Supplementary Materials:
Assimilation of radar freeboard and snow altimetry observations in the Arctic and Antarctic with a coupled ocean/sea ice modelling system

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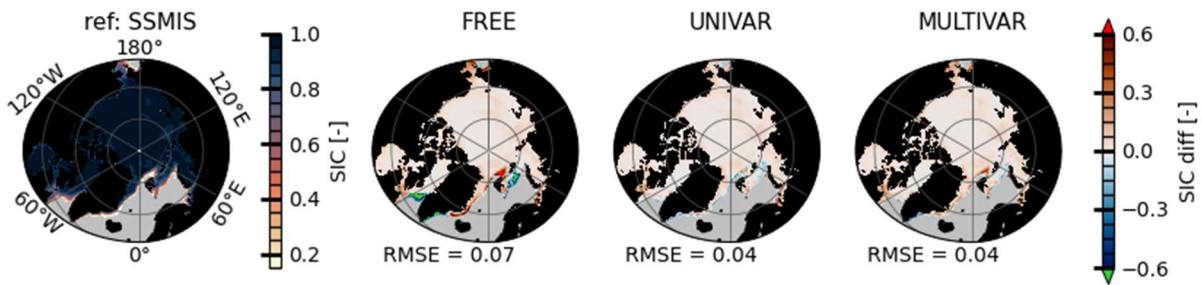2 CNES, Toulouse, 31400, France
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1 Additional figures
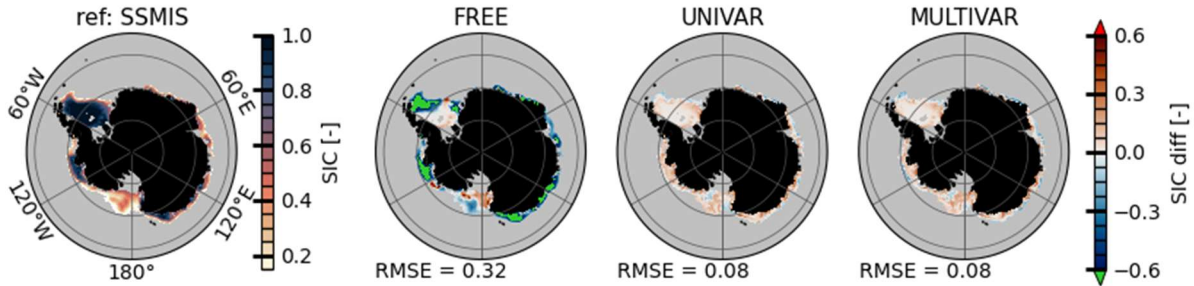
1.1 Sea ice concentration

Figure S1: Sea ice concentration in March 2018 in the Arctic (a) and in the Antarctic (b) for observation SSMIS (first column), and the difference between the experiments (FREE, UNIVAR and MULTIVAR) and the reference SSMIS observation on the following columns. Root mean squared errors (RMS) are provided under each map.

(a) September 2018 Arctic SIC differences relative to OSISAF SSMIS
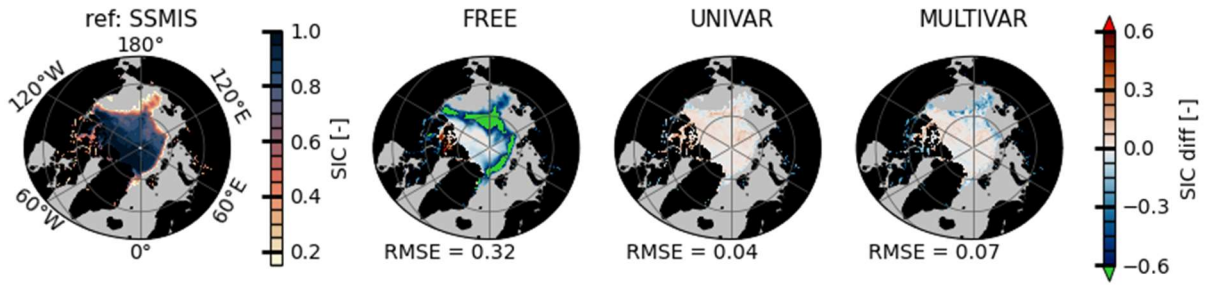
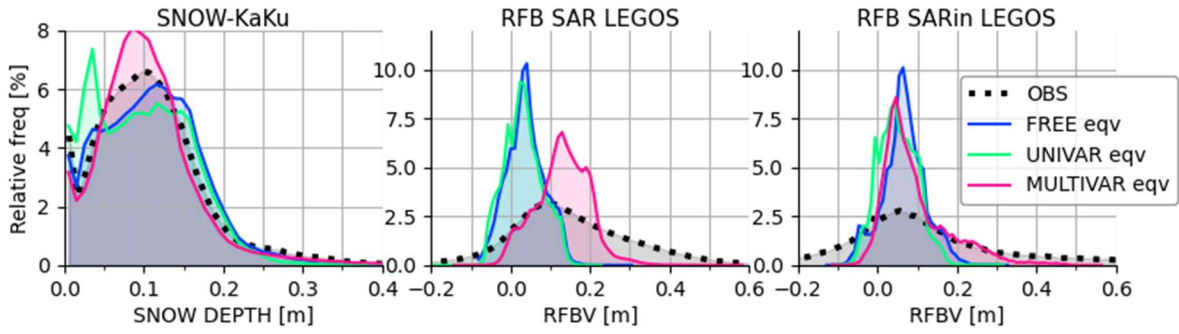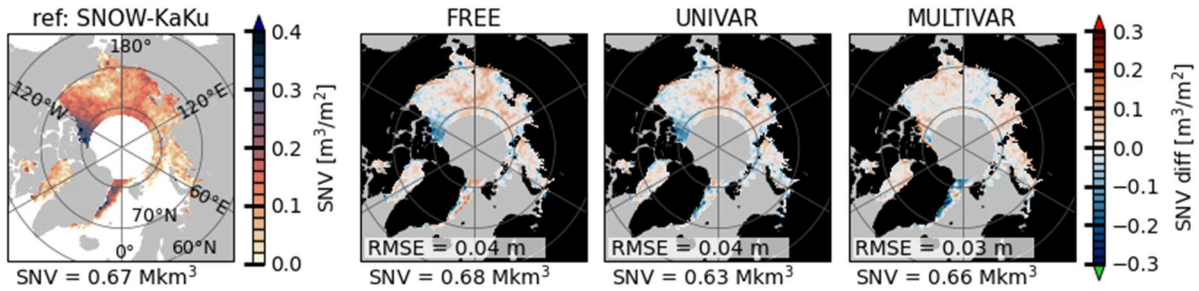(b) September 2018 Antarctic SIC differences relative to OSISAF SSMIS
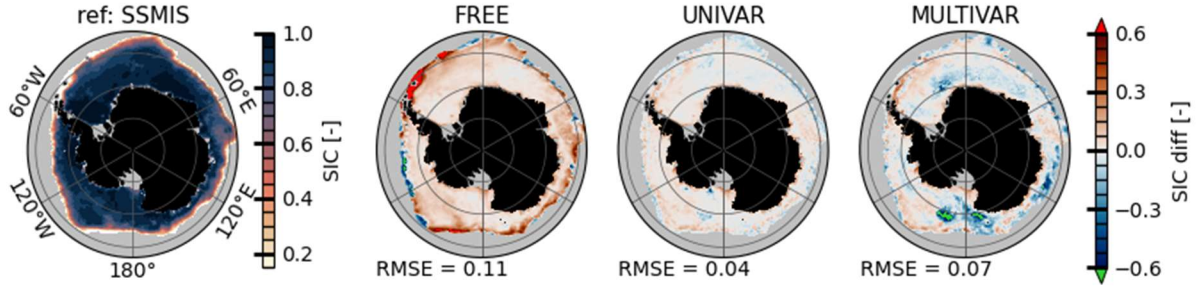
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Figure S2: Same as figure S1 for September 2018.
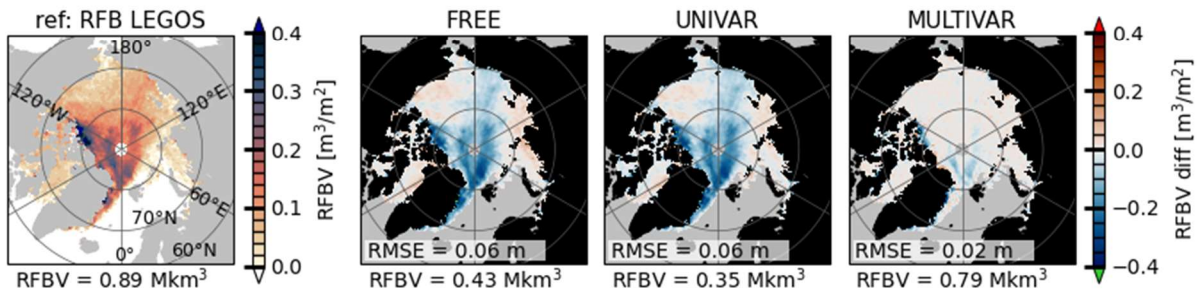
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1.2 Radar freeboard and snow

(a) January 2017 Arctic distributions

January 2017 Arctic snow volume differences relative to SNOW-KaKu

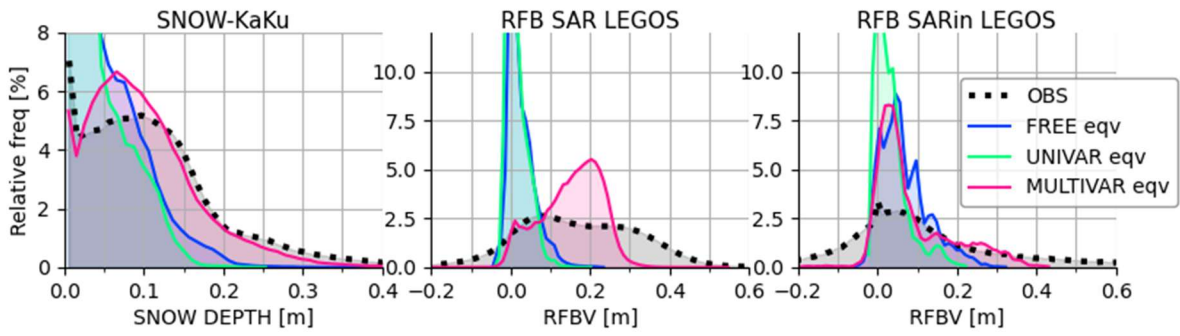January 2017 Arctic radar freeboard volume differences relative to RFB LEGOS

Figure S3: Top row panels: Probability density functions (%) of the snow thickness, the radar freeboard SAR and radar freeboard SARin observations (dotted black) and their model equivalent for the FREE (blue), UNIVAR (green) and MULTIVAR (pink) experiments in the Arctic for January 2017. Middle, resp. bottom, row panels: snow volume per unit area [m³/m2] , resp. radar freeboard volume per unit area, from SNOW-KaKu, resp. RFB LEGOS, (first column) and differences with FREE, UNIVAR and MULTIVAR experiments. Total snow and RFB volumes values and root mean squared difference (RMS) are provided under each map.

