I am very happy to read your work! Interestingly, I have conducted similar research using Lagrangian analysis, focusing on a case study of weather whiplash (https://doi.org/10.1088/1748-9326/ad9c9a). Back to your work, I have several questions and one possible suggestion that I would like to discuss with you!

Thank you so much for your insightful questions and suggestions regarding our paper. We've read your paper on the weather whiplash case study and found it to be very exciting. It's fascinating to see how Lagrangian analysis can be applied to the whiplash case study as well. Below, we've provided answers to the questions you raised in our paper.

1. **Question-1: Your work investigates the extreme day-to-day temperature (DTDT) variations and offers a physical understanding via a Lagrangian temperature budget. While Lagrangian analysis vividly depict synoptic motions, why not employ the "Eulerian temperature budget" instead? This approach directly examines the contributions of different physical processes to DTDT variations, i.e., partial T/partial t.**

   Answer: The Eulerian temperature budget focuses on changes in temperature at fixed spatial locations, breaking these changes down into contributions from various physical processes. While this approach is valuable for assessing local processes and may be well-suited for tropical regions during December–January–February, it may not capture the movement, evolution, and associated temperature changes of air parcels, particularly in the extratropics. As the advection effects are modulated by adiabatic and diabatic processes within transported air parcels, amplifying or dampening DTDT decreases (cold events) and increases (warm events). For instance, during warm events in North America, remote advection plays a significant role, with some additional contributions from surface heating. In contrast, in Australia, diabatic heating during transport is more dominant than local heating. In an Eulerian budget, this diabatic heating of the air parcels would not have been accounted for, since only local heating is captured. This underscores the importance of the Lagrangian framework for studying extreme DTDT by tracking air parcels and their trajectories, offering a comprehensive understanding of the underlying dynamics.

2. **Question-2: Your manipulation leading to Eq. A1 is very interesting. However, the "advection term" in Eq. A1 does not seem to have an intuitively clear connection to advection. Could you clarify this?**

Answer: $\overline{T}_{t}^{-3d}$ indicates the average temperature of the air parcels initialized on the day of the extreme events three days before their arrival at the target location and $\overline{T}_{t-1}^{-3d}$ the corresponding temperature for the air parcels initialized one day earlier. The expression $\delta_{T}^{-3d} = \overline{T}_{t}^{-3d} - \overline{T}_{t-1}^{-3d}$ thus represents the difference between the air parcel's temperatures three days before their arrival. Assuming that no further temperature changes occurred during the transport, the DTDT change would only be due to these initial differences of the air parcels, which means it would be caused by changes in the advection of air parcels with different original temperatures between the previous day and the day of the event. This is why we refer to this term as an advection term.

Suggestion-1: A study by Prof. Tapio Schneider provides a macroturbulence perspective (temperature gradient + mixing length) to understand synoptic temperature variability and its responses to global warming (https://doi.org/10.1175/JCLI-D-14-00632.1). It might be valuable to compare your findings with this work.

Answer: Thank you for the suggestion. This paper investigates the underlying physical processes of DTDT extremes and their linkage to the large-scale atmospheric circulation in the present climate. We will also explore how these processes influence projected DTDT extremes under climate warming in Part II. The study by Prof. Tapio Schneider shows that the reduction of meridional potential temperature gradients, driven by the polar amplification of global warming, leads to a decrease in synoptic temperature variance near the surface. Comparing our findings with this work will offer additional insights into the dynamics of temperature extremes and their response to global warming.