RC2

1 General comments

We would like to thank the reviewer for their careful reading of the manuscript and the helpful comments. The answers to the comments are written in bold, the reviewer's comment in normal font, and the sentences added to correct the manuscript in italic.

Review of "Assimilation of radar freeboard and snow altimetry observations in the Arctic and Antarctic with a coupled ocean/sea ice modelling system" by Chenal and co-authors.
I am mostly an expert of sea ice data assimilation, but not so much of altimeter remote sensing.

The manuscript presents "the first implementation of a multivariate sea ice assimilation scheme in both the Arctic and Antarctica within a global ¼° modelling and analysis system", as stated by the authors, which is correct to my knowledge, other comparable studies are cited in the text but, at much lower resolution.
The study takes a data assimilation system as used operationally and develops both a multivariate (multiple sea ice variables, though not ocean variables) assimilation scheme assimilating two new data sources on top of sea ice concentrations: radar freeboard and a novel snow thickness product. This represents an ambitious piece of work, of high technical complexity, associated with expensive computations.

The choice of satellite observations is of high standards and makes the experiments very relevant and timely in the present literature.
There are however some weaknesses that the authors should address before the paper is accepted for publication.
Thank you for your assessment of our paper. We answer each of the concerns in the following discussion.

The two assimilation experiments do not allow to evaluate separately the multivariate assimilation scheme from the effect of the assimilated data (multidata), as intermediate combinations of multivariate-monodata experiments would have allowed. The only exceptions are the locations and times when one or two input data types are missing. The paper is therefore shy on practical take-home messages. Rather than expensive additional experiments, the discussion of the benefits or weaknesses of the multivariate scheme could exploit better these special cases. In the present state of the manuscript, the two issues are mixed and hard to disentangle.
Thank you for this remark. Given the distinction monodata/multidata and monovariate/multivariate for the assimilation system, different sets of experiments could be done to evaluate the different system's impacts: monodata+monovariate (UNIVAR), multidata+multivariate (MULTIVAR), but also the in-between configuration monodata+multivariate. Only the first two configurations' results are presented in the manuscript.
However, another experiment with a monodata+multivariate configuration was performed: it consisted in assimilating the SIC OSISAF SSMIS product only (monodata) using the same multivariate assimilation system as the MULTIVAR experiment described in the manuscript (multivariate). As such, the information for the SIC SSMIS observation is propagated to the 5 increments (SIC, SIV, SNV, RFB, and SNT) by the Kalman filter, using the model covariances from the background error covariance matrix. We call this experiment Monodata/multivariate in this discussion. The authors chose not to present the results of this experiment as the manuscript is already long, we wanted to provide a clearer message as we believe that this experience does not add any value to the discussion in the paper. You can find in the supplementary materials (file: 2_Supplementary Materials) a short assessment of the Monodata/multivariate experiment, with illustrations. This experiment shows intermediate performances between monodata/univariate and multidata/multivariate for the sea ice concentration (Figure S16) and similar performance as monodata/univariate for the leads content (Figure S17). Similarly, this experiment provides intermediate performance for the RFB (Figures S18 and S19). The sea ice volume in this experiment increases the sea ice volume

compared to the monodata/univariate simulation but is significantly less than the multidata/multivariate experiment and far from the LEGOS altimetric observations, in both hemispheres (Figure S22). The sea ice volume estimate is significantly less with a mean of 6.75 km^3 and 5.74 km^3 for the Monodata/multivariate experiment in the Arctic and Antarctica respectively, compared to 8.73 km^3 and 10.80 km^3 for the MULTIVAR experiment, over the entire simulation period. The sea ice volume in Monodata/multivariate still increases the sea ice volume compared to the UNIVAR simulation, in both hemispheres. Further, RFB, SNV and SIV in this experiment have similar biases to those of the experiment without assimilation and the monodata/monovariate experiment. More work is needed to evaluate the impact of the monodata/multivariate method used here, which is another reason why we chose not to present this simulation in the paper.