Figure S4: Same as Figure S3 for November 2017.

(a) March 2018 Arctic distributions
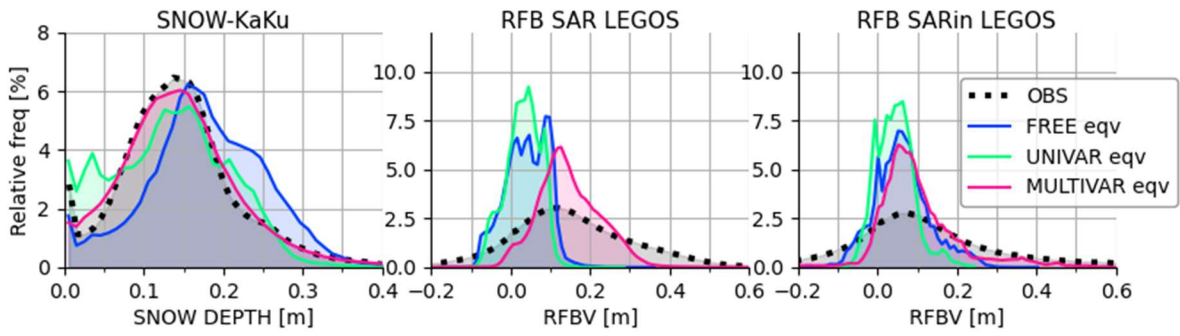
March 2018 Arctic snow volume differences relative to SNOW-KaKu
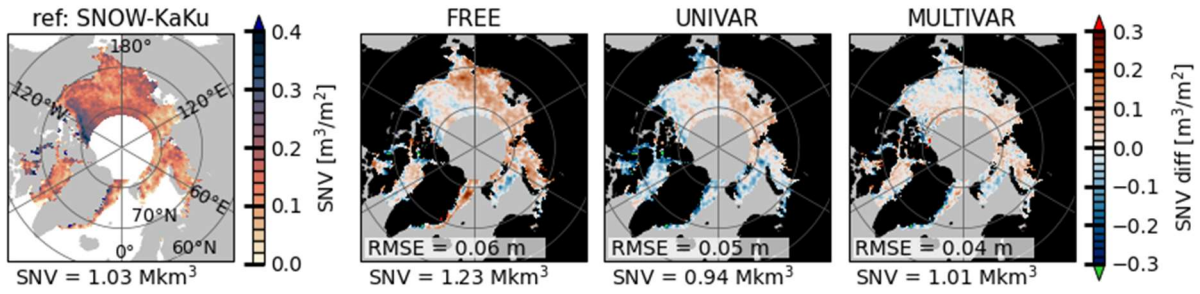
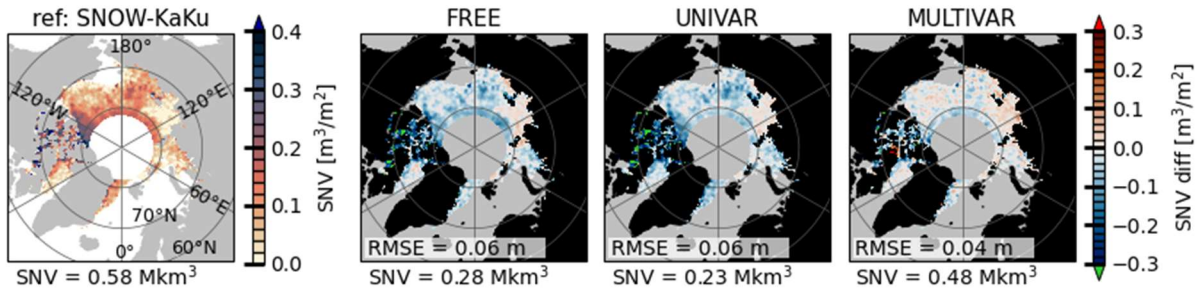March 2018 Arctic radar freeboard volume differences relative to RFB LEGOS
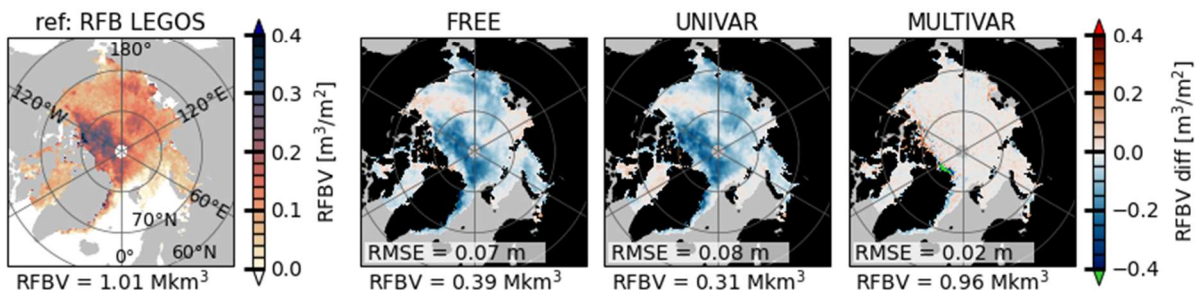
Figure S5: Same as Figure S3 for March 2018.

(a) December 2018 Arctic distributions

December 2018 Arctic snow volume differences relative to SNOW-KaKu

December 2018 Arctic radar freeboard volume differences relative to RFB LEGOS
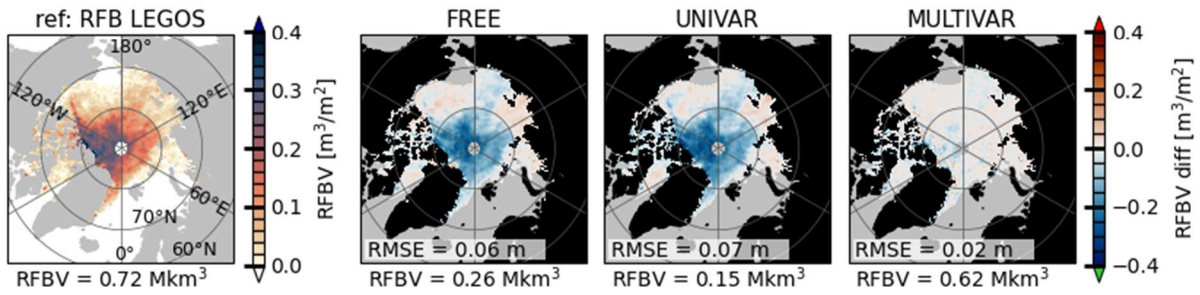
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Figure S6: Same as Figure S3 for December 2018.
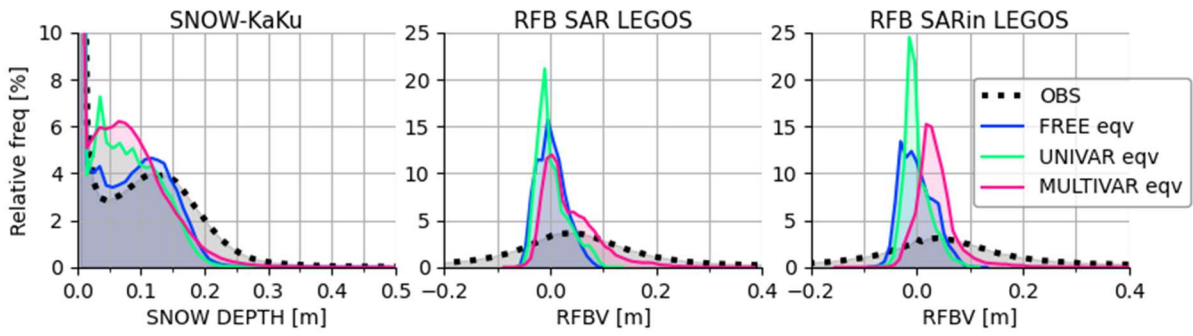
Figure S7: Same as Figure S3 for Antarctic and May 2017.

