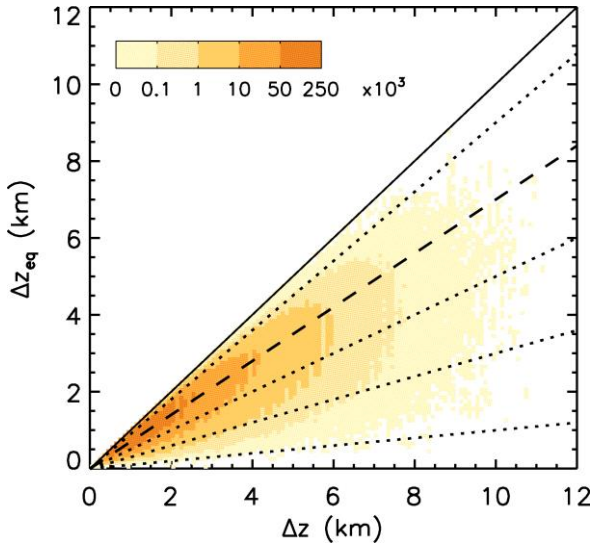
The examples shown in Fig. 5 illustrate that the IIR weighting function is in first approximation the CALIOP extinction profile normalized to the optical depth if the attenuation term in Eq. (12) is supposed to be close to 1 and $\varepsilon(i)$ is approximated to the corresponding τ_{abs} in Eq. (13). This IIR weighting function is also used to determine the cloud centroid radiance and the corresponding radiative temperature, T_r , (Garnier et al., 2021a), which is given in red in each panel. The temperatures in black

325 are T_{top} and T_{base} at the layer top and base altitudes, respectively. Because computing a centroid temperature would yield a temperature differing by less than a few tenths of a degree Kelvin (M2018), T_r can be seen as the temperature dividing the cloud optical depth τ into equal parts. In panel a where WF_{IIR} exhibits one main peak, the altitude corresponding to T_r is 15.9 km, near the WF_{IIR} maximum. The $T_r - T_{\text{top}}$ difference is 45 % of the thermal thickness. In panel b, the altitude corresponding to T_r is 9.1 km located between the two peaks of comparable amplitude, slightly closer to the upper one. T_r is slightly closer

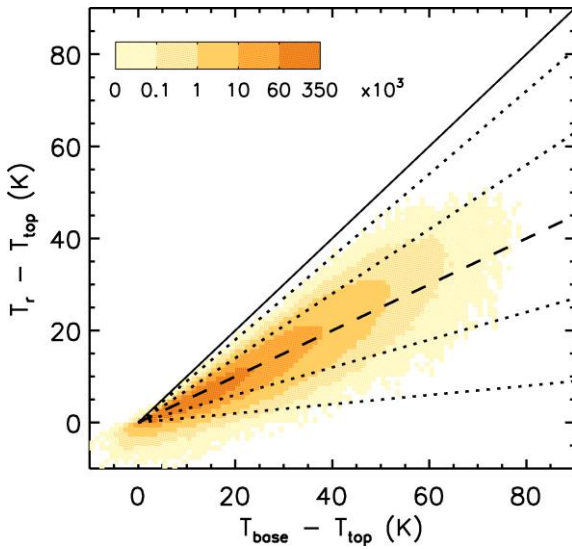
330 to the top with a $T_r - T_{\text{top}}$ difference of 36 % of the thermal thickness. As discussed in M2018 and illustrated in Fig. 7 showing $T_r - T_{\text{top}}$ against $T_{\text{base}} - T_{\text{top}}$ for all the sampled cirrus over oceans, $T_r - T_{\text{top}}$ represents typically 30 to 70 % of $T_{\text{base}} - T_{\text{top}}$. Using temperature difference as a proxy for altitude difference, it appears that T_r is on average at mid-cloud.



335 **Figure 5.** Two cirrus cloud examples showing the CALIOP extinction coefficient (α_{part}) profile in black and the IIR weighting function (WF_{IIR}) profile in red, with Δz_{eq} indicated in red. The temperatures in black on the right-hand side of each panel are at cloud top and base, and in red is the radiative temperature (T_r) at the corresponding altitude. These examples are extracted from CALIPSO granules (a) 2010-06-04T08-39-58ZN and (b) 2008-08-12T07-26-54ZD, with latitude and longitude given in the lower left corner of the respective panels.



340 **Figure 6.** Δz_{eq} vs. Δz for all sampled cirrus over oceans during 2008, 2010, 2012, and 2013. The colors represent the IIR pixels density. The dashed and dotted lines, from bottom to top, represent $\Delta z_{\text{eq}}/\Delta z$ of 0.1, 0.3, 0.5, 0.7, and 0.9.



345 **Figure 7.** $T_r - T_{top}$ vs. $T_{base} - T_{top}$ for all sampled cirrus over oceans during 2008, 2010, 2012, and 2013. The colors represent the IIR pixels density. The dashed and dotted lines, from bottom to top, represent $(T_r - T_{top})/(T_{base} - T_{top})$ of 0.1, 0.3, 0.5, 0.7, and 0.9. Using temperature difference as proxy for altitude difference, it appears that T_r is on average at mid-cloud.

2.3 Inclusion of additional tropical cirrus field campaigns

350 The M2018 CALIPSO retrieval was based on $XX - \beta_{eff}$ relationships (where XX refers to N_i/IWC , D_e , or $2/Q_{abs,eff}(12\mu m)$) developed from the SPARTICUS (Jensen et al., 2013a) and TC4 (Toon et al., 2010) cirrus cloud field campaigns. In this new retrieval, the ATTREX and POSIDON cirrus cloud field campaigns (Jensen et al., 2017; Schoeberl et al., 2019) conducted in the tropical western Pacific were also used for this purpose, along with the SPARTICUS and TC4 field campaigns. Cirrus clouds were sampled in POSIDON by the NASA WB-57 aircraft, which flew the SPEC Inc. Fast Cloud Droplet Probe (FCDP; 355 Glienke and Mei, 2020; Lawson et al., 2017), two-dimensional stereo (2D-S) probe (Lawson et al., 2006), and Cloud Particle Imager (CPI; Lawson et al., 2001). During ATTREX, they were sampled by the Global Hawk unmanned aircraft system, which flew the SPEC Inc. Hawkeye instrument to measure ice PSDs between $-50^\circ C$ and $-85^\circ C$ but mostly in the tropical tropopause layer (TTL) between $-65^\circ C$ and $-85^\circ C$ (Woods et al., 2018). The Hawkeye houses three instruments that measure the complete cloud PSD and the corresponding size-resolved cloud particle shapes; these are versions of the FCDP (), the 2D-S probe , and 360 the CPI (). The FCDP and 2D-S probe tips are designed to minimize ice particle shattering, and particle interarrival times are used to identify and remove clusters of particles resulting from shattering (Baker et al., 2009). The FCDP sampled particles between 1 and 50 μm while the 2D-S sampled ice particle maximum dimension D from 10 to 1280 μm (Woods et al., 2018), although $D > 1280 \mu m$ can be estimated up to 4 mm (Jensen et al., 2017). However, the first (5-15 μm) size-bin of the 2D-S probe and the last size-bin (45-50 μm) of the FCDP were not used for producing composite mean PSDs. Figure 8 shows 365 representative mean PSD examples from POSIDON (on left) and ATTREX (on right) along with information on corresponding

effective radius (R_{eff}), N_i , and IWC. The agreement between the FCDP and 2DS probes where they overlap (from 15 to 45 μm , indicated by the red and blue histograms) was good (as shown here) for most of the PSD measurements. The number of PSD during POSIDON having notably poorer agreement than those in Fig. 8 was 3 out of 66 PSDs in total, or 4.5%, with similar findings for ATTREX. Moreover, Jensen et al. (2013a) found good agreement for N_i between the 2D-S and another N_i probe (the Video Ice Particle Sampler or VIPS) when the first size-bin of the 2D-S probe was not considered.

For the SPARTICUS and TC4 campaigns, PSDs were measured only by the 2D-S probe. To determine whether ice particle concentrations in the first size-bin (i.e., $N(D)_1$) of the 2D-S probe should be used for calculating the $N_i/\text{IWC} - \beta_{\text{eff}}$, $N_i/A_{\text{PSD}} - \beta_{\text{eff}}$ and $1/Q_{\text{abs,eff}}(12\mu\text{m}) - \beta_{\text{eff}}$ relationships from these campaigns (that were used in this retrieval as described in Sect. 3), PSDs from the POSIDON campaign were qualitatively evaluated from PSD plots provided by SPEC, Inc. The good agreement noted above between the FCDP and 2D-S probes from 15 μm to 45 μm suggests that the FCDP measurements from 1 to 15 μm may also be realistic. Jensen et al. (2013a) states that “In nearly all of the 2D-S size distributions, the concentration in the

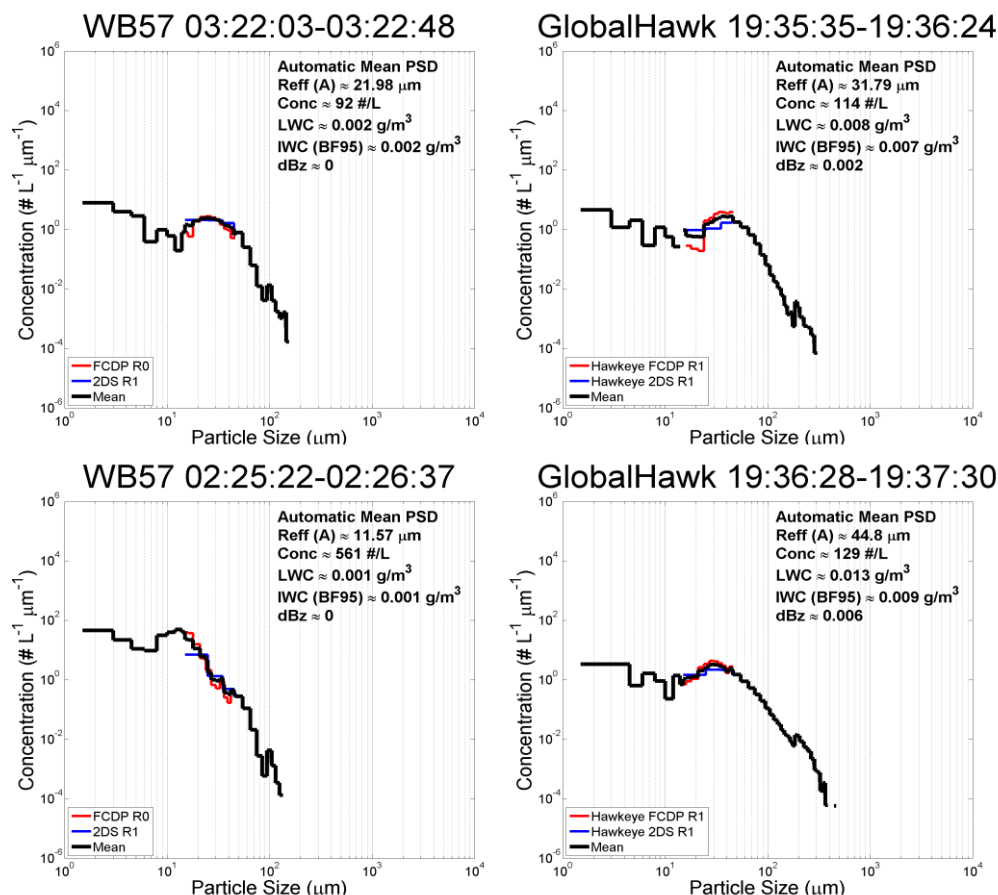


Figure 8. Representative mean composite PSD from the ATTREX (right) and POSIDON (left) field campaigns, sampled by the FCDP (1.5 – 45 μm) and 2D-S (15 – 1280 μm) probes. The two probes in their overlap region (red and blue histograms) yield relatively consistent values, providing confidence in these measurements.

first size bin (5-15 μm) is considerably larger than the concentrations in the next few larger bins, and the first bin often contributes significantly to the overall ice concentration.” We found this to be true of the ATTREX-POSIDON 2D-S measurements as well. For the POSIDON PSDs, $N(D)_1$ of the 2D-S was within a factor of ~ 2.5 of the combined corresponding FCDP bins for 23% of the PSDs but exhibited much higher factors ranging from 3 to 32 for the other PSDs. On average, the 2D-S $N(D)_1$ was a factor of 10.4 ± 8.1 greater than the ice particle concentration in the corresponding FCDP bins. Therefore, regarding the SPARTICUS and TC4 PSDs, we modified the measured PSDs by dividing the 2D-S $N(D)_1$ by 10.4 to approximately correct for this behaviour. While this correction would not always be valid for a single PSD measurement, it may be realistic for a large ensemble of PSDs. Relevant information for the SPARTICUS and TC4 field campaigns can be found in M2018. In M2018, different assumptions regarding $N(D)_1$ resulted in different retrieval formulations, but in the current approach, only one retrieval formulation is needed and presented.