We propose to add a few sentences in the methodology of the experiments in the paper to inform the reader that the Monodata/multivariate experiment was performed and that the results were not presented to avoid confusions,stay with the scope of the paper and because this experiment needs further investigations:

L.164: *"Assimilation systems can be described by the terms monodata or multidata, depending on the number of observations assimilated. Two different methods exist for the assimilation system: univariate and multivariate. They refer to the number of variables in the Kalman filter state vector, determining for which variables the increments are created. In a univariate configuration, the Kalman filter runs for each observation to create only one increment. In a multivariate configuration, multiple analysis increments are created at once, using the model covariances to simultaneously correct a number of variables in a coherent manner. Hence, different assimilation systems could be defined: monodata/univariate, monodata/multivariate and multidata/multivariate.*

L. 198: *the sentence "The different experiments presented in this paper show the evolution of the sea ice assimilation methods from a univariate and mono-data system, updating only SIC, to a multivariate and multidata setup." has been removed as the rationale for the methodology is explained in subsection 2.3.*

L. 237*:* *"To assess the impact of the multivariate and/or the multidata approach versus the more widespread SIC monodata/multivariate assimilation approach, we have considered the most relevant approaches that can be combined with a single-variety or multi-variety approach and the use of data in multi-data or single-data mode. We performed a monodata/multivariate experiment assimilating the SIC OSISAF SSMIS product only with the multivariate assimilation system described previously. The results of this experiment are presented in supplementary material (Section 2) to let the article focus on the major differences brought by the innovative multidata/multivariate configuration. We then restricted the study to the comparison of the results using the monodata/univariate and the multidata/multivariate configurations.*"

Second, some explanations are omitted, or left implicit, which hampers the fluidity of the arguments: I am for example missing a paragraph in the introduction that sets upfront what the experiments are expected to deliver (which improvements and which possible degradations) and what are yet open questions. In practical terms, this means that the research questions should be coming two paragraphs earlier in the introduction and should be linked more concretely to the results. The physical instability mechanisms of the Southern Ocean polynyas are also exposed too late in the paper and should have been given earlier in the introduction.

We agree and propose to add sentences, in the introduction part, on the possible degradation with the implementation of MULTIVAR given that covariances SIC/SIV are not necessarily informative. We also briefly mention the ice-ocean interactions and the existence of polynyas. L.107: "*Prior studies have shown that assimilating SIC alone significantly reduces concentration errors but yields limited improvement in ice thickness, despite strong correlations between both variables (Lisæter et al., 2003, Duliere and Fichefet, 2007). Up to now, as far as we know, no a priori links between SIC and the depth of the snow over sea ice have been studied. We therefore anticipate the following outcomes for each experiment: monodata/univariate assimilation should improve modeled SIC but may degrade SIT and SNT due to the necessary adjustment for SIV and SNV implemented in the analysis scheme (Table 2). Conversely, the multidata/multivariate assimilation is expected to better fit all assimilated variables (SIC, RFB, SNT), but may impact SIC accuracy due to uncertain SIC-SIT/SNT covariances. The different spatio-temporal resolutions of SIC, RFB, and SNT (e.g. daily gridded SIC vs. sparse altimeter tracks with seasonal gaps) may also introduce uncertainty into the impact of assimilation. Finally, few studies have focused on the constraints of the ice/snow system by assimilation in Antarctica, a region where the interaction between the ice and the upper ocean is much more dynamic than in the Arctic. In regions of open water surrounded by sea ice — known as polynyas — the ice-ocean interactions are particularly strong (e.g. Kjellsson et al., 2015, Cheon and*

Gordon, 2019) and difficult to reproduce by models (Mohrmann et al., 2021). The outcomes of the assimilation experiments could reveal whether improvements in SIC are offset by errors in SIT/SNT, how additional data sources interact, and how the scheme affects coupled ice–ocean behaviour."

Added references:

Cheon, W. G. and Gordon, A. L.: Open-ocean polynyas and deep convection in the Southern Ocean, Sci. Rep., 9, 6935, https://doi.org/10.1038/s41598-019-43466-2, 2019.

Dulière, V. and Fichefet, T.: On the assimilation of ice velocity and concentration data into large-scale sea ice models, Ocean Sci., 3, 321–335, https://doi.org/10.5194/os-3-321-2007, 2007.

Kjellsson, J., Holland, P. R., Marshall, G. J., Mathiot, P., Aksenov, Y., Coward, A. C., Bacon, S., Megann, A. P., and Ridley, J.: Model sensitivity of the Weddell and Ross seas, Antarctica, to vertical mixing and freshwater forcing, Ocean Model., 94, 141–152, https://doi.org/10.1016/j.ocemod.2015.08.003, 2015.