(a) May 2018 Antarctic distributions
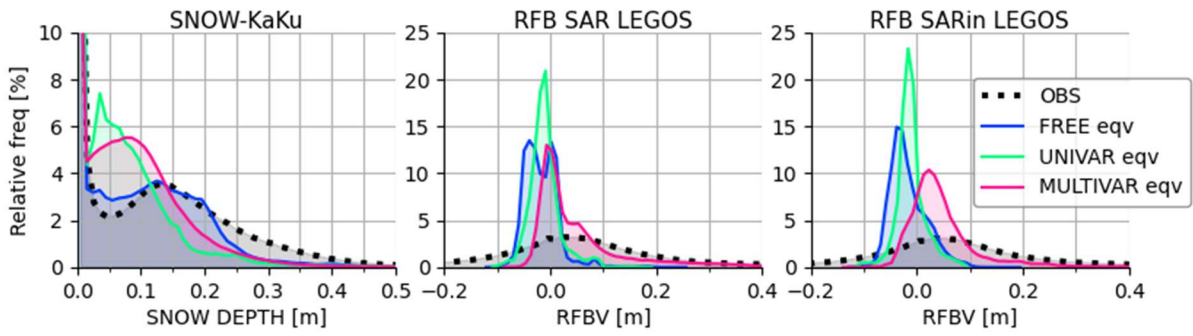
May 2018 Antarctic snow volume differences relative to SNOW-KaKu
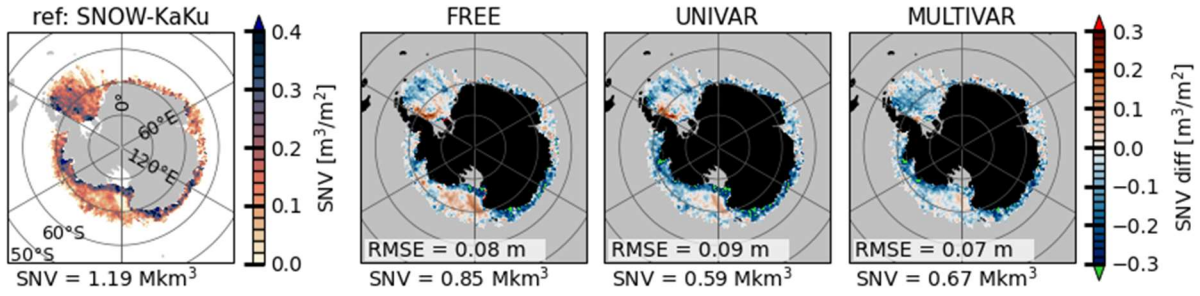
May 2018 Antarctic radar freeboard volume differences relative to RFB LEGOS
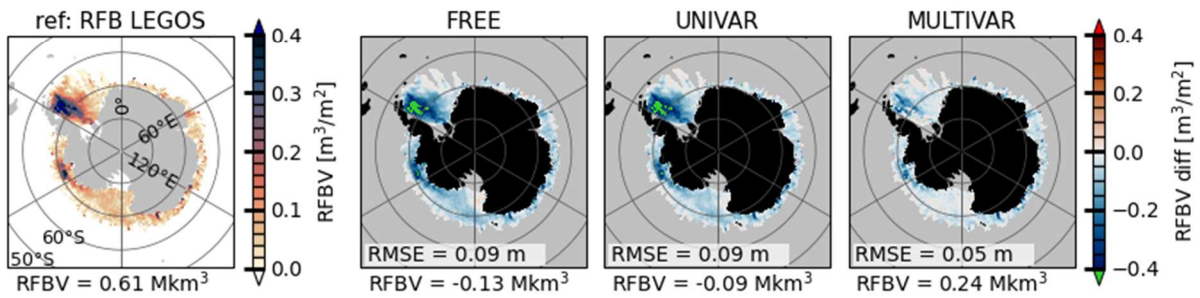
40   **Figure S8: Same as Figure S3 for Antarctic and May 2018.**

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(a) October 2018 Antarctic distributions
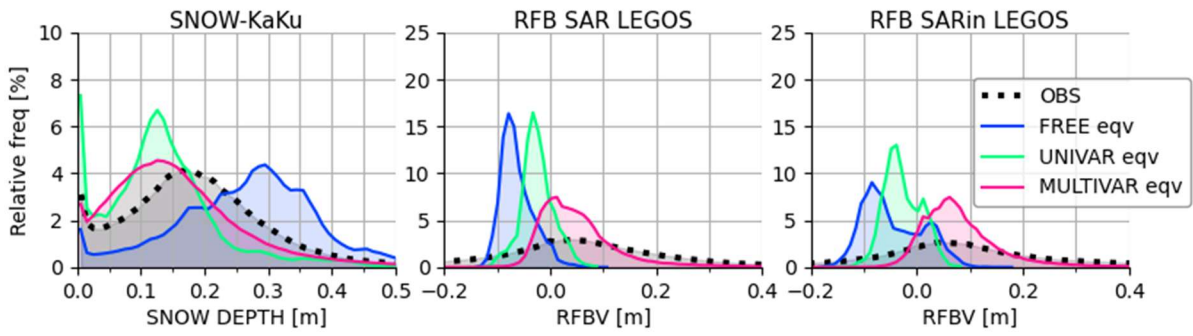
October 2018 Antarctic snow volume differences relative to SNOW-KaKu
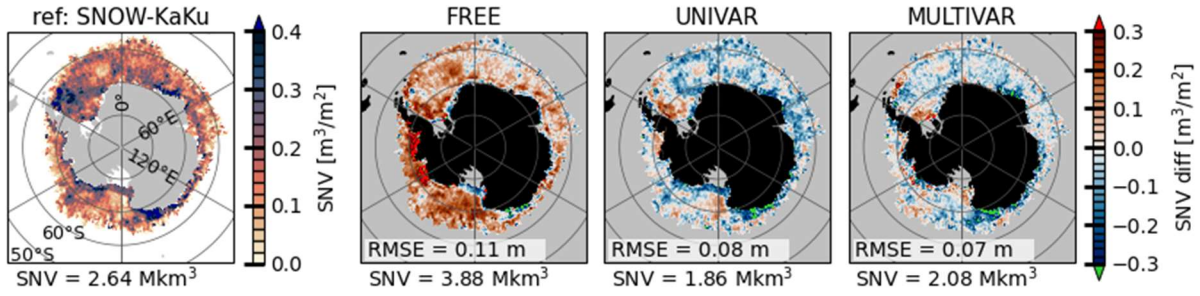
October 2018 Antarctic radar freeboard volume differences relative to RFB LEGOS

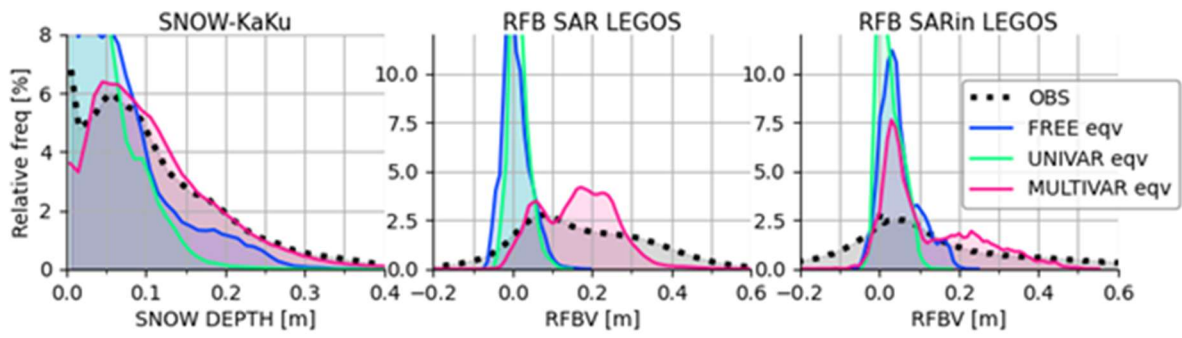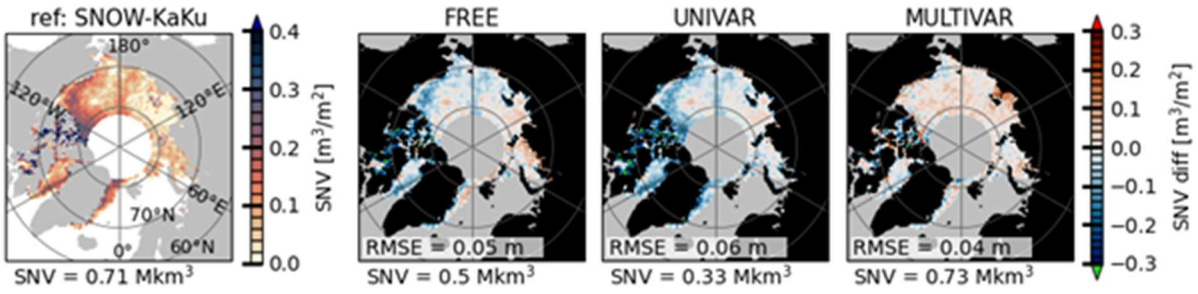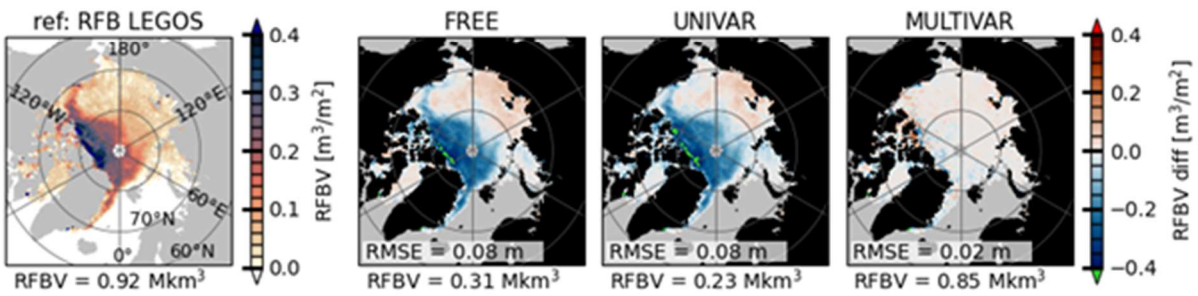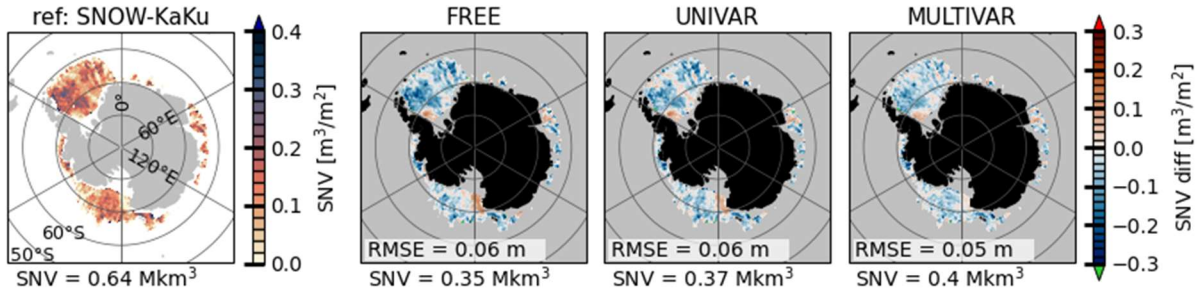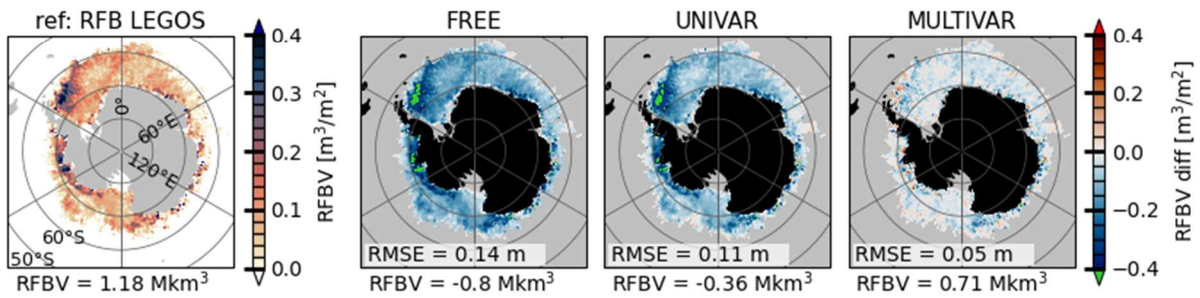**Figure S9: Same as Figure S3 for Antarctic and October 2018.**