2.4 Treatment of ice water content

As shown in Fig. 8, ATTREX and POSIDON PSDs were generally narrow, with maximum ice particle sizes generally less than 400 μm and often less than 150 μm . The Baker and Lawson (2006) ice particle area-mass expression that has normally been used to calculate the IWC for SPEC, Inc. PSD data predicts a spherical ice particle mass greater than predicted for spherical ice particles at bulk ice density (0.917 g cm^{-3}) when ice particle maximum dimension $D < 47 \mu\text{m}$ (which is non-physical). Since much of the PSD mass during the ATTREX and POSIDON campaigns is often associated with $D < 47 \mu\text{m}$, the ice particle mass-dimension expressions described in Erfani and Mitchell (2016; henceforth EM2016) were used for developing relationships in this retrieval scheme since the EM2016 mass-dimension relationships were designed to calculate the mass of small particle sizes $< 100 \mu\text{m}$ more realistically. These EM2016 relationships are shown in Fig. 9, along with relationships from Lawson et al. (2019) for marine anvils cirrus, from Mitchell et al. (2010), and from Weitzel et al. (2020). It is seen that these relationships are relatively consistent, especially for $D < 100 \mu\text{m}$ where uncertainties are greatest.

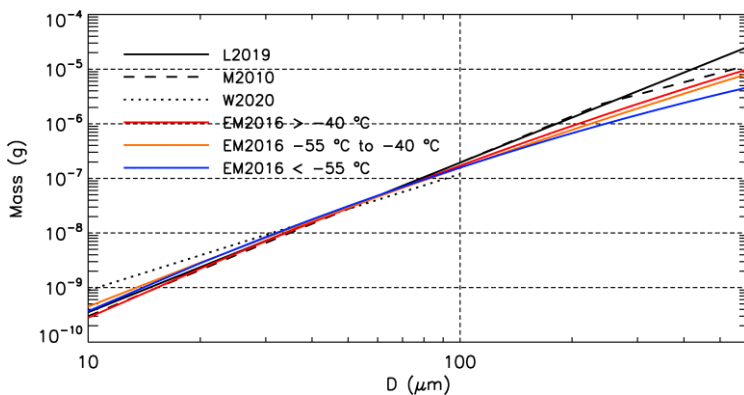


Figure 9. Relationships between mass m (g) and particle dimension D (μm) from Lawson et al. (2019) (L2019, solid black line), Mitchell et al. (2010) (M2010, dashed black line), Weitzel et al. (W2020, dotted black line), and from EM2016 for temperatures colder than 55 $^{\circ}\text{C}$ (blue), between -55 $^{\circ}\text{C}$ and -40 $^{\circ}\text{C}$ (orange) and warmer than -40 $^{\circ}\text{C}$ (red).

2.4.1 Mass-dimension relationship and β_{eff}

We recall that β_{eff} of the particle size distribution is the ratio of effective absorption efficiencies at 12.05 and 10.6 μm , where
 410 ‘effective’ refers to the scattering contribution (see Eqs, 4 and 5 of M2018). For this discussion, we can assume that $\beta_{\text{eff}} \sim \beta$,
 i.e. the ratio of absorption efficiencies at 12.05 and 10.6 μm . As discussed in Mitchell et al. (2002), the relevant dimension to
 characterize the absorption efficiency of the single particle at a given wavelength, λ , is the effective distance d_e defined as

$$d_e = \frac{m}{\rho_i \cdot A_p} . \quad (18)$$

Likewise, absorption efficiencies, $q_{\text{abs}}(\lambda)$, derived from MADA are uniquely related to d_e , as shown in Fig. 10a for the IIR
 415 channels using several ATTREX and POSIDON PSDs. The X-axis is $(3/2) \times d_e$, noting that this quantity is the effective
 diameter of the single particle. While A_p is directly measured, m is derived from mass-dimension or mass-area relationships,
 so that d_e depends on these relationships. The discontinuities in $q_{\text{abs}}(\lambda)$ result from changes in the MADA tunneling (i.e.,
 resonance) efficiencies that depend on ice particle shape and size (M2018; Sect. 2.3). The PSD absorption efficiency $Q_{\text{abs}}(\lambda)$
 is obtained after integration of $q_{\text{abs}}(\lambda)$ over the area distribution, $A(D)$, where D is particle dimension. Because $q_{\text{abs}}(\lambda)$ is
 420 uniquely related to d_e , $Q_{\text{abs}}(\lambda)$ can be written

$$Q_{\text{abs}}(\lambda) = \int_{\text{PSD}} q_{\text{abs}}(\lambda, d_e(D)) A(D) dD \quad (19)$$

and

$$\beta = \frac{Q_{\text{abs}}(12 \mu\text{m})}{Q_{\text{abs}}(10.6 \mu\text{m})} . \quad (20)$$

It appears that $Q_{\text{abs}}(\lambda)$ and β of a PSD depend on the variation of $A(D)$ with d_e , which again depends on the estimated mass of
 425 the single particle. This is illustrated in Figs. 10b-d with three examples from the ATTREX and POSIDON campaigns, where
 the same $A(D)$ is shown vs. $(3/2) \times d_e$ using the mass from EM2016 in black and from SPEC in red. The black distributions
 are shifted towards smaller d_e values compared to the red ones, yielding a larger β (and ultimately β_{eff}) value using the EM2016
 relationships. In these examples, the β_{eff} values are increased from 1.059 to 1.091 in panel a, from 1.088 to 1.192 in panel b,
 and from 1.191 to 1.316 in panel c.

430 2.4.2 IIR β_{eff} – temperature comparisons with SPEC and EM2016

To assess the impact of the m-D relationships on in situ β_{eff} , we compared β_{eff} of PSDs measured during the ATTREX (2014)
 and POSIDON campaigns with independent IIR β_{eff} retrievals (Fig. 11). Because one-to-one comparisons are not possible, we
 compared β_{eff} vs. temperature, which is layer radiative temperature for IIR. To match the field campaigns, IIR samples are in
 0° N - 20° N and 130° E - 160° E during February and March 2014 for ATTREX and October 2016 for POSIDON (Schoeberl
 435 et al., 2019). Comparisons in Fig. 11 are for IIR single-layer semi-transparent cirrus clouds having IAB > 0.01 sr^{-1} or $\tau > \sim$
 0.3. Most of the PSD β_{eff} derived using the SPEC relationships are smaller than median IIR β_{eff} , whereas using the EM2016

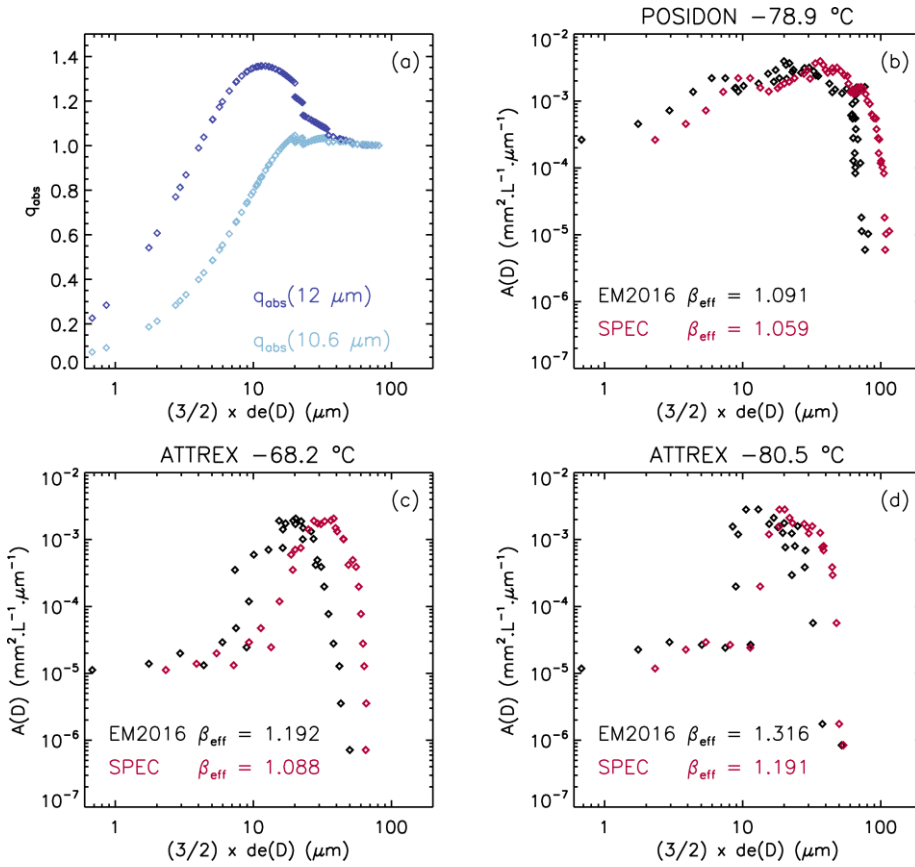


Figure 10. Panel (a) shows the unique relationship between absorption efficiency at $12 \mu\text{m}$ (dark blue) and $10.6 \mu\text{m}$ (light blue) and effective distance $d_e(D)$. Panels (b-d) show three examples of $A(D)$ vs. $(3/2) \times d_e(D)$ from the POSIDON (b) and the ATTREX (c-d) campaigns with $d_e(D)$ computed using particle mass from EM2016 (black) and SPEC (red). The smaller mass using EM2016 yields smaller $d_e(D)$ and larger β_{eff} values.

relationships brings PSD and IIR β_{eff} values in a good agreement. However, because the field campaigns targeted TTL cirrus clouds, most of the PSD temperatures are colder than 208 K whereas IIR sampling is sparse below 200 K. As the sampled region is over oceans, we repeated the experiment in Fig. 12 but this time by including cloud having $\tau > \sim 0.01$. IIR sampling of TTL clouds is improved in Fig. 12, so that the comparisons are more informative. Despite the increased random noise, which explains the larger occurrence of extreme IIR β_{eff} values in Fig. 12 than in Fig. 11, IIR and PSD β_{eff} are again in better agreement for the EM2016 relationships. The horizontal dashed light blue lines in the left panels of Figs. 11 and 12 indicate β_{eff} at the sensitivity limit (see Sect. 3).

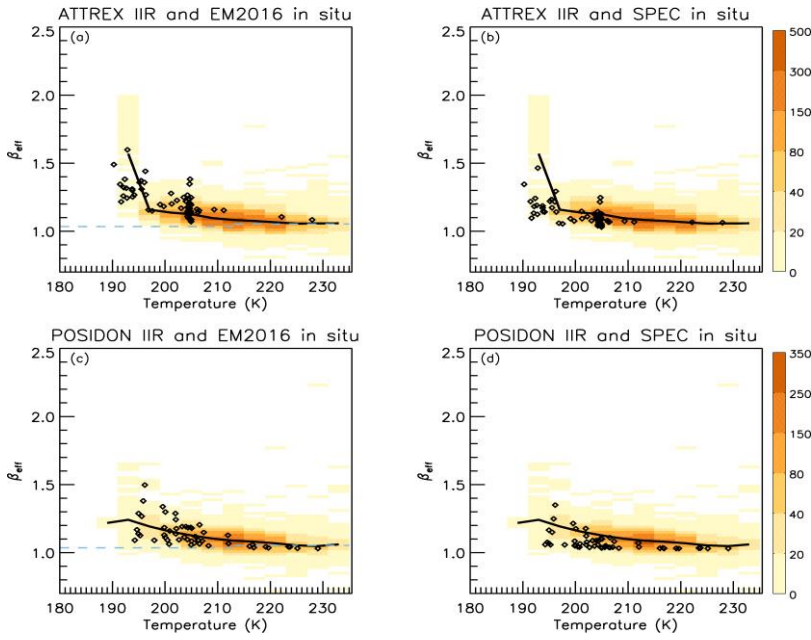


Figure 11. IIR β_{eff} vs. temperature in $0^\circ \text{N} - 20^\circ \text{N}$ and $130^\circ \text{E} - 160^\circ \text{E}$ during February and March 2014 (ATTREX, panels a and b) and October 2016 (POSIDON, panels c and d) compared with β_{eff} of PSDs (diamonds) measured during the denoted campaigns using the EM2016 (panels a and c) and the SPEC (panels b and d) mass-dimension relationships. The colors indicate IIR samples density and black curves represent median IIR β_{eff} . The horizontal dashed light blue lines in panels a and c indicate β_{eff} at the sensitivity limit (see Sect. 3). IIR optical depth $> \sim 0.3$.

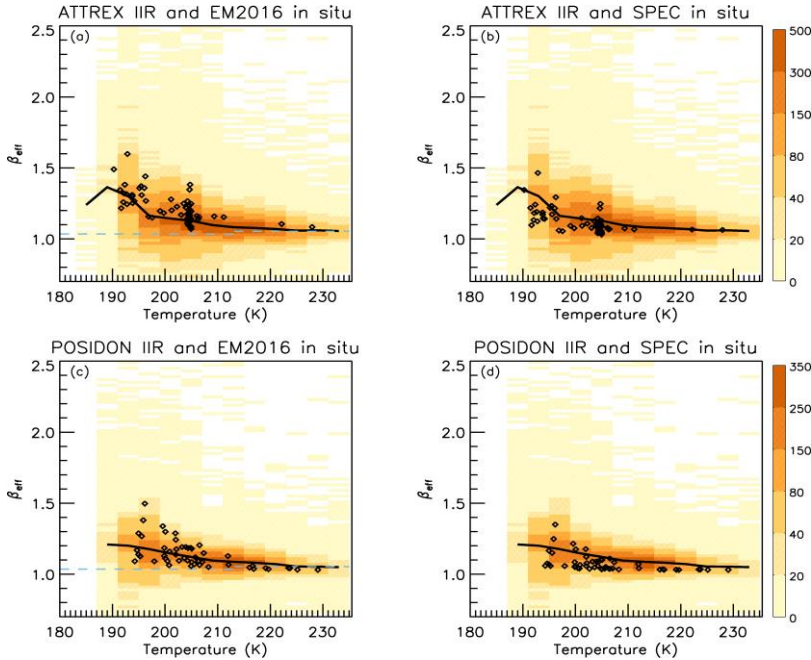


Figure 12. Same as Fig. 11 but using IIR optical depth $> \sim 0.01$.