Lisæter, K. A., Rosanova, J., and Evensen, G.: Assimilation of ice concentration in a coupled ice–ocean model, using the Ensemble Kalman filter, Ocean Dyn., 53, 368–388, https://doi.org/10.1007/s10236-003-0049-4, 2003.

Mohrmann, M., Heuzé, C., and Swart, S.: Southern Ocean polynyas in CMIP6 models, The Cryosphere, 15, 4281–4313, https://doi.org/10.5194/tc-15-4281-2021, 2021.

The experimental setup may have omitted some information that is important to understand the results later on. I was missing a clear indication of where the RFB and KaKu snow data are assimilated until I found Figures 3 and 4. More importantly, I am missing an explanation of what the assimilation does in the absence of one or two of these datasets. Perhaps introducing the observations before the data assimilation method could make the logic more fluid. The authors do not lay out the limitations of the data assimilation method upfront, so the reader discovers them by surprise as the negative results appear. For example, when the MULTIVAR scheme deteriorates the SIC in Figure 2. I would expect - by elimination - that the assimilation of RFB is responsible for this bias. Since the multivariate (negative) correlation between RFB and SIC stems from a long model simulation, it could be that this long simulation is to be blamed. See for example Counillon and Bertino (2009) although in an ocean-only application (as an example, you do not have to cite this reference).

Thank you for your comment.

We agree and put the description of the assimilated data before the assimilation methodology. Moreover, a table (see Table 2 below) has been added to provide a clearer view of the availability, spatially and in time, of the assimilated observations. We have added the comments on Table 2 in the subsections describing SIC, RFB and snow thicknesses observations:

L. 203:*" Ocean and Sea Ice Satellite Application Facility (OSISAF) OSI-450 (OSI SAF, 2022) (Table 2)."*

L. 224: *"The data are only available during winter in both hemispheres, November to April in the Arctic and May to October in the Antarctic (Table 2). Apart from north of 88°N, CS2 satellite tracks cover the entire ice domain of each hemisphere in about a month: during each assimilation cycle, important areas remain unobserved, especially at lower latitudes (Antarctica)."*

L.230: *"The data are provided in monthly gridded files, available during the same winter periods as RFB, in each hemisphere (Table 2)."*

L. 231:" *Due to SARAL orbital characteristics, no data are available for latitudes higher than 81.5°N".*

L.199:

Observations	*SIC SSMIS*	*RFB-LEGOS*	*SNOW-KaKu*
Producer	*EUMETSAT OSI-SAF*	*LEGOS*	*LEGOS*
Temporal resolution	*Daily*	*20 Hz*	*Monthly → weekly (linear interpolation)*
Temporal coverage	*All-time*	*Winter: November to April in the Arctic; May to October in the Antarctic.*	
Spatial resolution	*40 km (effective resolution); 25 km (grid resolution).*	*Along-tracks*	*12.5 km (grid resolution).*
Spatial gaps	*None (reprocessed).*	*Central Arctic (latitude > 88°N); in-between satellite tracks.*	*Central Arctic (latitude > 81.5°N); coastal areas.*
nsel	*400*	*4000*	*400*

Table 2: Assimilated observation products and their specificities.

Figure below shows the full extent of the SAR/SARin instruments measuring regions used to get the radar freeboard data. The SNOW KaKu product that is assimilated in the MULTIVAR experiment only uses the SAR mode measurements, so snow depth along the coastlines is not available in that observation product. Moreover, the Ka radar measurements used for the SNOW KaKu product comes from the SARAL-AltiKa satellite, which is limited to 81.5°N due to its orbit.