1.3 Sea ice volume: comparison to CS2SMOS and SMOS products

January 2017 Arctic sea ice volume

	LEGOS zone				Outside LEGOS zone			
	SIT > 1 m		SIT < 1 m		SIT > 1 m		SIT < 1 m	
	RMS	MD	RMS	MD	RMS	MD	RMS	MD
FREE	0.29	-0.18	0.21	0.17	0.59	-0.57	0.19	0.16
UNIVAR	0.33	-0.25	0.16	0.01	0.71	-0.71	0.13	-0.01
MULTIVAR	0.26	0.19	0.16	0.13	0.44	-0.37	0.18	0.16
LEGOS_og	0.21	0.11	0.19	0.1	-	-	-	-
LEGOS_mD	0.42	0.39	0.23	0.16	-	-	-	-
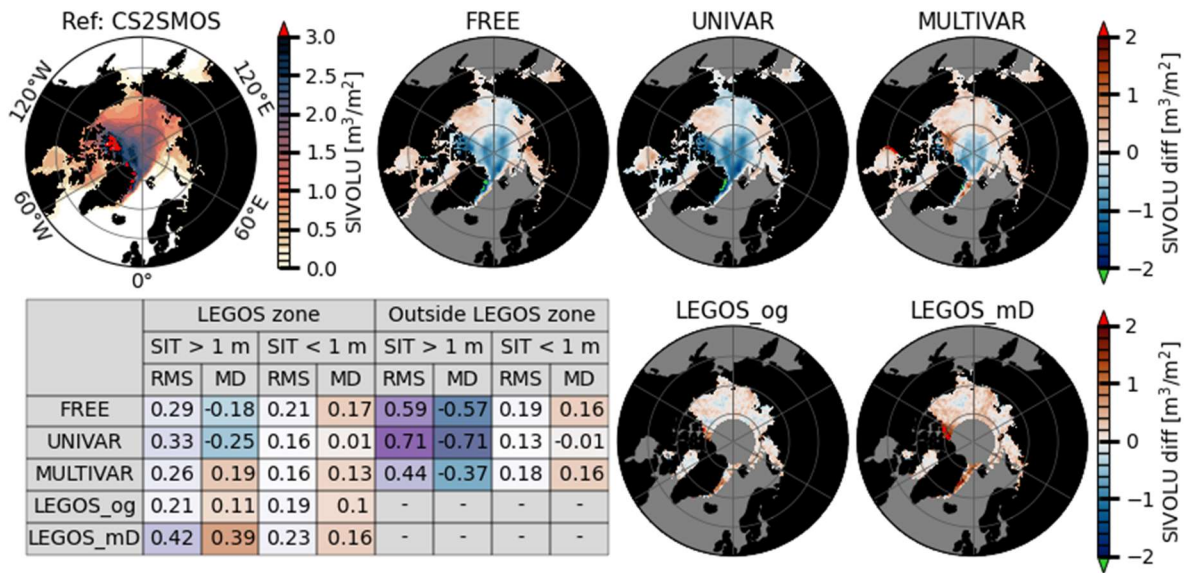
Figure S10: Sea ice volume in the Arctic in January 2017 for CS2SMOS dataset (reference) and its difference with the FREE, UNIVAR, and MULTIVAR experiments (first line) and the observations LEGOS_og (original) and LEGOS_mD (with model constant densities). Table: root mean square error (RMS) and mean difference (MD) between FREE, UNIVAR, MULTIVAR, LEGOS_og, LEGOS_md and CS2SMOS data, calculated on the LEGOS zone and outside the LEGOS zone and for CS2SMOS sea ice thickness of less than or greater than 1m. The table colors highlight the values close to 0 (white) and the extremes (green for the RMS, and blue/red for the negative/positive MD).

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November 2017 Arctic sea ice volume

	LEGOS zone				Outside LEGOS zone			
	SIT > 1 m		SIT < 1 m		SIT > 1 m		SIT < 1 m	
	RMS	MD	RMS	MD	RMS	MD	RMS	MD
FREE	0.48	-0.45	0.13	-0.04	0.97	-0.96	0.12	-0.01
UNIVAR	0.71	-0.7	0.2	-0.16	1.16	-1.16	0.13	-0.1
MULTIVAR	0.36	0.24	0.16	0.12	0.29	-0.09	0.13	0.1
LEGOS_og	0.27	0.17	0.29	0.22	-	-	-	-
LEGOS_mD	0.64	0.62	0.38	0.32	-	-	-	-
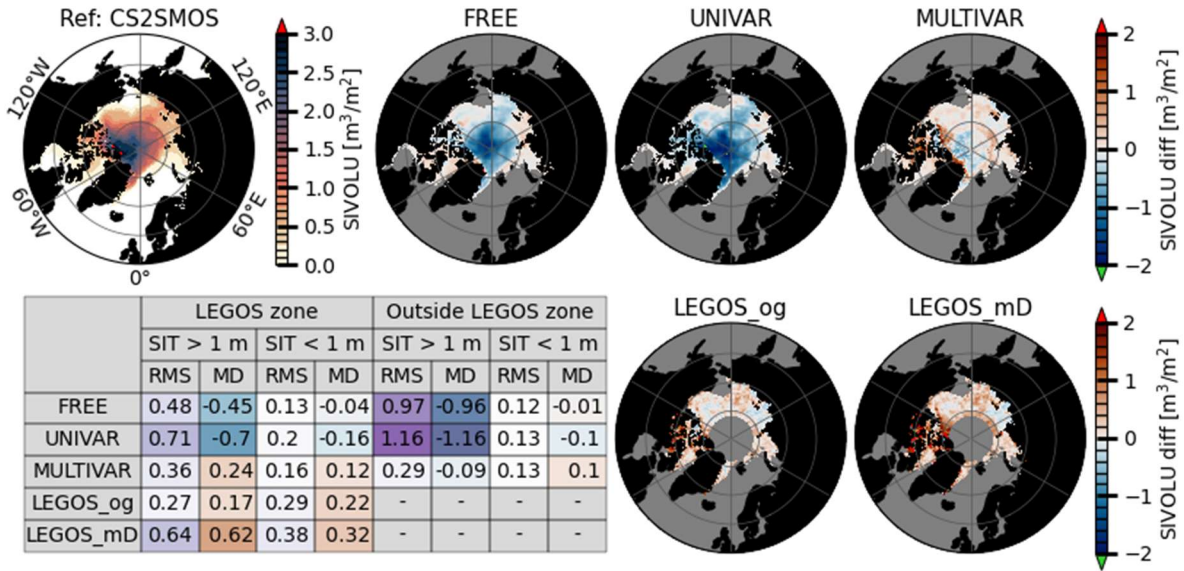
Figure S11: Same as Figure S10 for November 2017.

March 2018 Arctic sea ice volume

	LEGOS zone				Outside LEGOS zone			
	SIT > 1 m		SIT < 1 m		SIT > 1 m		SIT < 1 m	
	RMS	MD	RMS	MD	RMS	MD	RMS	MD
FREE	0.24	-0.02	0.28	0.22	0.45	-0.26	0.36	0.35
UNIVAR	0.37	-0.33	0.22	-0.04	0.63	-0.56	0.15	0.03
MULTIVAR	0.4	0.3	0.4	0.37	0.61	0.52	0.28	0.26
LEGOS_og	0.27	0.09	0.39	0.36	-	-	-	-
LEGOS_mD	0.42	0.35	0.52	0.5	-	-	-	-

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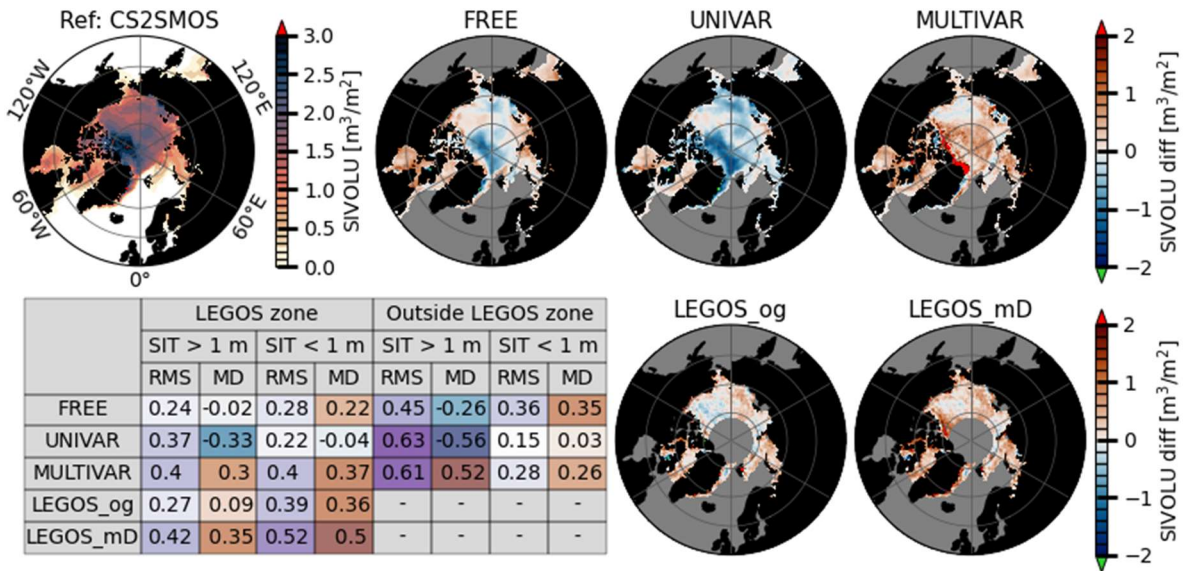
Figure S12: Same as Figure S10 for March 2018.