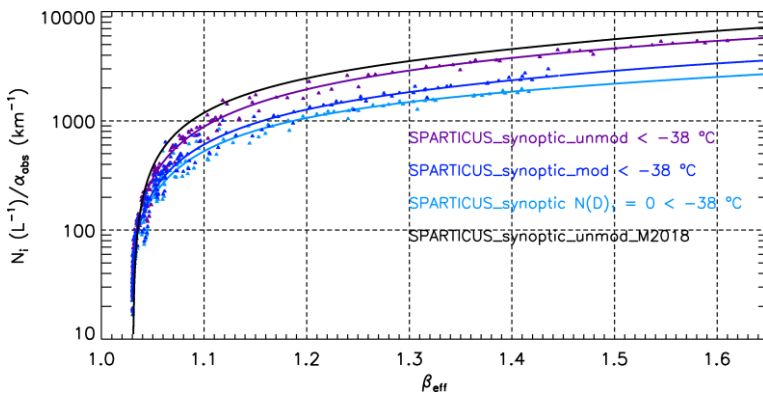
3.1 Correction of the smallest bin of the 2D-S probes and mass-dimension relationship

As discussed in Sect. 2.3, a modification of the smallest bin of the PSDs is needed for the SPARTICUS and TC4 campaigns where only the 2D-S probe was used. The correction was determined from the analysis of PSDs measured during the ATTREX and POSIDON campaigns since the FCDP was used over the ice particle size-range corresponding to the smallest 2D-S bin.

465 In addition, based on the findings presented in Sect. 2.4, we now use the EM2016 mass-dimension relationships.

It is instructive to examine the impact of these changes in terms of the N_i retrieval as described in Eq. (2). The field campaign dependence (and thus the 2D-S probe dependence) of N_i enters through the β_{eff} dependent terms in Eq. (2), namely through the $N_i/A_{\text{PSD}} - \beta_{\text{eff}}$ and the $[1/Q_{\text{abs,eff}}(12 \mu\text{m})] - \beta_{\text{eff}}$ relationships. From Eq. (2), the product of these two ratios is N_i/α_{abs} which is plotted in Fig. 13, showing the impact of the mass-dimension relationships and of the $N(D)_1$ assumption on the N_i retrieval.

470 There are three assumptions: (1) $N(D)_1$ is unmodified, meaning the $N(D)_1$ measurement is correct, (2) $N(D)_1$ is modified, divided by 10.4 as discussed in Sect. 2.3, and (3) $N(D)_1 = 0$. These three assumptions were evaluated in Fig. 13 using 2D-S PSD data from the SPARTICUS field campaign measured at temperatures less than -38°C using the EM2016 relationships. Assumption (1) as derived in M2018 is also shown in black, showing that using the EM2016 relationships (in purple) reduces retrieved N_i . Taking the modified assumption (in navy blue) to be most realistic, it is seen that either assumption (1) or (3) can
475 produce significant errors. Moreover, this reveals the N_i retrieval sensitivity to the size bin for the smallest ice particles. It was fortuitous that both the FCDP and 2D-S probes were flown during the ATTREX and POSIDON field campaigns, which enabled the estimation of a correction factor. Hence forwards only the modified assumption is used for $N(D)_1$ for both SPARTICUS and TC4.



480

Figure 13. Sensitivity of the SPARTICUS N_i retrieval using the EM2016 mass-dimension expressions to assumptions concerning the first size-bin of the 2D-S probe, $N(D)_1$, which can either be unmodified (purple), modified (by dividing $N(D)_1$ by 10.4, navy blue), or set equal to zero (light blue). The black curve is the relationships for $N(D)_1$ unmodified from M2018. See text for details.

485 3.2 Relating β_{eff} to N_i/IWC , N_i/A_{PSD} , D_e , and $1/Q_{\text{abs}}(12\ \mu\text{m})$

The $X - \beta_{\text{eff}}$ relationships used in the retrieval Eqs. (2), (4), and (7) where X is N_i/A_{PSD} , N_i/IWC , and $1/Q_{\text{abs}}(12\ \mu\text{m})$, respectively, are shown in Fig. 14 along with the β_{eff} dependence of D_e which is based on Eq. (4). The solid lines in panels (a), (b), and (d) are second order polynomial curve fits based on the indicated field campaigns where both X and β_{eff} are calculated from PSD measurements and MADA. The solid lines in panel c are based on Eq. (4). PSDs from the ATTREX and
 490 POSIDON field campaigns are mostly from TTL cirrus and were sampled using the same instruments in the tropical western Pacific; therefore, they were combined as a single dataset. While the SPARTICUS data was subdivided into anvil cirrus and synoptic cirrus (i.e., any cirrus not associated with convection), only the curve fits for synoptic cirrus were used since they represented both cirrus types well. All the PSDs used to produce Fig. 14 were measured at temperatures less than -38°C . The IWCs and β_{eff} values were calculated using the m-D expressions in EM2016.

495 A plot similar to Fig. 14 but showing the curve fits only is included in the Supplement to this article (Fig. S1) and the coefficients used to produce these curves are reported in Table 3, where β_{eff} is the independent “x” variable and the retrieved microphysical ratio is the dependent “X” variable. Sometimes a linear extrapolation had to be defined to extend the validity of the formulation over the full range of β_{eff} . These $X - \beta_{\text{eff}}$ relationships in Table 3 are only valid when $\beta_{\text{eff}} < 10$ (and are evaluated at $\beta_{\text{eff}} = 10$ if $\beta_{\text{eff}} > 10$). In practice, β_{eff} almost never exceeds 10 and rarely exceeds 2.

500 As noted in M2018, our retrieval of N_i and D_e is the most sensitive to β_{eff} when the PSD includes a large proportion of small ice crystals and β_{eff} is relatively large. The vertical dashed lines in Fig. 14 indicate the β_{eff} sensitivity limit for each field campaign dataset, which are reported in Table 4. If the retrieved β_{eff} lies below this value, the retrieved quantity in the X-column of Table 3 is evaluated at the β_{eff} sensitivity limit. Since D_e is essentially a product of two of these ratios, it is constant at the sensitivity limit, as shown in Table 4. However, when the retrieved property has an additional dependence on α_{ext} and
 505 therefore $\tau_{\text{abs}}(12.05\ \mu\text{m})$, that property is not a constant at the sensitivity limit since $\tau_{\text{abs}}(12.05\ \mu\text{m})$ is not subject to this limit. This is illustrated in Table 4, where the N_i retrieval equation is expressed in terms of the extinction coefficient for visible light α_{ext} . Two values of α_{ext} are given that bracket the α_{ext} range commonly found in cirrus, and corresponding N_i values are given for each α_{ext} and β_{eff} sensitivity limit, where N_i/A_{PSD} is evaluated at the sensitivity limit for each campaign.

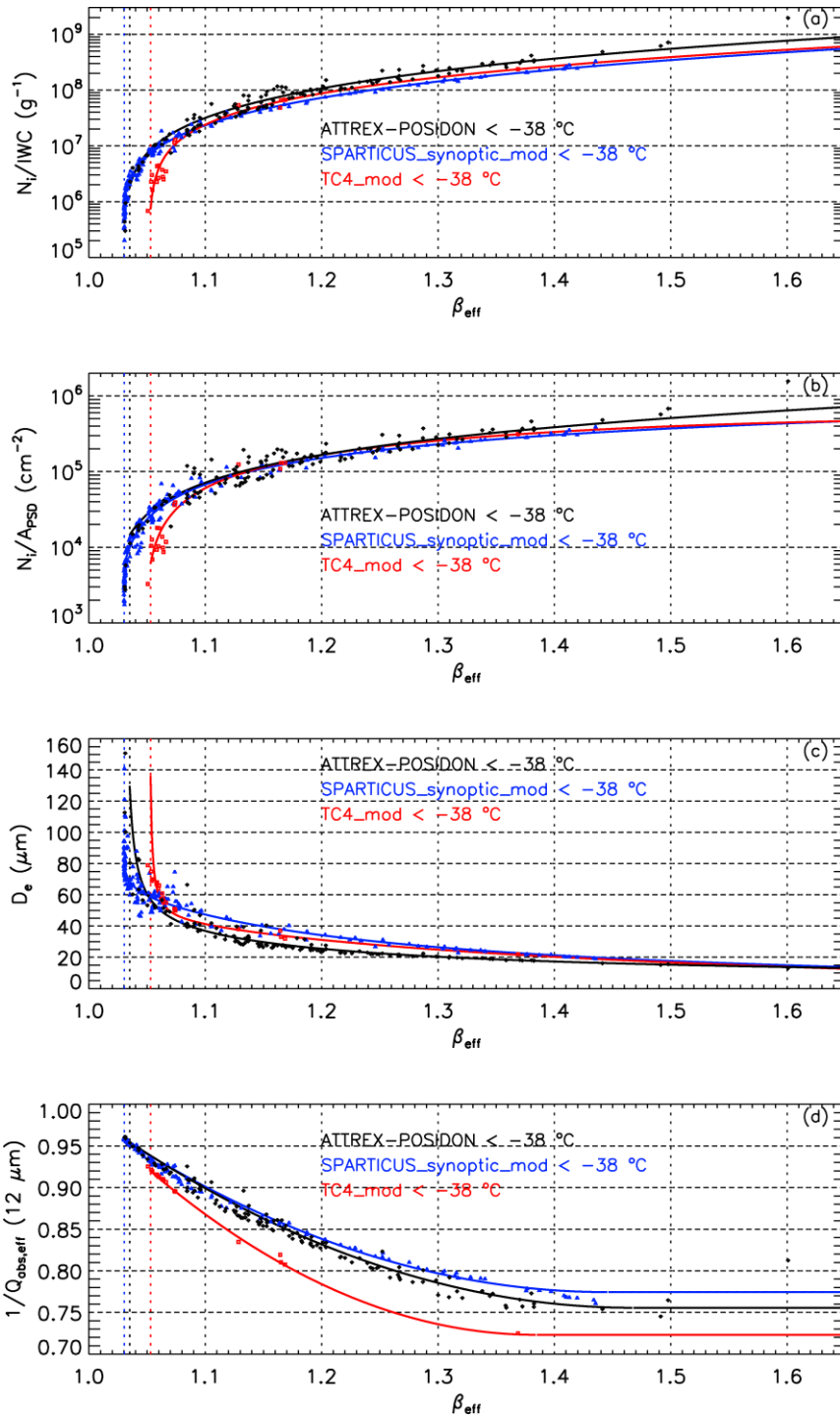


Figure 14. The dependence of this retrieval on β_{eff} is comprised of the above four types of relationships. The curve fits shown correspond to the ATTREX-POSIDON (black), SPARTICUS (navy blue) and TC4 (red) field campaigns where SPARTICUS is based on synoptic cirrus clouds and $N(D)_1$ was modified for SPARTICUS and TC4. Data points were calculated from the PSD samples having temperature less than -38°C with colors indicating the respective field campaign.

Table 3. Regression curve variables and coefficients for polynomials of the form $X = a_0 + a_1 x + a_2 x^2$ used in the CALIPSO retrieval. Units for N_i/IWC and N/A_{PSD} are in g^{-1} and cm^{-2} , respectively.