May 2017 Antarctic sea ice volume, comparison with SMOS product

	LEGOS zone		Outside LEGOS zone	
	RMS	MD	RMS	MD
FREE	0.21	-0.17	0.13	-0.03
UNIVAR	0.23	-0.2	0.11	-0.07
MULTIVAR	0.19	0.0	0.13	0.03
LEGOS_og	0.42	0.35	-	-
LEGOS_mD	0.57	0.53	-	-
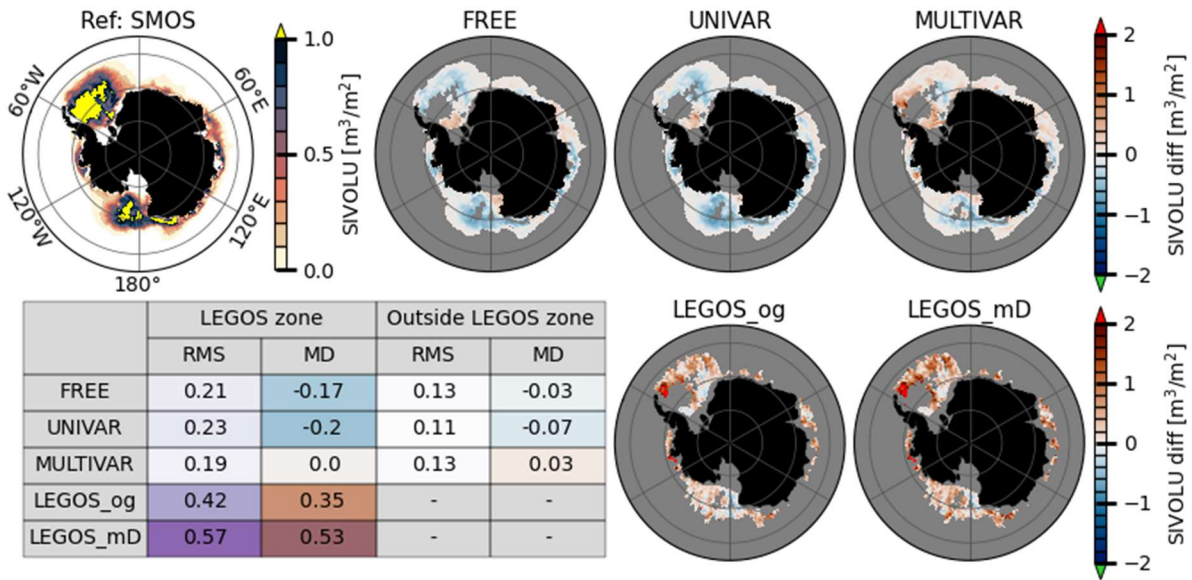
Figure S13: Same as Figure S10 for May 2017 in Antarctic.

May 2018 Antarctic sea ice volume, comparison with SMOS product

	LEGOS zone		Outside LEGOS zone	
	RMS	MD	RMS	MD
FREE	0.22	-0.1	0.05	0.0
UNIVAR	0.26	-0.24	0.02	-0.01
MULTIVAR	0.24	0.1	0.04	0.02
LEGOS_og	0.62	0.58	-	-
LEGOS_mD	0.84	0.81	-	-

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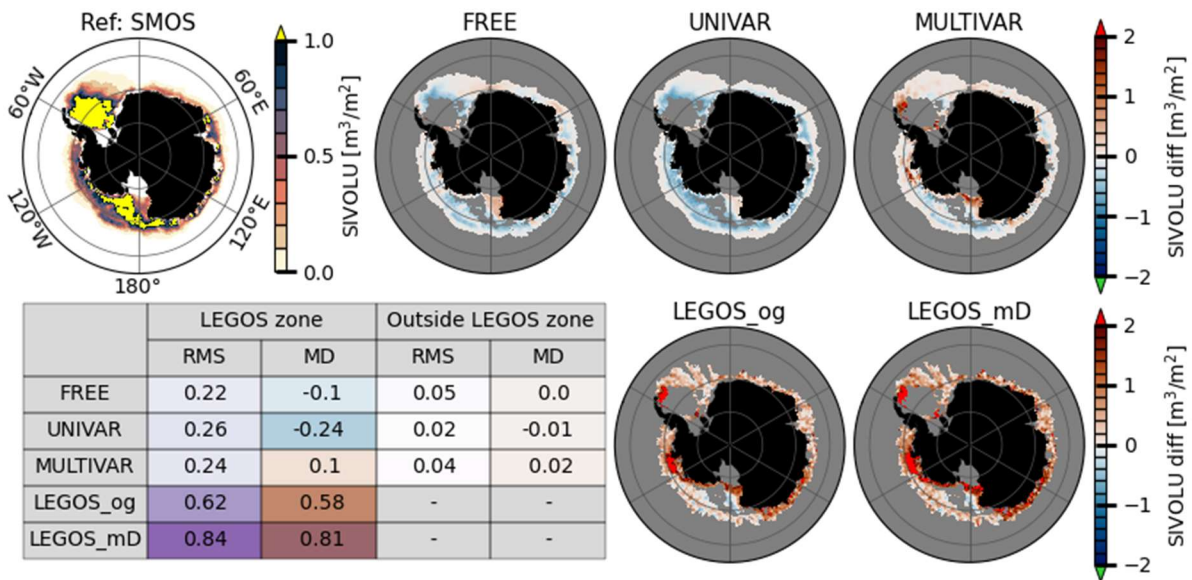
Figure S14: Same as Figure S10 for May 2018 in Antarctic.

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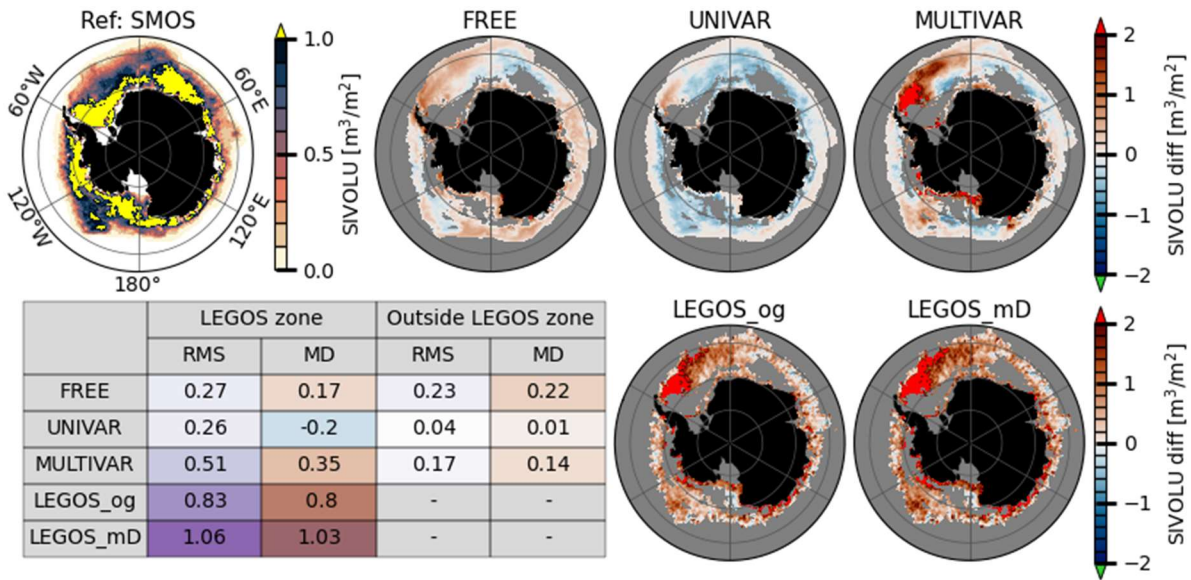
Figure S15: Same as Figure S10 for September 2018 in Antarctic.

2 Monodata/multivariate experiment

We performed a monodata/multivariate experiment assimilating the SIC OSISAF SSMIS product only with the multivariate assimilation system described previously. This experiment shows intermediate performances between monodata/univariate and multidata/multivariate for the sea ice concentration (Figure S16) and similar performance as monodata/univariate for the leads content (Figure S17). Similarly, this experiment provides intermediate performance for the RFB (Figures S18 and S19). The sea ice volume in this experiment increases the sea ice volume compared to the monodata/univariate simulation but is significantly less than the multidata/mutlivariate experiment and far from the LEGOS altimetric observations, in both hemispheres (Figure S22).

2.1 Performance of the assimilation system
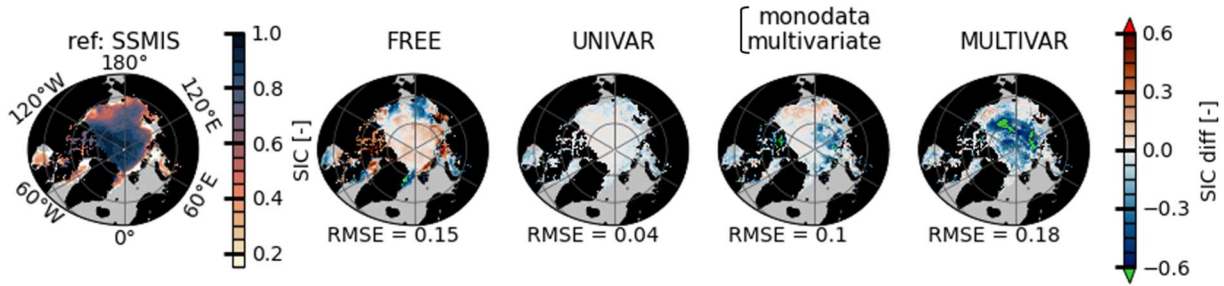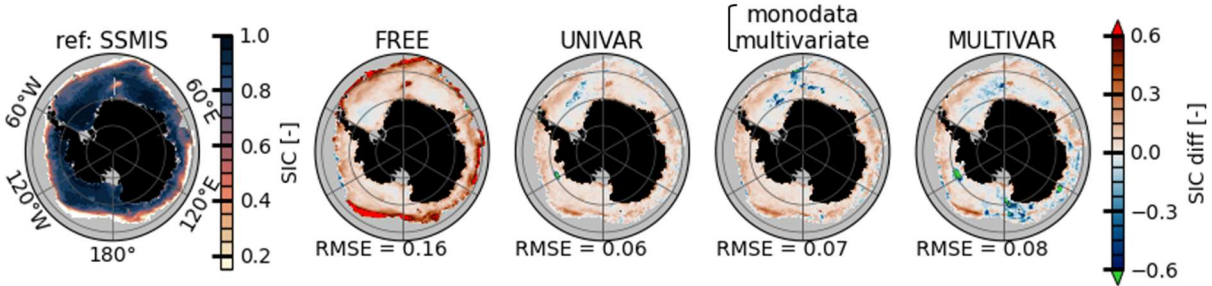
Figure S16: Same as Figure 1. in the paper. July 2018 in the Arctic (a) and September 2017 in the Antarctic (b) maps of the sea ice concentration, representing the observation SSMIS on the first column, and the difference between the experiments and the reference SSMIS observation on the following columns. The simulations are, in that order: FREE, UNIVAR, monodata/multivariate and MULTIVAR. Root mean squared errors (RMS) are provided under each map.