X	x	a_0	a_1	a_2
SPARTICUS				
N_i/IWC	β_{eff}	0.84597×10^9	-1.88517×10^9	1.03391×10^9
N_i/A_{PSD}	$\beta_{eff} \leq 2.1$	-1.21251×10^6	1.459×10^6	-0.268493×10^6
	$\beta_{eff} > 2.1$	-0.28446×10^5	3.3133×10^5	0
$1/Q_{abs,eff}$ (12 μm)	$\beta_{eff} \leq 1.45$	2.99	-3.065	1.06
	$\beta_{eff} > 1.45$	0.774	0	0
TC4				
N_i/IWC	β_{eff}	0.566052×10^9	-1.52366×10^9	0.93712×10^9
N_i/A_{PSD}	$\beta_{eff} \leq 1.65$	-2.03022×10^6	2.67666×10^6	-0.705499×10^6
	$\beta_{eff} > 1.65$	-1.09499×10^5	3.48513×10^5	0
$1/Q_{abs,eff}$ (12 μm)	$\beta_{eff} \leq 1.38$	4.15	-4.95	1.7875
	$\beta_{eff} > 1.38$	0.723	0	0
ATTREX-POSIDON				
N_i/IWC	β_{eff}	1.56577×10^9	-3.36428×10^9	1.79055×10^9
N_i/A_{PSD}	β_{eff}	-0.3480×10^6	-0.1437×10^6	0.4772×10^6
$1/Q_{abs,eff}$ (12 μm)	$\beta_{eff} \leq 1.47$	3.045	-3.12	1.063
	$\beta_{eff} > 1.47$	0.755	0	0

520 Table 4: Maximum retrieved D_e (μm) and minimum retrieved N_i (L^{-1}) at β_{eff} sensitivity limit.

	β_{eff} sensitivity limit	D_e (μm)	$N_i(L^{-1}) = 0.01 \frac{N_i}{A_{PSD}} (cm^{-2}) \frac{\alpha_{ext}(km^{-1})}{2}$	
			$\alpha_{ext} = 0.01 km^{-1}$	$\alpha_{ext} = 1 km^{-1}$
SPARTICUS	1.0304	77.7	0.29	29
TC4	1.053	136.2	0.60	60
ATTREX- POSIDON	1.035	129.8	0.72	72

3.3 Strategy for a global retrieval scheme

The $X - \beta_{\text{eff}}$ relationships from the 3 field campaigns shown in Fig. 14 are overall consistent, but they exhibit differences. The $D_e - \beta_{\text{eff}}$ relationship for TC4 differs significantly from that of the ATTREX-POSIDON campaigns, even though these three
525 campaigns were conducted in the tropics, and they both differ from the SPARTICUS relationship obtained at mid-latitudes. For a given β_{eff} larger than 1.05, ATTREX-POSIDON yields the smallest D_e and SPARTICUS the largest one. By expressing PSDs in terms of d_e (effective photon path) as described in Sect. 2.4.1, Fig. 15 shows that TTL PSDs differ substantially from SPARTICUS PSDs over a narrow range of β_{eff} (i.e., β_{eff} is approximately constant). Since β_{eff} is essentially the ratio of two
530 absorption coefficients involving the integration of PSD area, integrals of PSD area are shown. Each panel in Fig. 15 shows a SPARTICUS PSD and a PSD taken from either the ATTREX or POSIDON campaign, having similar β_{eff} values. In the bottom are the corresponding D_e values. It is seen that over a very narrow range of β_{eff} , D_e changes considerably (along with PSD shape), suggesting that the $D_e - \beta_{\text{eff}}$ relationship is subject to changes in PSD shape. The number of TC4 PSDs were much less than for SPARTICUS, precluding the pairing of PSDs of similar β_{eff} . Nonetheless, it appears likely that PSD shape differences may be responsible for the different $D_e - \beta_{\text{eff}}$ relationships regarding the anvil cirrus sampled during TC4 and the
535 TTL cirrus sampled during ATTREX-POSIDON.

Supporting evidence relating to differences between anvil and TTL cirrus is found in Gasparini et al. (2018), which contrasted in situ cirrus dominating at $T < -55^\circ\text{C}$ (including TTL cirrus) with liquid origin cirrus dominating when $-55^\circ\text{C} < T < -40^\circ\text{C}$, where the latter are either anvil cirrus formed from deep convection or are glaciated mixed phase clouds. Consistent with Gasparini et al. (2018), Heymsfield et al. (2014) found a D_e discontinuity in cirrus clouds in the tropics and at the top of
540 mid-latitude clouds between $\sim -60^\circ\text{C}$ and -65°C , with much smaller D_e at these colder temperatures (their Fig. 11).

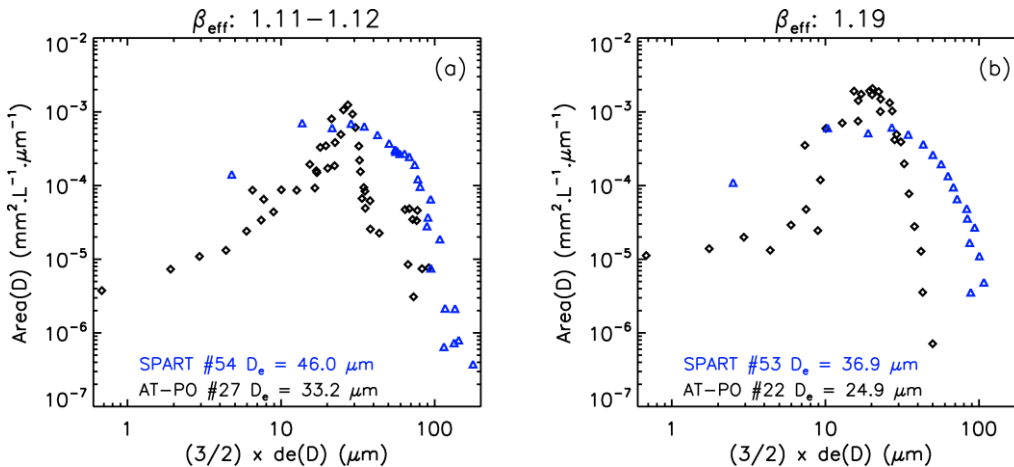


Figure 15. Comparisons of area PSDs from the ATTREX-POSIDON (black) and SPARTICUS (blue) campaigns having very similar β_{eff} values but considerably different D_e values, illustrating how differences in PSD shape between the two field campaigns can yield
545 different $D_e - \beta_{\text{eff}}$ relationships.

Table 5: Combination of the empirical relationships from the various campaigns. The T_{cold} and T_{warm} temperature limits were chosen based on the sampled temperatures during the respective campaigns.

Cloud temperature, T_r	$T_r \leq T_{\text{cold}} = -65\text{ }^{\circ}\text{C}$	$T_r \geq T_{\text{warm}} = -60\text{ }^{\circ}\text{C}$	$-65\text{ }^{\circ}\text{C} < T_r < -60\text{ }^{\circ}\text{C}$
Tropics: 30°S - 30°N	ATTREX-POSIDON Cold TTL cirrus, narrow PSD	TC4	Temperature interpolation for each relationship: $N_i / A_{\text{PSD}}\text{-}\beta_{\text{eff}}$, $N_i / \text{IWC}\text{-}\beta_{\text{eff}}$, and $1/Q_{\text{abs,eff}}(12\text{ }\mu\text{m})\text{-}\beta_{\text{eff}}$
Extra-tropics	ATTREX-POSIDON Narrow PSD	SPARTICUS	

550 To accommodate these findings, we developed a latitude- and temperature-dependent scheme for our retrieval as described in Table 5. That is, for clouds having radiative temperature $T_r < -65^{\circ}\text{C}$, the ATTREX-POSIDON $D_e - \beta_{\text{eff}}$ relationship was used at any latitude. When $T_r > -60^{\circ}\text{C}$, the TC4 $D_e - \beta_{\text{eff}}$ relationship was used in the tropics (30°S - 30°N) and the SPARTICUS $D_e - \beta_{\text{eff}}$ relationship was used outside the tropics. Between -60°C and -65°C , a temperature interpolation between the two relevant formulations was implemented. The same practice applies to the other $X - \beta_{\text{eff}}$ relationships as shown in Table 3.

555 This temperature dependence mostly affects the tropics as shown in Fig. 16, featuring seasonal maps of the fraction of IIR pixels with cirrus clouds having $T_r < -65^{\circ}\text{C}$ relative to all pixels with cirrus clouds (where cirrus clouds are defined as having $T_r \leq 235\text{ K}$). These fractions are for $0.01 < \tau < \sim 3$ sampled only over oceans. This fraction was evaluated over both land and ocean using $\sim 0.3 < \tau < \sim 3$ in the Supplement (Fig. S2) where it is shown that in the tropics, the fraction over land is comparable to that over the tropical western Pacific. Over the tropics, this fraction can easily exceed 60% or 70%, while outside the tropics,

560 this fraction is generally $< 5\%$, with exceptions over the Antarctic (JJA and SON) and over Greenland (DJF) as shown in the Supplement.

The $X\text{-}\beta_{\text{eff}}$ relationships reported in Table 3 together with the combination strategy summarized in Table 5 can be used to reproduce the findings shown in Sect. 4 and in Part 2 of this study.

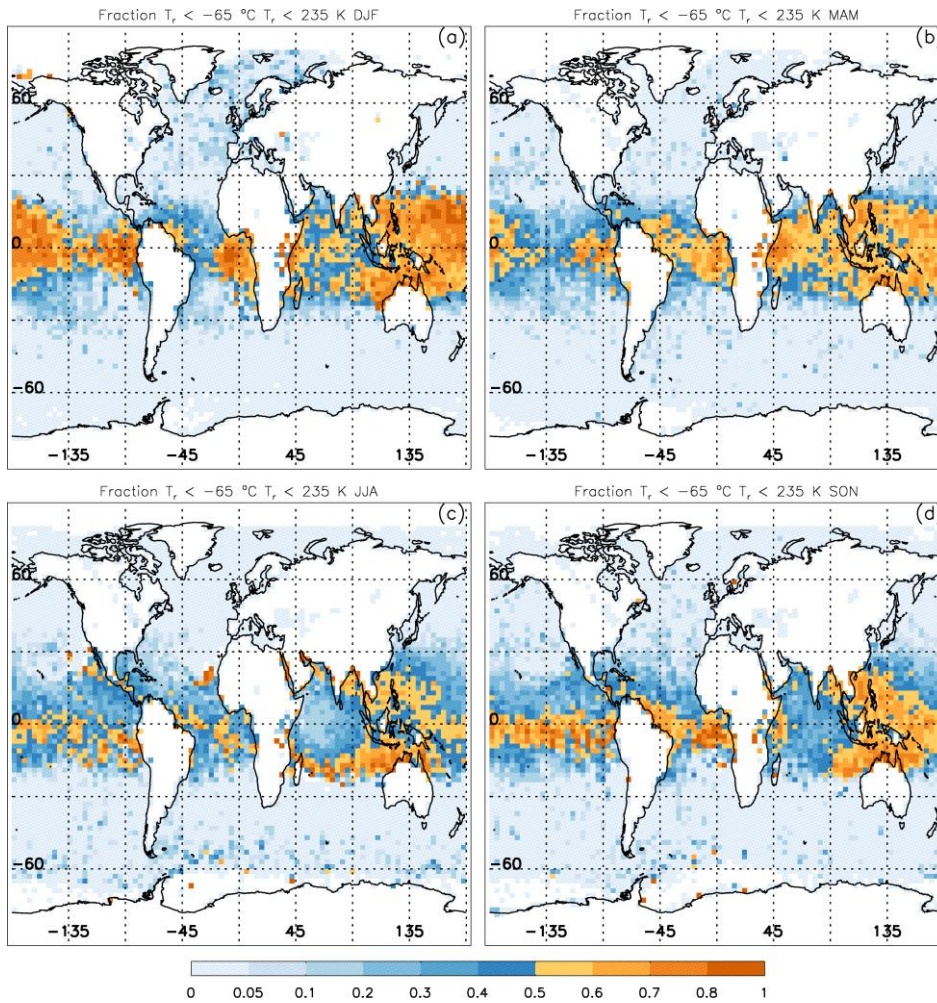


Figure 16. Seasonal maps of the fraction of cirrus cloud pixels ($T_r \leq 235$ K) having $T_r < -65^\circ\text{C}$ (208 K) over oceans only, where $\sim 0.01 < \tau < \sim 3$. This is the fraction of cirrus clouds for which the ATTREX-POSIDON formulation is used in this retrieval. The four panels are for (a) December-January-February (DJF), (b) March-April-May (MAM), (c) June-July-August (JJA), and (d) September-October-November (SON) during 2008, 2010, 2012 and 2013.

3.4 Retrieval uncertainties

Uncertainties in retrieved β_{eff} , noted $\Delta\beta_{\text{eff}}$, translate into uncertainties in N_i/A_{PSD} , N_i/IWC , $1/Q_{\text{abs,eff}}(12\ \mu\text{m})$, and ultimately D_e . While $\Delta\beta_{\text{eff}}$ increases as optical depth decreases, the resulting uncertainty in X , noted ΔX , depends also on the $\partial X/\partial\beta_{\text{eff}}$ slope of the X - β_{eff} relationships at retrieved β_{eff} . An additional contribution to the uncertainty in N_i , IWC , and α_{ext} is the uncertainty in $\tau_{\text{abs}}(12\ \mu\text{m})$. The uncertainties in $\tau_{\text{abs}}(12\ \mu\text{m})$ and β_{eff} are estimated following the same rationale as in M2018. Details are given in Appendix B which includes the equations used to estimate the uncertainties in N_i , D_e , IWC , α_{ext} , and R_v . Note that additional uncertainties in the X - β_{eff} relationships are difficult to estimate and are not included in this assessment.

We will see in the following section that relative uncertainties in N_i typically exceed 100 % when $0.01 < \tau < \sim 3$. These large random uncertainties of individual retrievals can be mitigated by accumulating many samples. Median values of an ensemble of retrievals should not be too impacted by the samples having β_{eff} smaller than the sensitivity limit for which N_i/A_{PSD} , N_i/IWC , $1/Q_{\text{abs,eff}} (12 \mu\text{m})$, and ultimately D_e are set to constant values.

3.5 Comparison with previous work

For comparison with the previous work (M2018), Figure 17a shows the N_i/α_{abs} ratio from the ATTREX-POSIDON (black), SPARTICUS (navy blue), and TC4 (red) relationships developed in this study vs. the N_i/α_{abs} ratio from SPARTICUS $N(D)_1$ unmodified established in M2018, which, out of the 4 formulations examined in M2018, yielded the largest N_i values (Fig. 5 in M2018). Also shown in Fig. 17a is TC4 $N(D)_1 = 0$ from M2018 (dashed orange) which yielded the lowest N_i values. We see that N_i from this study is about half N_i from M2018 SPARTICUS unmodified for both SPARTICUS and TC4 which are similar to M2018 $N(D)_1 = 0$, while ATTREX-POSIDON is half to two third. Panel b in Fig. 17 compares the D_e retrievals.

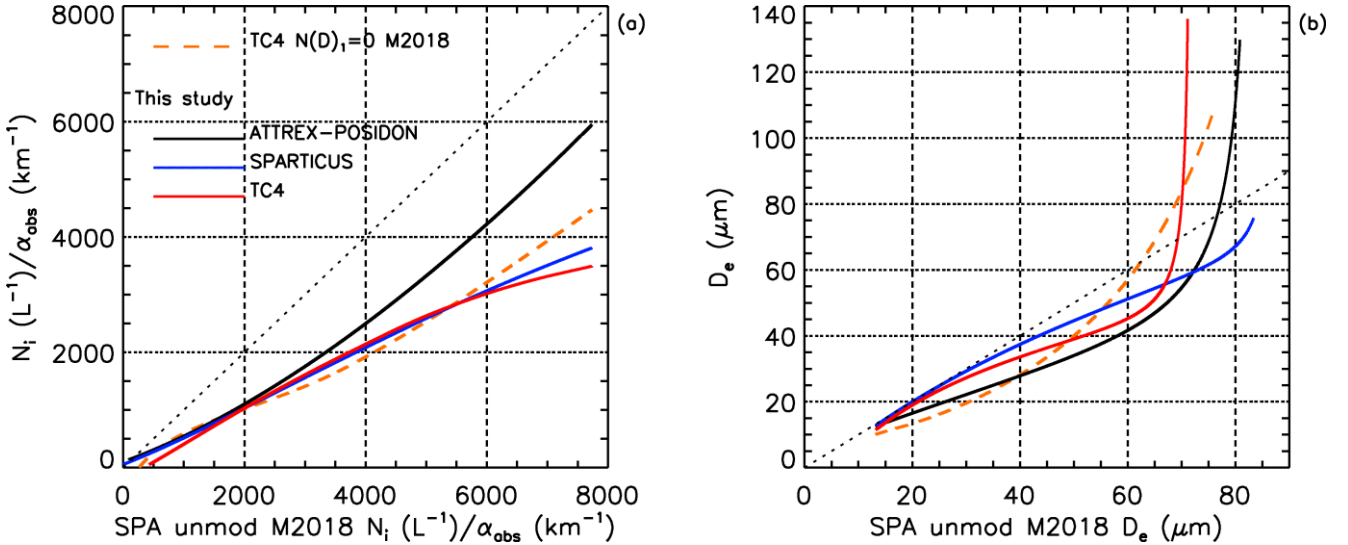


Figure 17. Comparison of (a) N_i and (b) D_e retrievals in this study (black: ATTREX-POSIDON, navy blue: SPARTICUS, red: TC4) and from TC4 $N(D)_1 = 0$ in M2018 (dashed orange) with retrievals from SPARTICUS $N(D)_1$ unmodified in M2018.

4 Testing of the retrieval

Since this CALIPSO-IIR retrieval was developed from cirrus cloud field campaign measurements, we compared the satellite retrievals of N_i , D_e , IWC, and α_{ext} during the period of the field campaigns over their respective regions with these same properties that were measured in situ during the field campaigns. In this section, the retrieval is tested against aircraft measurements from the ATTREX and POSIDON field campaigns for the tropics (together ATPO) and against SPARTICUS

aircraft measurements for the midlatitudes. In addition, the Krämer et al. (2020) global climatology of cirrus cloud properties, based on numerous cirrus cloud field campaigns, will be compared against the corresponding properties from this retrieval.

4.1 Comparisons with ATTREX and POSIDON PSD data

Since cirrus clouds sampled during these field campaigns were over ocean, their aircraft-measured properties can be compared against corresponding retrieved properties for cirrus having $\sim 0.01 < \tau < \sim 3$, as shown for ATTREX in Fig. 18 and for POSIDON in Fig. 19. These retrievals were confined to the field campaign domain (in the tropical western Pacific) and to the campaign sampling period (Feb.-March 2014 for ATTREX and Oct. 2016 for POSIDON). The retrieval sample density is given by the color bar while the black diamonds indicate the aircraft in situ PSD measurements for a given property. The blue-dashed curves in the D_e plots indicates the fraction of cirrus clouds for which D_e could be “reliably” retrieved (i.e., $\beta_{\text{eff}} > \beta_{\text{eff}}^{\text{sensitivity limit}}$). D_e retrievals for which $\beta_{\text{eff}} < \beta_{\text{eff}}^{\text{sensitivity limit}}$ comprise the high sample densities between $130 \mu\text{m}$ and $136 \mu\text{m}$. As PSDs broaden at higher temperatures, D_e increases and $\beta_{\text{eff}} < \beta_{\text{eff}}^{\text{sensitivity limit}}$ occurs more often, which is evident from POSIDON in situ D_e and the blue-dashed curve. Overall, the ATTREX and POSIDON retrievals appear consistent with the corresponding in situ values. Similar comparisons for optically thicker cirrus where $\sim 0.3 < \tau < \sim 3$ are given in the Supplement (Figs. S3 and S4). Table 6 shows median retrieved properties and relative uncertainty estimates for ATTREX for cirrus having $\sim 0.2 - 0.3 < \tau < \sim 3$ (left) and $\sim 0.01 < \tau < \sim 3$ (right). A similar table for the POSIDON campaign is shown in the Supplement (Table S1). In Table 6, median $\Delta\beta_{\text{eff}}$ ranges from 0.03 to 0.44 where median $\tau = 0.05$ at $T_r = 193$ K, $\Delta\text{Ni}/\text{Ni} = 1.88$ and $\Delta D_e/D_e = 0.98$. The smallest median $\Delta\text{Ni}/\text{Ni}$ is 0.35 at $T_r = 193$ K when only the thicker clouds are sampled and median τ is somewhat small (0.23), but this is compensated by the fact that $\beta_{\text{eff}} = 1.57$ where the sensitivity of the technique is very favorable. In contrast, median β_{eff} is 1.056-1.058 at 233 K, where the sensitivity of the technique is less favorable, which explains the occurrence of relative uncertainties larger than 2.4 despite the small $\Delta\beta_{\text{eff}} = 0.03$. Again, these uncertainty estimates characterize random uncertainties of individual retrievals and are reduced for statistical analyses involving a large number of samples.

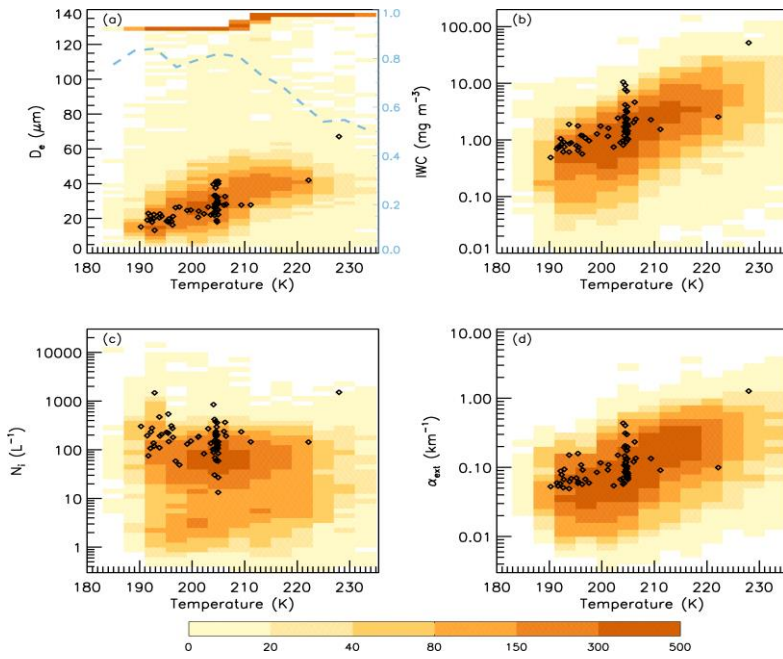


Figure 18. Pixel sampling densities (given by the color bar) for retrievals of (a) D_e , (b) IWC , (c) N_i , and (d) a_{ext} taken during the period of the ATTREX field campaign (February-March 2014) over the ATTREX domain (0° N - 20° N and 130° E - 160° E) where $\sim 0.01 < \tau < 3$. Black diamonds indicate corresponding aircraft PSD measurements of these properties. The right vertical axis of the D_e plot indicates the fraction of cirrus clouds sampled having β_{eff} greater than the β_{eff} sensitivity limit given by the blue-dashed curve while the high sample densities having D_e between $130 \mu\text{m}$ and $136 \mu\text{m}$ are from samples having β_{eff} lower than the β_{eff} sensitivity limit (i.e., non-quantifiable D_e). The change in D_e from 130 to 136 is due to the temperature interpolation (ATPO to TC4).

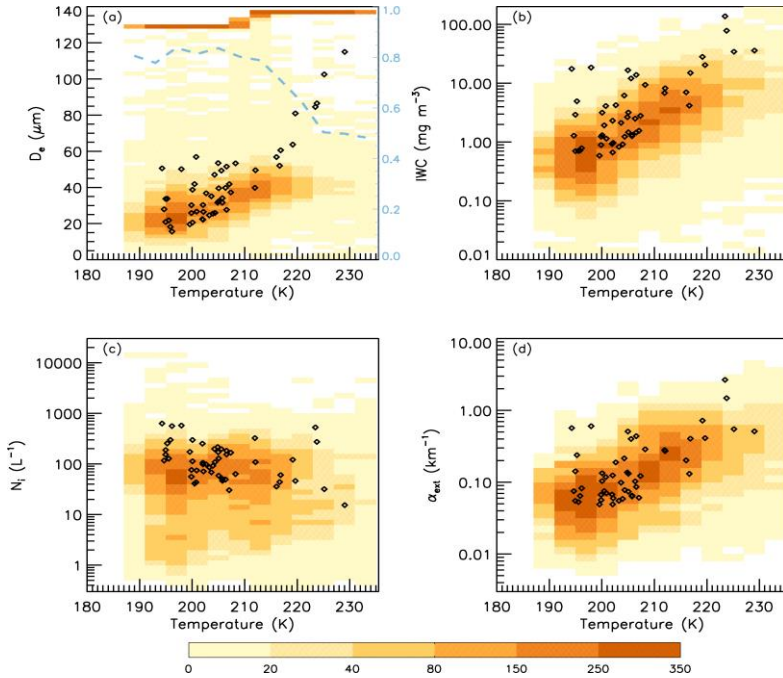


Figure 19. Same as Fig. 17 but for the POSIDON field campaign in October 2016.

Table 6: Median values and estimated uncertainties of various retrieved properties at 193, 213, and 233 K for $\sim 0.2 - 0.3 < \tau < \sim 3$ (left) and $\sim 0.01 < \tau < \sim 3$ (right) during the ATTREX campaign.

ATTREX	$\sim 0.2 - 0.3 < \tau < \sim 3$			$\sim 0.01 < \tau < \sim 3$		
T_r (K)	193	213	233	193	213	233
Pixel count	144	2011	253	3065	3654	467
$\tau_{\text{abs}}(12.05 \mu\text{m})$	0.14	0.32	0.53	0.03	0.18	0.24
$\Delta\tau_{\text{abs}}(12.05 \mu\text{m})$	0.016	0.019	0.028	0.016	0.019	0.023
τ	0.23	0.56	0.95	0.05	0.32	0.42
$\Delta\tau$	0.025	0.040	0.051	0.029	0.039	0.046
β_{eff}	1.569	1.083	1.058	1.301	1.083	1.056
$\Delta\beta_{\text{eff}}$	0.18	0.03	0.03	0.44	0.05	0.05
$\alpha_{\text{vis}} (\text{km}^{-1})$	0.21	0.25	0.55	0.05	0.17	0.30
$\Delta\alpha_{\text{vis}}/\alpha_{\text{vis}}$	0.13	0.07	0.05	0.63	0.12	0.11
$D_e (\mu\text{m})$	14	44	64	20	44	78
$\Delta D_e/D_e$	0.19	0.19	2.44	0.98	0.45	> 3.00
IWC (mg m^{-3})	0.9	4.2	12.7	0.4	2.9	6.7
$\Delta\text{IWC/IWC}$	0.29	0.26	2.48	1.54	0.61	> 3.00
$N_i (\text{L}^{-1})$	664	52	41	71	34	28
$\Delta N_i/N_i (\text{L}^{-1})$	0.35	0.84	> 3.00	1.88	1.51	> 3.00

4.2 Comparisons with SPARTICUS PSD data

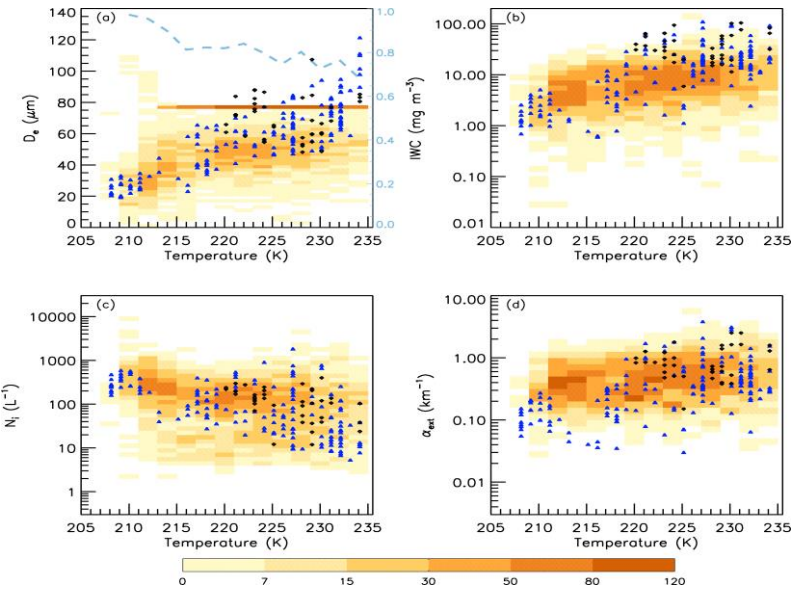


Figure 20. Pixel sampling densities (given by the color bar) for retrievals of (a) D_e , (b) IWC, (c) N_i , and (d) α_{ext} taken during the period of the SPARTICUS field campaign (January through April 2010) over the SPARTICUS domain (31- 42 °N; 90-103 °W) where $\sim 0.3 < \tau < \sim 3$. Black diamonds indicate corresponding aircraft PSD measurements of these properties. The right vertical axis of the D_e plot indicates the fraction of cirrus clouds sampled having β_{eff} greater than the β_{eff} sensitivity limit given by the blue-dashed curve while the high sample densities having $D_e \sim 78 \mu\text{m}$ are from samples having β_{eff} lower than the β_{eff} sensitivity limit (i.e., non-quantifiable D_e).