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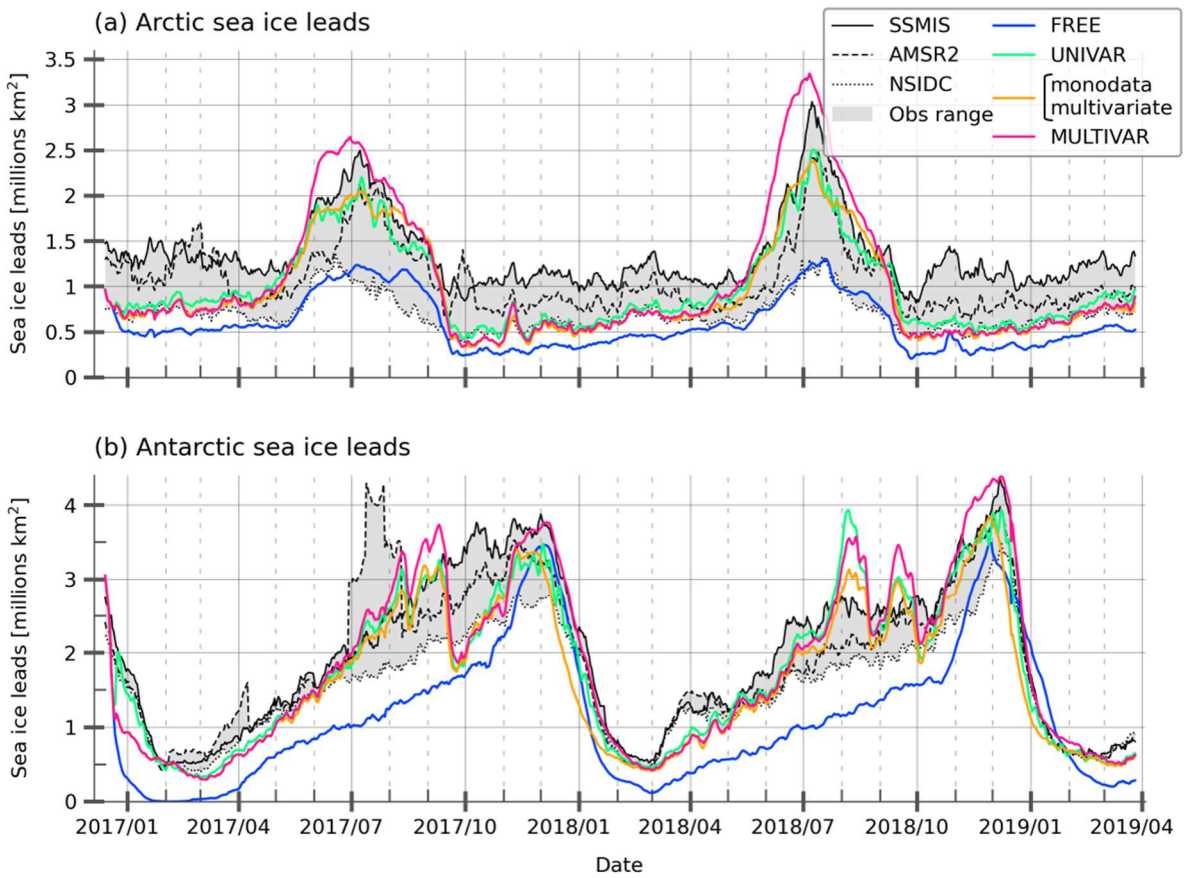
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Figure S17: Same as Figure 2 in the paper. Daily time evolution of Arctic (a) and Antarctic (b) surface covered by sea ice leads in millions of km² for SSMIS (black), AMSR2 (dashed black), NSIDC (dotted black) satellite data with the surface range covered by them (shaded grey) and for FREE (blue), UNIVAR (green), monodata/multivariate (orange) and MULTIVAR (pink) experiments.

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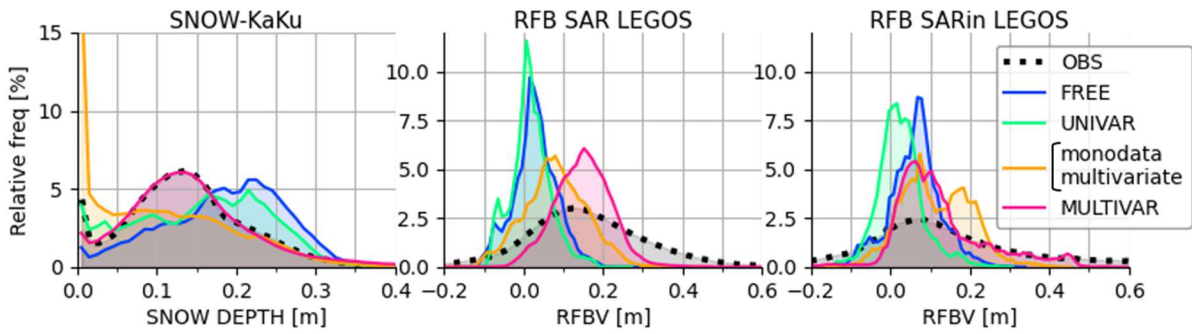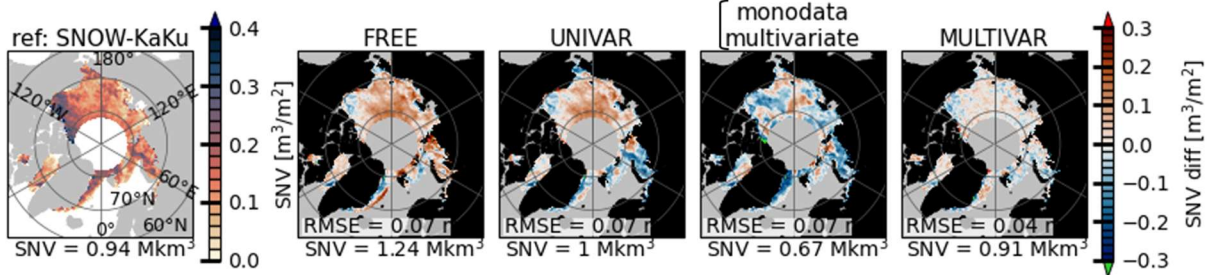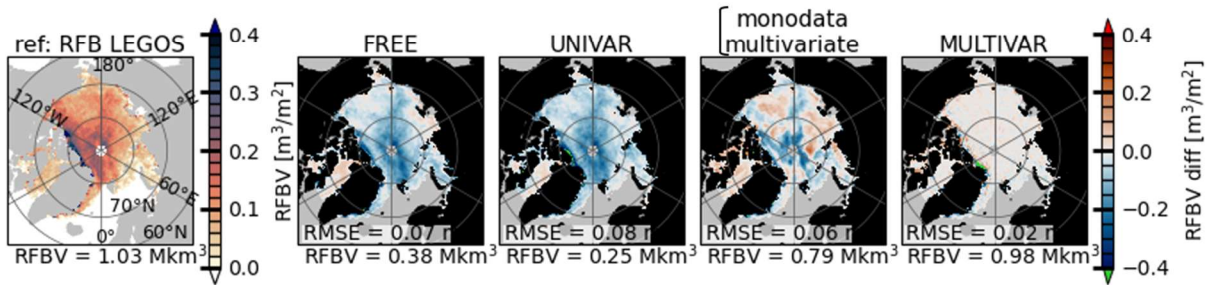
Figure S18: Same as Figure 3 in the paper. Top panels (a): Probability density functions (%) of the snow thickness, the radar freeboard SAR and radar freeboard SARin observations (dotted black) and their model equivalent for the FREE (blue), UNIVAR (green), monodata/multivariate (orange) and MULTIVAR (pink) experiments in the Arctic for April 2017. Middle (b), resp. bottom (c), row panels: snow volume per unit area [m³/m2] , resp. radar freeboard volume per unit area, from SNOW-KaKu, resp. RFB LEGOS, (first column) and differences with FREE, UNIVAR, monodata/multivariate and MULTIVAR experiments. Total snow and RFB volumes values and root mean squared difference (RMS) are provided under each map.

(a) May and October 2017 Antarctic distributions

(b) October 2017 Antarctic snow volume differences relative to SNOW-KaKu

(c) October 2017 Antarctic radar freeboard volume differences relative to RFB LEGOS

Figure S19: Same as Figure 4 in the paper. Top panels (a): Probability density functions (%) of the snow thickness, the radar freeboard SAR and radar freeboard SARin observations (dotted black) and their model equivalent for the FREE (blue), UNIVAR (green), monodata/multivariate (orange) and MULTIVAR (pink) experiments in the Antarctic for May and October 2017. Middle (b), resp. bottom (c), row panels: snow volume per unit area, resp. radar freeboard volume per unit area, from SNOW-KaKu, resp. RFB LEGOS, (first column) and differences with FREE, UNIVAR, monodata/multivariate and MULTIVAR experiments in October 2017. Total snow and RFB volumes values and root mean squared difference (RMS) are provided under each map.
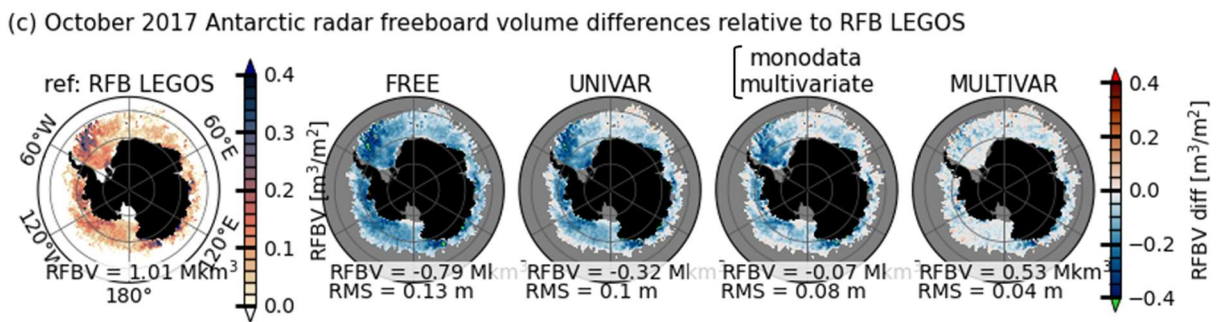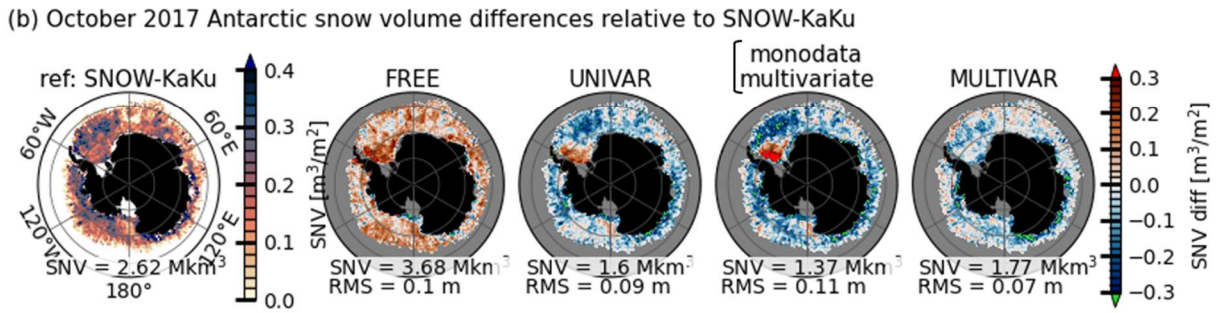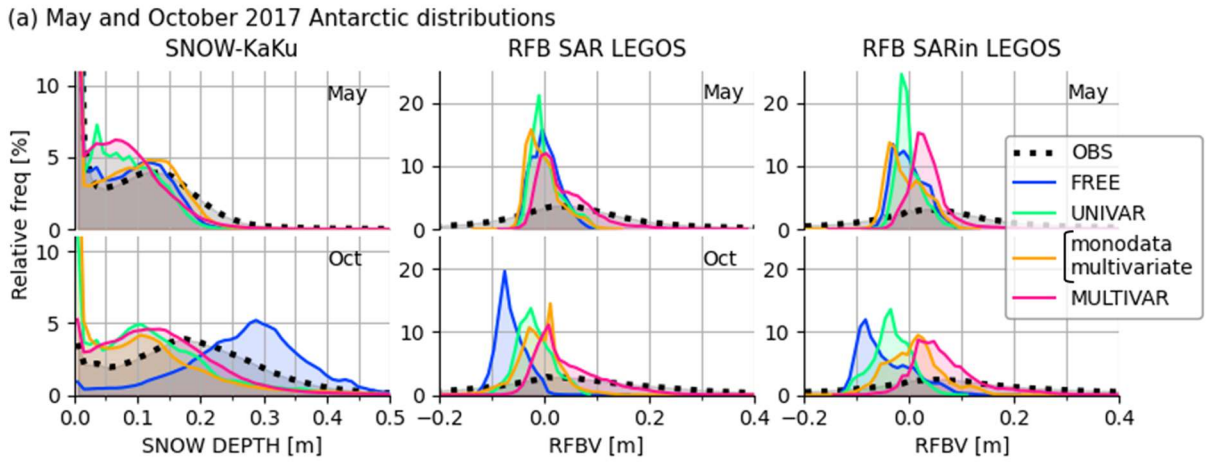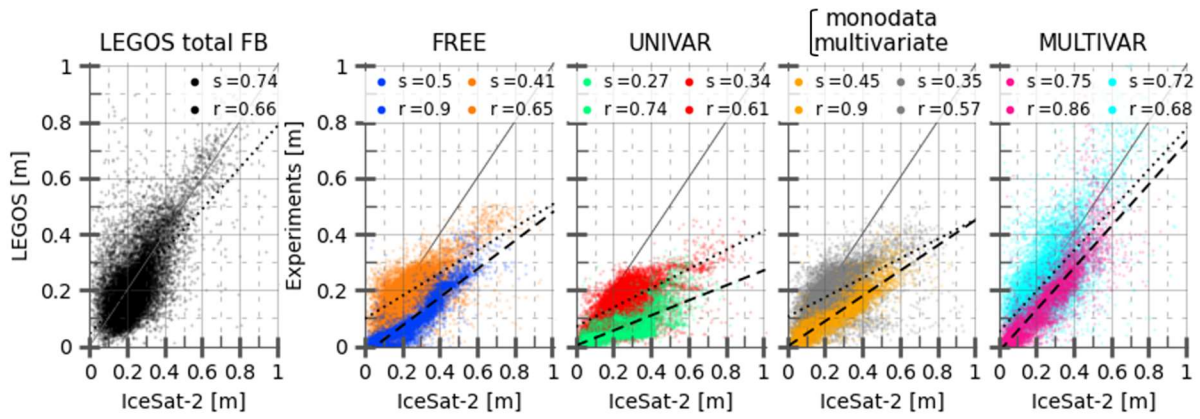
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2.2 Validation with independent datasets

Figure S20: Same as Figure 5 in the paper. Scatterplots of the monthly Arctic ICESat-2 total freeboard against FREE, UNIVAR, monodata/multivariate, MULTIVAR experiments and LEGOS RFB/SND-KaKu data computed with model densities (black) for October 2018, beginning on the 14/10/2018 (experiments respectively in blue, green, orange and pink; no LEGOS data), and for January-February 2019 (experiments respectively in orange, red, grey and cyan). The x=y line (grey) and linear regressions for Oct 2018 (dashed black) and Jan-Feb 2019 (dotted black) are shown. Values of the linear slopes (s) and the r-values (r) are provided and all statistics are significant.
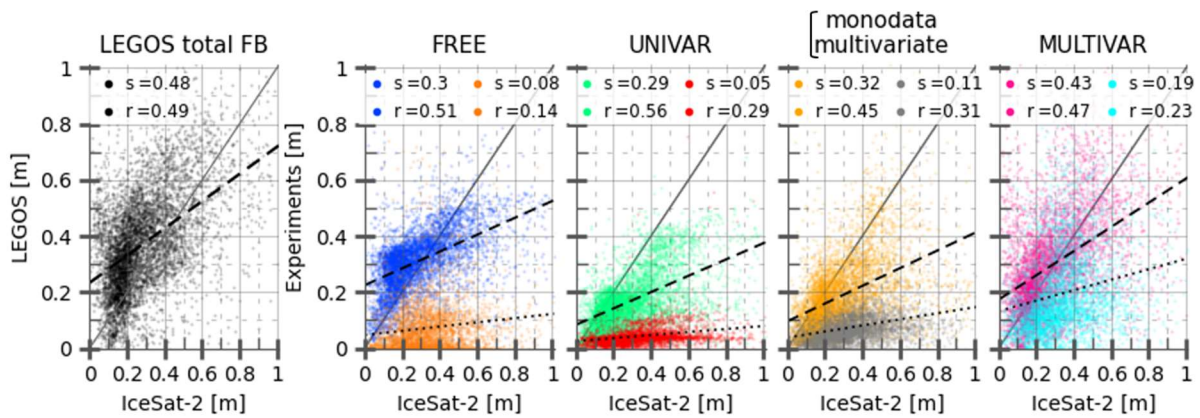
Figure S21: Same as Figure 6 in the paper. Idem Figure S20 but for Antarctica.
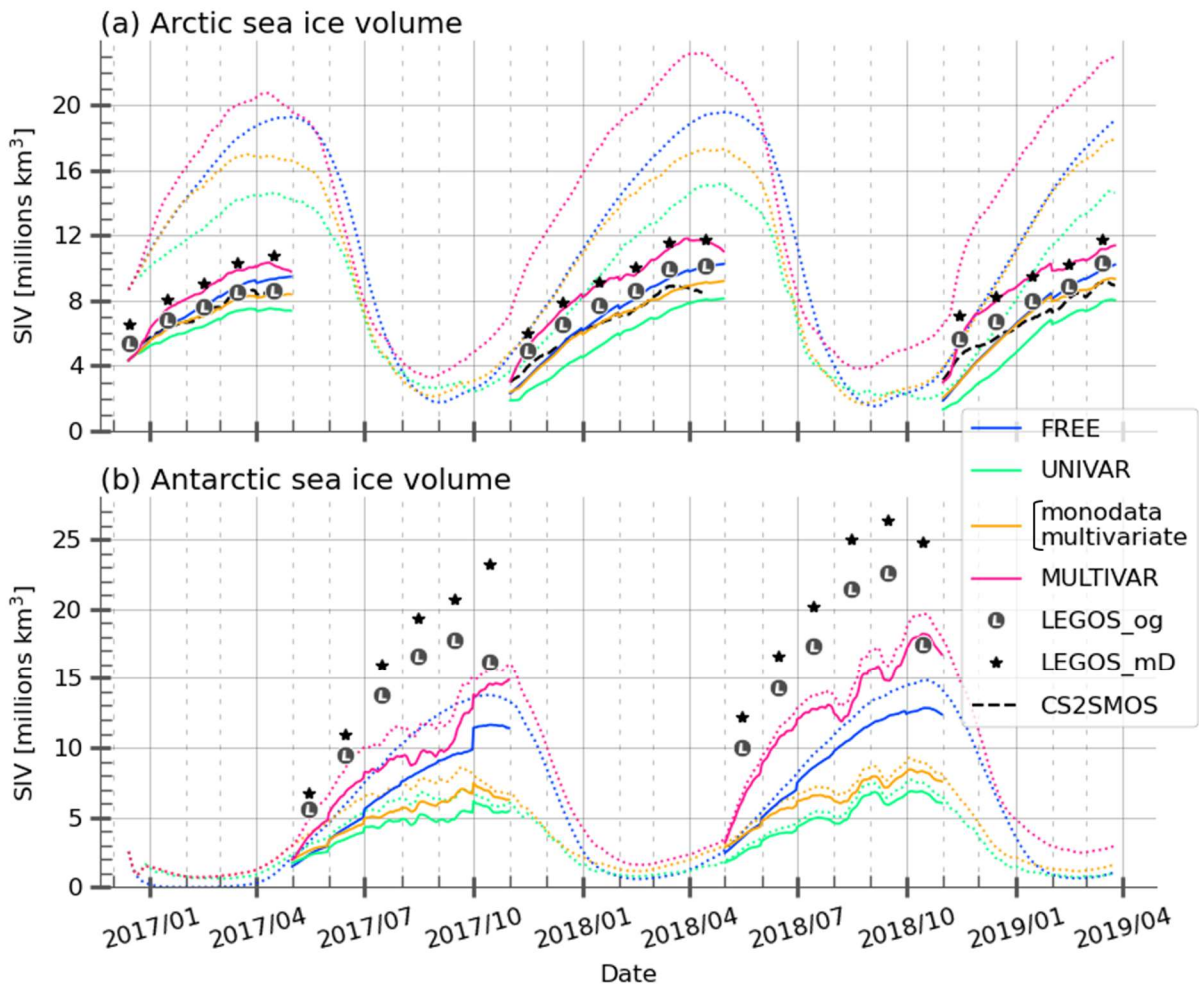
Figure S22: Same as Figure 7 in the paper. Time evolution of Arctic (a) and Antarctic (b) sea ice volume. The daily values are presented for the simulations FREE (blue), UNIVAR (green), monodata/mutivariate (orange) and MULTIVAR (pink), integrated over the whole h hemisphere (dotted) and over the observation domain (plain lines). SIV observations used for comparison are computed over the LEGOS observation domain: LEGOS original SIT (LEGOS_og, grey L in circles), SIT constructed from LEGOS observations of RFB and snow and the model constant ice and snow densities (LEGOS_mD, black stars), and CS2SMOS AWI data in the Arctic (black dashes). The SIVOLU is computed using either SIC data provided by the supplier or the SIC OSISAF SSMIS data.