As mentioned, $N(D)_1$ of the SPARTICUS PSD data was divided by 10.4 to correct $N(D)_1$ based on a comparison of $N(D)_1$ with corresponding FCDP values from the POSIDON campaign. These corrected SPARTICUS PSDs are used in this section to compare in situ cirrus cloud properties with corresponding retrieved values. As with the ATTREX and POSIDON campaigns, these retrievals are from the SPARTICUS domain (31 – 42 °N and 90 – 103 °W) during the campaign measurement period (January through April 2010). Since these retrievals are over land, they were restricted to the thicker cirrus where $\sim 0.3 < \tau < \sim 3$. These comparisons are shown in Fig. 20. As before, the blue-dashed curve in panel a indicates the fraction of D_e retrievals having $\beta_{\text{eff}} >$ the β_{eff} sensitivity limit which corresponds to $D_e \approx 78 \mu\text{m}$ (shown by the narrow band of high pixel sampling densities). At the highest cirrus temperatures (T_r), in situ D_e tends to be higher than retrieved D_e (where $\beta_{\text{eff}} >$ the β_{eff} sensitivity limit). This may be partly due to aircraft sampling of relatively thick cirrus clouds below the mid-cloud level (i.e., at higher temperatures) where PSDs are broader (with larger D_e) due to longer ice particle growth times through vapor diffusion and aggregation. In contrast, retrieved D_e characterizes a layer and might reflect the presence of smaller crystals above the aircraft flight level. Similar reasons may explain why in situ IWCs tend to be higher than retrieved IWCs at higher T_r . Overall, the retrievals in Fig. 19 exhibit reasonable agreement with SPARTICUS in situ measurements, similar to the ATTREX and POSIDON comparisons.

The difficulty to directly compare IIR layer retrievals and aircraft in situ data is illustrated in the SPARTICUS case study shown in Fig. 21 for 30 March 2010. Following the CALIPSO track, the Learjet flew northwards (leg 1, triangles) with measurements at 11 km altitude 7 to 3 minutes before the CALIPSO overpass and then southwards (leg 2, diamonds) with measurements at 11.6 km altitude 6.5 to 8 minutes after. CALIPSO detected a single layer cirrus of top altitude near 12.6 km. The colors in panel a represent the altitude-dependent CALIOP extinction profiles scaled to IIR τ . The colors inside the triangles and diamonds indicate the PSD extinctions larger than 0.01 km^{-1} after averaging over a 30-s period. At the top of panel a is IIR cloud layer α_{ext} , which was derived from τ and Δz_{eq} shown in panel b. As discussed in Sect. 2.2.5, Δz_{eq} represents the portion of a layer contributing the most to the cloud emissivity. The solid black line in panel a is the radiative altitude corresponding to T_r , which to a first approximation corresponds to the mid-cloud altitude (see Fig. 7). IIR D_e in red in panel c (with vertical bars indicating $D_e \pm \Delta D_e$) is lower than the in situ values, which is explained by the fact that both flight legs were below the radiative altitude. That is, in the lower half of an ice cloud, mean ice particle size tends to be larger and N_i lower relative to the upper half due to diffusional growth and aggregation (e.g., Mitchell, 1988; 1994; Field and Heymsfield, 2003). Only a lower portion of the cloud was detected by the CloudSat radar (shown by the stars) between latitudes 36.5° and 36.78°. IIR D_e is the smallest (around $20 \mu\text{m}$) south of 36.5° and north of 36.78° where there is no radar detection, indicating crystals smaller than about $40 \mu\text{m}$. Moreover, the absence of radar detection outside this CloudSat domain (defined by the stars) indicates ice particles smaller than $\sim 40 \mu\text{m}$, revealing a vertical gradient in ice particle size. Regarding N_i (panel d showing $N_i \pm \Delta N_i$), the large IIR N_i values in red between 300 and 850 L^{-1} are explained by higher N_i near cloud top. Regarding uncertainties, ΔN_i is overall equal to about 80 L^{-1} and its noticeable increase up to 300 L^{-1} in the northernmost part of the cloud is due to the decrease of τ . The same observation applies to ΔD_e which is between 3 and $11 \mu\text{m}$. To summarize, while the vertically resolved extinction retrievals exhibit reasonable agreement with the in situ extinction measurements, the bulk cloud

layer retrievals often do not exhibit similar agreement, and this appears to be due to vertical gradients in D_e and N_i and aircraft sampling location. This case study has been classified as ridge crest cirrus which have higher N_i than the other cirrus cloud classes described in Muhlbauer et al. (2014). In this regard, the retrievals here are consistent with this category of cirrus cloud.

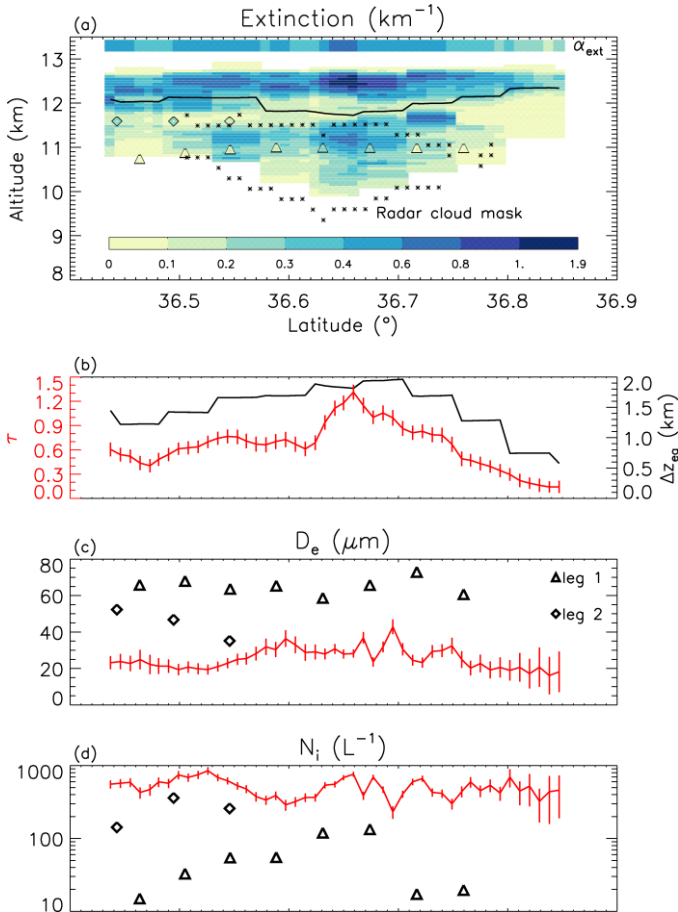


Figure 21. Comparison of IIR retrievals and in situ observations on 30 March 2010 during the SPARTICUS field campaign (CALIPSO granule 2010-03-30T19-27-25ZD). (a) extinction profile derived from the CALIOP lidar, IIR layer α_{ext} , and PSD extinctions in leg 1 (triangles) and leg 2 (diamonds). The stars denote the boundaries of the CloudSat radar GEOPROF cloud mask; (b) IIR τ (red) and Δz_{eq} (black, right axis); (c) IIR (red) and in situ (triangles and diamonds) D_e ; (d) same as (c) but for N_i . The vertical bars in red in panels b-d represent the IIR estimated uncertainties.

4.3 Comparisons with a global cirrus cloud property climatology

A recent study by Krämer et al. (2020) has expanded the in situ cirrus cloud property database described in Krämer et al. (2009) by a factor of 5 to 10 (depending on cloud property). Here we compare the temperature dependence of R_v , N_i , and IWC from the CALIPSO-IIR retrievals and from the Krämer et al. (2020) climatology. Since the aircraft measurements used in

Krämer et al. (2020) often did not allow D_e to be calculated (and thus D_e was not reported), we use R_v as a measure of ice particle size for comparison purposes since R_v is reported in Krämer et al. (2020). However, R_v and D_e are unique quantities where D_e cannot be calculated from R_v (and vice-versa). Since D_e partly determines a cloud's radiative properties, D_e and R_v are intercompared in Appendix C based on in situ data and for different PSD shape assumptions using a PSD model that
695 assumes a simple gamma PSD distribution. While natural PSDs exhibit shapes more complex than these gamma PSDs, this modelling exercise suggests the relation between D_e and R_v depends on PSD shape.

Figure 3 in Krämer et al. (2020) shows that aircraft measurements are mostly between 20° S and 63° N. Thus, the IIR retrievals were averaged over oceans for 20° S- 0° , 0° - 30° N, and 30° N- 63° N for 4 years (2008, 2010, 2012 and 2013). Since the Krämer et al. (2020) data has no seasonal dependence, IIR retrievals were averaged over all seasons. The results are shown
700 in Fig. 22, where the IIR results, using T_r for the temperature, are in red for samples with $\tau > \sim 0.3$ and in blue using $\tau > \sim 0.01$. In situ data (black curves) in panels a and c are the climatological values. In panel d showing IWC, the black curve is an estimate of median in situ IWC derived from median in situ R_v and median in situ N_i using Eq. (5). The retrieved values of R_v , N_i , and IWC for $\tau > 0.01$ (blue curves) are generally within the ± 25 percentile range of corresponding in situ values.

The large spread of IIR data when τ can be as low as ~ 0.01 (blue) compared to $\tau > \sim 0.3$ (red) is due in part to larger random
705 uncertainties in clouds having optical depth < 0.3 , which represent the majority of the samples at $T < 215$ K (panel b). We note however that median R_v from the red and blue curves are similar, suggesting no systematic bias introduced by the retrievals at $\tau < \sim 0.3$. IIR and in situ median R_v agree reasonably well at $T > 210$ K and below 190 K. IIR R_v increases steadily with temperature and can be lower than in situ R_v by up to $7 \mu\text{m}$ between 190 and 205 K.

Differences between the optically thicker ($\tau > 0.3$, red) and thinner ($\tau > 0.01$, blue) N_i and IWC retrievals may be due to
710 differences in ice nucleation processes (i.e., het and hom) as described in Part 2, with hom occurring more often in the optically thicker cirrus clouds, promoting higher N_i and IWC. If true, it may be important during cirrus cloud field campaigns to attempt to characterize the cirrus in terms of τ to make in situ cloud property comparisons with cirrus cloud remote sensing and climate modelling results more meaningful. Fortunately, Krämer et al. (2020) contains a disclaimer stating, "Because of the dangerous nature of measurements under such conditions, the frequency of convective – and also orographic wave cirrus – is
715 underrepresented in the entire in situ climatology." And related to this, there is a statement about the higher in situ N_i in Krämer et al. (2009) resulting from flights in the "lee wave cirrus behind the Norwegian mountains". Orographic gravity waves (OGWs) produce relatively high updrafts more conducive to hom and tend to produce optically thicker cirrus clouds with higher N_i that can be spatially extensive (M2018). The sparsity of OGW cirrus in situ sampling in Krämer et al. (2020) may help explain the tendency of IIR N_i being slightly higher than in situ N_i in Fig. 22.

720 The retrieved median N_i in Fig. 22 (blue curve) exhibit similar magnitudes as a function of temperature to those of the DARDAR N_i retrieval (Sourdeval et al., 2018), which are compared against the median N_i of the Krämer et al. climatology in Fig. 15 of Krämer et al. (2020). The main difference between the DARDAR N_i retrieval and this one is that median DARDAR N_i is higher for $T < 220$ K, with DARDAR $N_i \sim 100 \text{ L}^{-1}$ for $T < 205$ K.

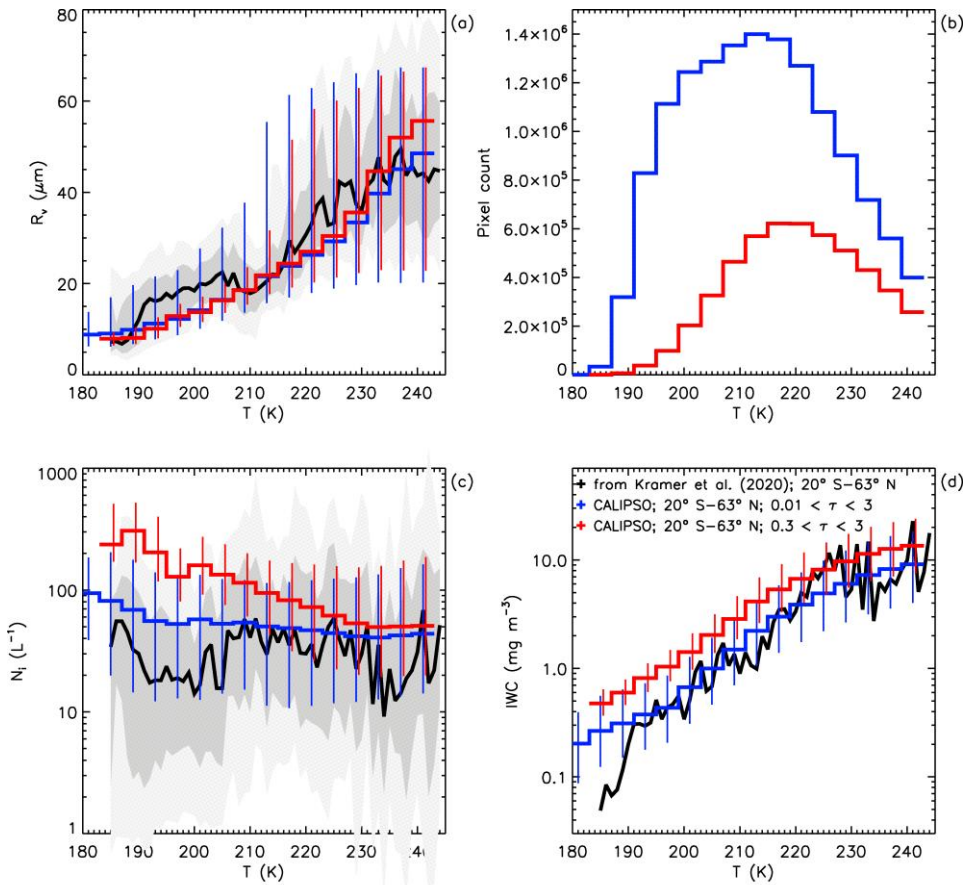


Figure 22. Temperature dependence of median values of (a) R_v (μm), (c) N_i (L^{-1}), and (d) IWC (mg m^{-3}) from the IIR retrievals (red: $\sim 0.3 < \tau < \sim 3$; blue: $\sim 0.01 < \tau < \sim 3$) and from the Krämer et al. (2020) in situ climatology (black curves). The vertical bars indicate the IIR 25th and 75th percentiles, except in panel (b) which shows the number of IIR sampled pixels. The light shade of grey in panels (a) and (c) is between the 10th and 90th percentiles and the superimposed darker shade of grey is between the 25th and 75th percentiles for the in situ data.

5 Conclusions

This study has utilized the CALIPSO IIR and CALIOP lidar in new ways, resulting in new methods for retrieving N_i , D_e , IWC, IWP, α_{ext} , and τ . The following improvements contributed to this CALIPSO retrieval:

- 1) By expanding the sampling range to include optically thinner cirrus clouds ($0.01 < \tau < 3$) over oceans, the sampling has become more representative of all cirrus clouds over oceans. The sampling over land, snow, and sea ice remains limited to thicker cirrus clouds having $\tau > 0.3$ because of larger uncertainties in IIR absorption optical depth retrievals.
- 2) The retrieval of N_i has become more accurate by using the N_i/A_{PSD} ratio, which is directly measured by aircraft probes.

- 3) The computation of in situ β_{eff} used in the X - β_{eff} relationships was improved using mass-dimension relationships that appear more realistic.
- 4) The retrieval of D_e has become more accurate by using the ratios N_i/A_{PSD} and N_i/IWC , where IWC is estimated using the more realistic mass-dimension relationships.
- 5) Improvements in D_e accuracy transfer to improvements in IWC and IWP accuracy via Eqs. 6 and 9, respectively.
- 6) The relationship between D_e and β_{eff} was not unique, where PSDs having the same β_{eff} can have different D_e due to PSD shape differences between TTL cirrus and cirrus at higher temperatures. For this reason, separate X - β_{eff} relationships were developed for TTL and anvil (or synoptic) cirrus, with a temperature interpolation linking these two temperature regimes. This mostly affects the tropics where cirrus clouds are abundant in the TTL (see Fig. 16). The X - β_{eff} relationships for the SPARTICUS synoptic and the TC4 anvil cirrus yield similar N_i retrievals (see Fig. 17).
- 7) By comparing the FCDP and 2D-S probes in their overlap region, the first size-bin of the 2D-S probe was corrected to a first approximation, resulting in improved X - β_{eff} relationships.
- 8) In general, the physical properties of cirrus clouds differ when comparing optically thicker ($0.3 < \tau < 3$) cirrus clouds with all cirrus clouds ($0.01 < \tau < 3$), where N_i and IWC are higher in the optically thicker cirrus clouds.

This study should be extended to more field campaigns, in particular at high latitude, to further investigate the variability in the X - β_{eff} relationships, which seems more important for D_e than for N_i . In view of (8), cirrus cloud field campaigns should indicate, if possible, the type of cirrus clouds being sampled, especially outside the tropics where orographic wave (OGW) cloud cirrus (often having $\tau > 0.3$) are common (M2018). A global/seasonal analysis of the frequency of occurrence of these OGW cirrus clouds, developed through satellite remote sensing, would also be useful for testing the representation of cirrus clouds in climate models, given their distinct optical properties.

Given the apparent dependence of N_i and IWC on τ , the agreement between the two remote sensing methods (DARDAR and CALIPSO) and the Krämer et al. (2020) climatology appears reasonable. That is, cirrus associated with strong updrafts (i.e., anvil cirrus near convection and OGW cirrus) are generally avoided during cirrus field campaigns for safety reasons (Krämer et al., 2020) and therefore may not be accurately represented by in situ sampling-based climatology. It may be possible that the high median DARDAR N_i ($\sim 100 \text{ L}^{-1}$) for $T < 205 \text{ K}$ (Krämer et al. 2020, Fig. 15) relative to in situ climatological N_i in Fig. 20 results from the DARDAR sampling of thick anvil cirrus near convection where hom affects N_i more profoundly. This CALIPSO retrieval does not sample such cirrus (i.e., $\tau > 3$) and thus would retrieve a lower median climatological N_i . Nonetheless, tropical cirrus clouds having $\tau < 3$ are probably representative of tropical cirrus in terms of their areal coverage, which matters most for cloud radiative effects.

This CALIPSO retrieval provides layer properties based on layer β_{eff} and the IIR weighting function derived from the CALIOP extinction profiles at 532 nm. Future work could aim at estimating in-cloud vertical profiles of IWC, D_e , and N_i . This would require knowledge of the in-cloud variation of β_{eff} , which could be inferred from *a priori* assumptions regarding variations of D_e further constrained by co-located CloudSat radar observations when available.

The application of this CALIPSO retrieval for studying the physics of cirrus clouds is exemplified in Part 2 of this article. In particular, a method for estimating the fraction of cirrus clouds strongly affected by hom is presented as well as a new conceptual model for cirrus cloud formation and evolution.

6 APPENDIX A: Inter-channel optical depth differences

Both over land and over oceans, the solid lines in Fig.3 and Fig. 4 tend to 0 as IAB tends to 0, as expected. The median $\tau_{\text{abs}12-10}$ differences are reported in Table A1. To estimate whether these differences are realistic, Table A1 also includes an approximate β_{eff} derived from the median $\tau_{\text{abs}12-10}$ and median $\tau_{\text{abs}}(12.05 \mu\text{m})$ reported in Table 1 as

$$\beta_{\text{eff proxy}} = \frac{\text{median } \tau_{\text{abs}}(12.05 \mu\text{m})}{\text{median } \tau_{\text{abs}}(12.05 \mu\text{m}) - \text{median } \tau_{\text{abs}12-10}} . \quad (\text{A1})$$

This approximate β_{eff} is the ratio of two very small optical depths (smaller than 0.01) and is therefore very sensitive to small inter-channel biases. It is as expected larger than 1, except at 0° N-30° N over land in DJF where it is only slightly smaller. We estimate that the upper range of realistic values for β_{eff} is ~ 1.5 -2, so that $\beta_{\text{eff}} = 3.2$ at 60°-82° N over oceans in DJF is unambiguously overestimated. Decreasing $\tau_{\text{abs}12-10}$ from 0.0029 to 0.0010 would bring β_{eff} to 1.3, suggesting a positive 12-10 inter-channel bias ≤ 0.002 at 60°-82° N over oceans in DJF. Note that the impact of such an inter-channel bias decreases sharply as optical depth increases (Garnier et al., 2021a). For instance, at $\tau_{\text{abs}}(12.05 \mu\text{m}) = 0.05$, corresponding to IAB ~ 0.004 in Fig. 2, $\beta_{\text{eff}} = 1.2$ in Fig. 4 could correspond to true $\beta_{\text{eff}} \sim 1.145$, i.e. β_{eff} could be overestimated but less than 0.06. At $\tau_{\text{abs}}(12.05 \mu\text{m}) = 0.15$, corresponding to IAB ~ 0.01 in Fig. 2, $\beta_{\text{eff}} = 1.15$ in Fig. 4 could be overestimated by less than 0.02.

Table A1: Median IIR $\tau_{\text{abs}}(12.05 \mu\text{m}) - \tau_{\text{abs}}(10.6 \mu\text{m})$ (i.e., $\tau_{\text{abs}12-10}$) at CALIOP IAB $\sim 7.6 \times 10^{-4} \text{ sr}^{-1}$ using all retrievals (cf solid lines in Figs. 3 and 4) and an approximation for β_{eff} (Eq. (A1)).

	Land				Oceans			
	DJF		JJA		DJF		JJA	
Latitude	$\tau_{\text{abs}12-10}$	$\beta_{\text{eff proxy}}$	$\tau_{\text{abs}12-10}$	$\beta_{\text{eff proxy}}$	$\tau_{\text{abs}12-10}$	$\beta_{\text{eff proxy}}$	$\tau_{\text{abs}12-10}$	$\beta_{\text{eff proxy}}$
60°-82° N	0.0024	1.17	0.0010	1.10	0.0030	3.20	0.0016	1.22
30°-60° N	0.0020	1.45	0.0007	1.10	0.0018	1.39	0.0008	1.15
0°-30° N	-0.0002	0.97	0.0008	1.34	0.0009	1.20	0.0006	1.09
30°-0° S	0.0015	1.87	0.0015	1.48	0.0007	1.15	0.0006	1.11
60°-30° S	0.0013	1.17	0.0015	1.22	0.0012	1.16	0.0015	1.23
82°-60° S	0.0011	1.06	0.0021	1.21	0.0031	1.86	0.0027	1.90

7 APPENDIX B: Retrieval uncertainty analysis

7.1 Ice particle number concentration, N_i

The retrieval equation for the ice particle number concentration, N_i , is

$$N_i = \left[\frac{N_i}{A_{\text{PSD}}} \right]_{\beta_{\text{eff}}} \frac{\tau_{\text{abs}}(12.05 \mu\text{m})}{[Q_{\text{abs,eff}}(12 \mu\text{m})]_{\beta_{\text{eff}}} \Delta z_{\text{eq}}} . \quad (\text{B1})$$

The quantities N_i/A_{PSD} and $1/Q_{\text{abs,eff}}(12 \mu\text{m})$ are retrieved from β_{eff} using regression curves and the coefficients given in Table 3. By writing $x = \beta_{\text{eff}}$, they are computed as

$$\left(\frac{N_i}{A_{\text{PSD}}} \right) (\text{cm}^{-2}) = \sum_{i=0}^{i=2} b_i x^i \quad (\text{B2})$$

and

$$\frac{1}{Q_{\text{abs,eff}}(12 \mu\text{m})} = \sum_{i=0}^{i=2} c_i x^i . \quad (\text{B3})$$

Equation (B1) can be re-written as

$$N_i (\text{L}^{-1}) = 0.01 \times f(x) (\text{cm}^{-2}) \times \alpha_{\text{abs}} (\text{km}^{-1}) \quad (\text{B4})$$

with

$$f(x) = \left(\sum_{i=0}^{i=2} b_i x^i \right) \left(\sum_{i=0}^{i=2} c_i x^i \right) \quad (\text{B5})$$

and

$$\alpha_{\text{abs}} = \frac{\tau_{\text{abs}}(12.05 \mu\text{m})}{\Delta z_{\text{eq}}} . \quad (\text{B6})$$

Assuming a negligible error in ΔZ_{eq} , and writing $\tau_{\text{abs}}(12.05 \mu\text{m})$ as τ_{12} and $\tau_{\text{abs}}(10.6 \mu\text{m})$ as τ_{10} for more clarity, so that $x = \tau_{12}/\tau_{10}$, the derivative of N_i can be written

$$\frac{dN_i}{N_i} = \frac{1}{f} \frac{\partial f}{\partial x} x \left(\frac{d\tau_{12}}{\tau_{12}} - \frac{d\tau_{10}}{\tau_{10}} \right) + \frac{d\tau_{12}}{\tau_{12}} . \quad (\text{B7})$$

In Eq. (B7), the derivative of $x = \beta_{\text{eff}}$ is

$$dx = d\beta_{\text{eff}} = x \left(\frac{d\tau_{12}}{\tau_{12}} - \frac{d\tau_{10}}{\tau_{10}} \right) . \quad (\text{B8})$$

Errors in τ_{12} and in τ_{10} are computed by propagating errors in i) the measured brightness temperatures T_m , ii) the background brightness temperatures T_{BG} , and iii) the blackbody brightness temperatures T_{BB} (Garnier et al., 2015; Garnier et al., 2021a, M2018). For each of the 3 temperature (T) components (T_{BG} , T_{BB} , and T_m), $\frac{\partial \tau}{\partial T}$ in channels 12 and 10 is computed as

$$815 \quad \frac{\partial \tau}{\partial T} = \frac{1}{1-\varepsilon} \frac{\partial \varepsilon}{\partial T} \quad (\text{B9})$$

where the effective emissivity in the channel, ε , and the three associated $\frac{\partial \varepsilon}{\partial T}$ terms are reported in the IIR Version 4 product.