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April 2017 Arctic sea ice volume

| | LEGOS zone | | | | Outside LEGOS zone | | | |
| | SIT > 1 m | | SIT < 1 m | | SIT > 1 m | | SIT < 1 m | |
	RMS	MD	RMS	MD	RMS	MD	RMS	MD
FREE	0.25	-0.01	0.46	0.39	0.44	-0.3	0.48	0.45
UNIVAR	0.32	-0.26	0.28	-0.0	0.65	-0.62	0.16	0.02
monodata multivariate	0.27	-0.11	0.26	0.11	0.51	-0.35	0.23	0.16
MULTIVAR	0.31	0.22	0.36	0.31	0.38	0.05	0.27	0.22
LEGOS_og	0.27	-0.12	0.35	0.32	-	-	-	-
LEGOS_mD	0.35	0.27	0.46	0.44	-	-	-	-
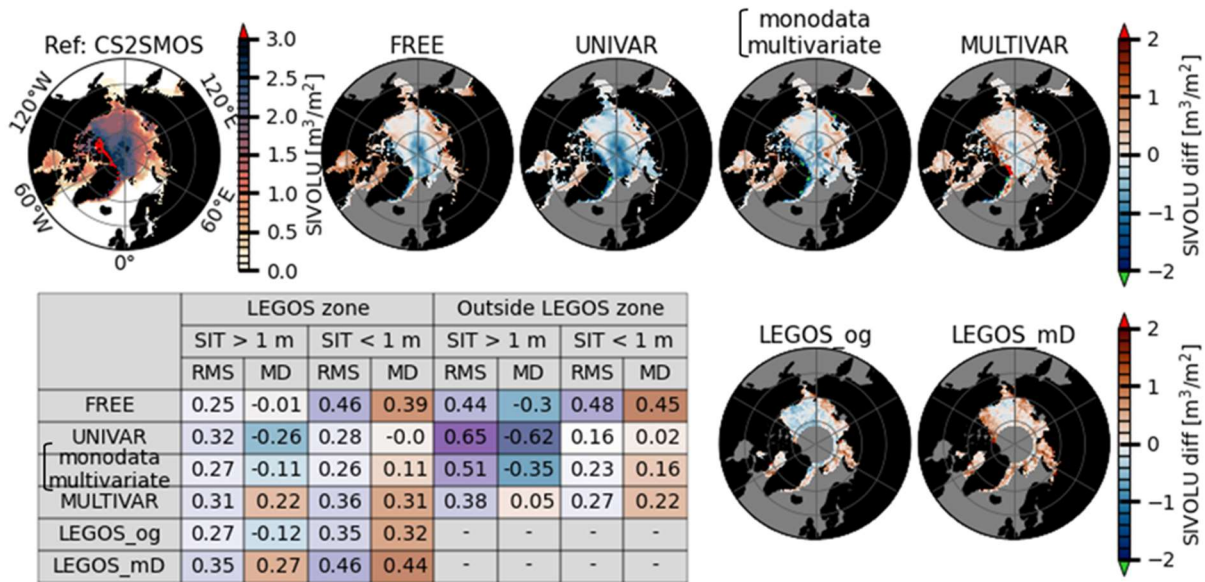
Figure S23: Same as Figure 8 in the paper. April 2017 sea ice volume in the Arctic for CS2SMOS dataset (reference) and its difference with the FREE, UNIVAR, monodata/multivariate and MULTIVAR experiments (first line) and the observations LEGOS_og (original) and LEGOS_mD (with model constant densities). Table: root mean square error (RMS) and mean difference (MD) between FREE, UNIVAR, MULTIVAR, LEGOS_og, LEGOS_md and CS2SMOS data, calculated on the LEGOS zone and outside the LEGOS zone and for CS2SMOS sea ice thickness of less than or greater than 1m. The table colours highlight the values close to 0 (white) and the extremes (green for the RMS, and blue/red for the negative/positive MD). The LEGOS zone corresponds to areas where the KaKu snow depth is available.

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September 2017 Antarctic sea ice volume, comparison with SMOS product

	LEGOS zone		Outside LEGOS zone	
	RMS	MD	RMS	MD
FREE	0.25	0.16	0.32	0.3
UNIVAR	0.26	-0.2	0.11	-0.03
monodata/multivariate	0.25	-0.12	0.11	-0.02
MULTIVAR	0.38	0.21	0.19	0.07
LEGOS_og	0.78	0.73	-	-
LEGOS_mD	0.98	0.94	-	-
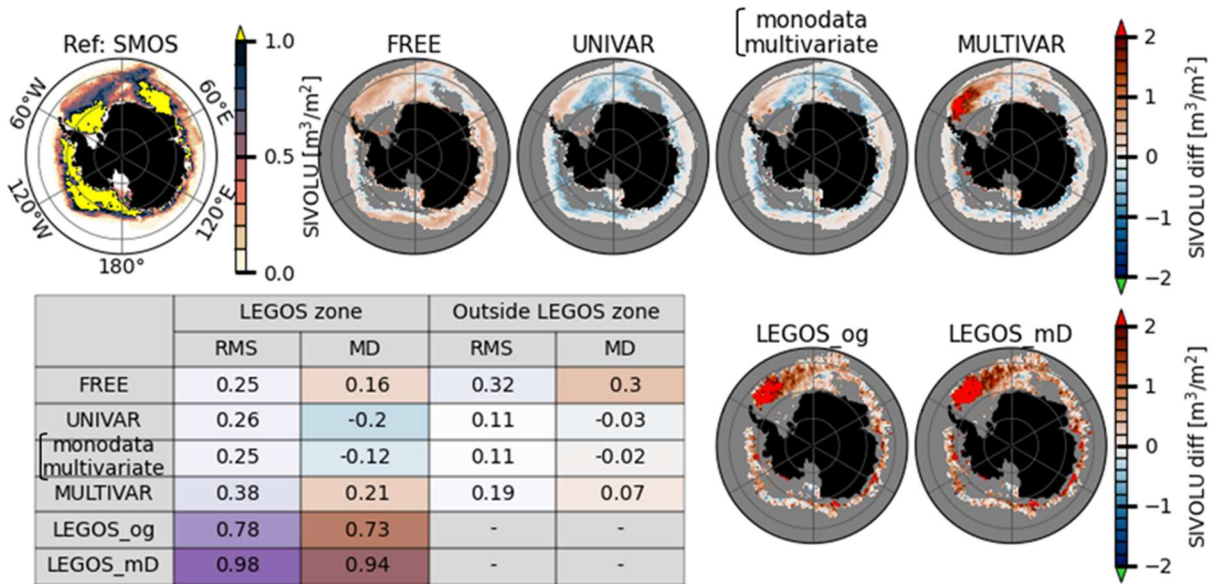
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Figure S24: Same as Figure 9 in the paper. September 2017 sea ice volume maps in the Antarctic for the SMOS data (reference) and its difference to the FREE, UNIVAR, monodata/multivariate and MULTIVAR experiments (first line) and to the observations LEGOS_og (original) and LEGOS_mD (with model constant densities). The colorbar shows only which only measures the ice that is thinner than 1 m (thicker ice is represented in yellow). Table: root mean square error (RMS) and mean difference (MD)
130 **between FREE, UNIVAR, monodata/multivariate and MULTIVAR, LEGOS_og, LEGOS_mD and SMOS data, calculated on the LEGOS zone and outside the LEGOS zone. The table colours highlight the values close to 0 (white) and the extremes (green for the RMS, and blue/red for the negative/positive MD). The LEGOS zone corresponds to areas where the KaKu snow depth is available.**

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