The uncertainties ΔT_{m10} in T_{m10} at 10.6 μm and ΔT_{m12} in T_{m12} at 12.05 μm are random errors set to 0.3 K, which are statistically independent (Garnier et al., 2015; Garnier et al., 2021a). Because the same cloud temperature is used to compute τ_{12} and τ_{10} , the uncertainty ΔT_{BB} in T_{BB} is the same at 10.6 and at 12.05 μm . A random error of $\pm 2\text{K}$ is estimated to include
820 errors in the atmospheric model. Finally, it was shown in Garnier et al. (2021a) that the uncertainty ΔT_{BG} in T_{BG} can be considered identical in both channels. ΔT_{BG} is estimated to be 1 K over oceans and 3 K over land.

Finally, the relative uncertainty $\Delta N_i/N_i$ is written as

$$\begin{aligned} \left(\frac{\Delta N_i}{N_i}\right)^2 = & \left[\frac{1}{f} \frac{\partial f}{\partial x} x \left(\frac{\partial \tau_{12}}{\tau_{12} \partial T_{BG}} - \frac{\partial \tau_{10}}{\tau_{10} \partial T_{BG}} \right) + \frac{\partial \tau_{12}}{\tau_{12} \partial T_{BG}} \right]^2 \Delta T_{BG}^2 + \left[\frac{1}{f} \frac{\partial f}{\partial x} x \left(\frac{\partial \tau_{12}}{\tau_{12} \partial T_{BB}} - \frac{\partial \tau_{10}}{\tau_{10} \partial T_{BB}} \right) + \frac{\partial \tau_{12}}{\tau_{12} \partial T_{BB}} \right]^2 \Delta T_{BB}^2 + \\ & \left[\left(\frac{1}{f} \frac{\partial f}{\partial x} x + 1 \right) \frac{\partial \tau_{12}}{\tau_{12} \partial T_{m12}} \right]^2 \Delta T_{m12}^2 + \left[\left(\frac{1}{f} \frac{\partial f}{\partial x} x \right) \frac{\partial \tau_{10}}{\tau_{10} \partial T_{m10}} \right]^2 \Delta T_{m10}^2. \end{aligned}$$
825 (B10)

7.2 Effective diameter, D_e

$$D_e = \frac{3}{2\rho_i} \left[\frac{N_i}{A_{PSD}} \right]_{\beta_{\text{eff}}} \left[\frac{IWC}{N_i} \right]_{\beta_{\text{eff}}} \quad (\text{B11})$$

with $\rho_i = 0.917 \text{ g.cm}^{-3}$. Again, (N_i/A_{PSD}) is given by Eq. (B2), and (IWC/N_i) is retrieved from $x = \beta_{\text{eff}}$ as

$$\frac{IWC}{N_i}(g) = 1 / \sum_{i=0}^{i=2} d_i x^i. \quad (\text{B12})$$

830 It comes that Eq. (B11) can be re-written

$$D_e(\mu\text{m}) = 10^4 \times \left(\frac{3}{2\rho_i} \right) \times g(x) (g \text{ cm}^{-2}) = 10^4 \left(\frac{3}{2\rho_i} \right) \frac{\sum_{i=0}^{i=2} b_i x^i}{\sum_{i=0}^{i=2} d_i x^i}. \quad (\text{B13})$$

Using the same notations as previously, it comes that

$$\frac{dD_e}{D_e} = \frac{1}{g} \frac{\partial g}{\partial x} x \left(\frac{d\tau_{12}}{\tau_{12}} - \frac{d\tau_{10}}{\tau_{10}} \right). \quad (\text{B14})$$

Finally, the relative uncertainty $\Delta D_e/D_e$ is written as

$$\begin{aligned} 835 \quad \left(\frac{\Delta D_e}{D_e}\right)^2 = & \left[\frac{1}{g} \frac{\partial g}{\partial x} x \left(\frac{\partial \tau_{12}}{\tau_{12} \partial T_{BG}} - \frac{\partial \tau_{10}}{\tau_{10} \partial T_{BG}} \right) \right]^2 \Delta T_{BG}^2 + \left[\frac{1}{g} \frac{\partial g}{\partial x} x \left(\frac{\partial \tau_{12}}{\tau_{12} \partial T_{BB}} - \frac{\partial \tau_{10}}{\tau_{10} \partial T_{BB}} \right) \right]^2 \Delta T_{BB}^2 \\ & + \left[\frac{1}{g} \frac{\partial g}{\partial x} x \frac{\partial \tau_{12}}{\tau_{12} \partial T_{m12}} \right]^2 \Delta T_{m12}^2 + \left[\frac{1}{g} \frac{\partial g}{\partial x} x \frac{\partial \tau_{10}}{\tau_{10} \partial T_{m10}} \right]^2 \Delta T_{m10}^2. \end{aligned} \quad (\text{B15})$$

7.3 Ice water content, IWC

$$\text{IWC} = N_i \left[\frac{\text{IWC}}{N_i} \right]_{\beta_{\text{eff}}} \quad (\text{B16})$$

Using Eq. (B4), Eq. (B16) can be re-written

$$840 \quad \text{IWC} (\text{mg m}^{-3}) = 10^4 \times h(x) (\text{g cm}^{-2}) \times \alpha_{\text{abs}} (\text{km}^{-1}) \quad (\text{B17})$$

with

$$h(x) = f(x) / \sum_{i=0}^{i=2} d_i x^i. \quad (\text{B18})$$

It comes that the relative uncertainty $\Delta \text{IWC} / \text{IWC}$ is given by Eq. (B10) by replacing $f(x)$ with $h(x)$.

7.4 Visible IIR equivalent extinction coefficient, α_{ext}

$$845 \quad \alpha_{\text{ext}} = 2 \left[\frac{1}{Q_{\text{abs,eff}}(12 \mu\text{m})} \right]_{\beta_{\text{eff}}} \frac{\tau_{\text{abs}}(12.05 \mu\text{m})}{\Delta z_{\text{eq}}} \quad (\text{B19})$$

Equation (B19) can be written

$$\alpha_{\text{ext}} (\text{km}^{-1}) = k(x) \times \alpha_{\text{abs}} (\text{km}^{-1}) = 2 \left(\sum_{i=0}^{i=2} c_i x^i \right) \times \alpha_{\text{abs}} (\text{km}^{-1}) \quad (\text{B20})$$

Again, the relative uncertainty $\Delta \alpha_{\text{ext}} / \alpha_{\text{ext}}$ is given by Eq. (B10), using now $k(x)$ instead of $f(x)$.

7.5 Volume radius, R_v

$$850 \quad R_v = \left(\frac{3}{4\pi\rho_i} \right)^{1/3} \left[\frac{\text{IWC}}{N_i} \right]_{\beta_{\text{eff}}}^{1/3} \quad (\text{B21})$$

Which can be written as

$$R_v (\mu\text{m}) = 10^4 \times \left(\frac{3}{4\pi\rho_i} \right)^{1/3} \times l(x) = 10^4 \left(\frac{3}{4\pi\rho_i} \right)^{1/3} \left(\sum_{i=0}^{i=2} d_i x^i \right)^{-1/3}. \quad (\text{B22})$$

Eq. (B22) is of the same form as Eq. (B13) and the relative uncertainty $\Delta R_v / R_v$ is given by Eq.(B15) by replacing $g(x)$ with $l(x)$.

8 APPENDIX C: Relating R_v to D_e

Figure C1 shows the relationship between D_e and R_v for the PSDs measured at temperatures colder than -38°C during the ATTREX-POSIDON, SPARTICUS (synoptic cirrus clouds only) and TC4 field campaigns. We recall that PSDs measured

during SPARTICUS and TC4 were modified by dividing $N(D)_1$ measured in the first bin by a correction factor equal to 10.4 (see Sect. 2.3). For reference, the curves in grey show relationships assuming a simple gamma PSD distribution expressed as

$$N(D) = N_0 D^\nu e^{-\lambda D}, \quad (C1)$$

where D is the ice particle maximum dimension, ν is the PSD dispersion parameter and λ is the PSD slope in $\log N(D) - D$ space and where ν is varied between -0.5 and 4.0. R_v and D_e of a PSD were computed using mass-dimension and area-dimension relationships from EM2016 for anvil cirrus clouds between -55 and -40 °C in combination with a temperature-dependent PSD scheme for tropical anvil cirrus clouds (Mitchell et al., 1999) where only the large ice particle mode was used. They are independent of the multiplying term, N_0 . The small-particle end of the PSD is governed by ν , with decreasing contributions from these smaller particles as ν increases. The simulated PSDs reproduce the general behaviour seen in the in situ data, and they illustrate the dependence of the D_e - R_v relationship on the PSD shape. The exponential form of Eq. (C1) (i.e., $\nu = 0$) gives an approximate representation for those cirrus clouds having relatively high concentrations of small ice crystals, assuming that the mid-to-large sizes follow an exponential distribution and D_e is $> 50 \mu\text{m}$. For narrow PSDs having $D_e < 50 \mu\text{m}$, ν tends to be $\gg 0$.

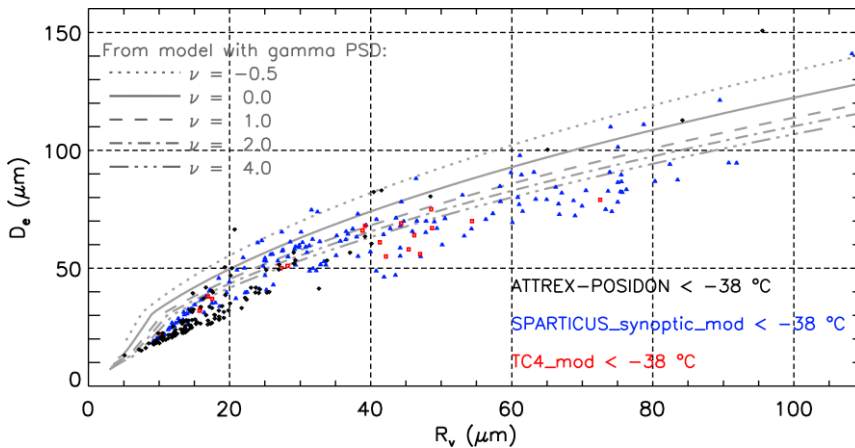


Figure C1. D_e against R_v for the PSDs measured at $T < -38^\circ \text{C}$ during the ATTREX-POSIDON (black), SPARTICUS (navy blue) and TC4 (red) field campaigns where SPARTICUS is based on synoptic cirrus clouds and $N(D)_1$ was modified for SPARTICUS and TC4. The grey curves are from a model with gamma PSD for 5 values of the PSD dispersion parameter, ν , between -0.5 and 4, illustrating that PSD shape affects the D_e - R_v relationship.

9 Data availability

The CALIPSO IIR Level 2 data products used in this study are available at the NASA Langley Atmospheric Science Data Center and can be retrieved from https://doi.org/10.5067/IIR/CALIPSO/CAL_IIR_L2_Track-Standard-V4-51. The CALIPSO

Lidar Level 2 cloud profiles used in this study are available at the NASA Langley Atmospheric Science Data Center and can be retrieved from https://doi.org/10.5067/CALIOP/CALIPSO/CAL_LID_L2_05kmCLay-Standard-V4-51. These CALIPSO data products are also available from the AERIS/ICARE Data and Services Center in France (<https://www.icare.univ-lille.fr/>). SPARTICUS in situ data is available from the ARM Data Archive at <https://www.arm.gov/data/>. ATTREX, POSIDON, and TC4 in situ data is available at <https://espoarchive.nasa.gov/archive/browse>.

10 Author contribution

DM and AG conceived the study; DM analysed the in situ aircraft data and contributed to the writing of the paper; AG accessed and analysed the CALIPSO data and contributed to the analysis of the in situ aircraft data and to the writing of the paper. SW accessed and processed the ATTREX and POSIDON aircraft data.

11 Competing interests

The authors declare that they have no conflict of interest.

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