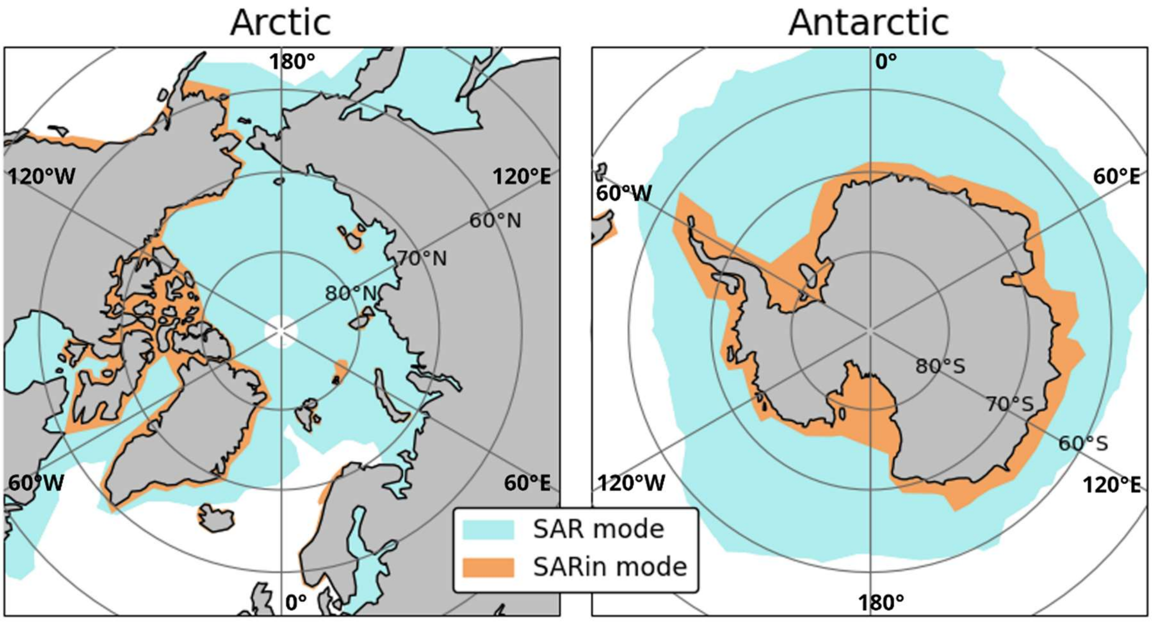

Figure RC2fig1: Geographical masks used for SAR and SARIn modes (version 5.0) for CryoSat-2 measurements in the Arctic and Antarctica.SAR and SARin modes activation regions in the Arctic and Antarctica. Data from ESA: https://earth.esa.int/eogateway/instruments/siral/geographical-mode-mask.

In the absence of one or two of the datasets, the assimilation system builds the increments using the covariances calculated from a long simulation. This long simulation is indeed biased (the detailed analysis of this long simulation is not presented in the paper). A sentence is added in the "Assimilation scheme" section to explain more clearly this process: L.171: "*Similarly, the model covariances are used to create increments even if the full set of observed data to be assimilated is not available. These covariances, constructed from a "long" free simulation, will therefore potentially spread the biases (partially presented in this paper) to all the variables concerned.***"**

Some of the minor design choices could have been better justified, for example the threshold on snow depths, which is not justified and comes back as a limitation later on.

Since submitting this article, an experiment has shown that the snow threshold is not responsible for the unexpected results obtained with snow in the UNIVAR simulation. We agree with your comment and decide to remove the comments related to this algorithm's small specificity. Specifically, we removed the sentences L.188: "*If the updated SNT exceeds a threshold defined as half the analysis SIT, it is capped to avoid unrealistic values. In such cases, the total snow volume may decrease compared to the forecast.***"; L.552: "***with a dynamic threshold on the SNT***", L.555: "***This result shows that our threshold is not appropriate in most of the Antarctic, and in some regions in the Arctic.***" and L.556: "***A modification of the SNT threshold would improve the snow assimilation algorithm in that sense.***"; L.736: "***analysis snow depth threshold***".**

The snow thickness is the difference between AltiKa and CryoSAT-2 RFBs, so it is not independent from the CryoSAT-2 RFB, whereas the assimilation system assumes observations errors are independent. Although I do not see an immediate solution for that, this may be acknowledged in Section 2.2.3.

That is right. A sentence has been added to the manuscript to account for this information L.235: "*Multiple processing are applied to the Ku-band CryoSat-2 measurements to create the SNOW-KaKu product: a degraded version of the SAR measurements (pseudo-LRM mode) is used to get a similar footprint as the SARAL-AltiKa measurements, a 25 km radius median smoothing is applied, and the data is gridded at a monthly frequency, as described by Garnier et al. (2021). However, the SNOW-KaKu product remains not fully independent from RFB-LEGOS measurements.***".**

The text puts the same emphasis on results that are trivial (i.e. that assimilation runs agree better with the assimilated data) and those that are less obvious. The discussion could flow more easily if the description of the experiments had a few sentences with the a priori expectations from each run.

See answer above with a paragraph added in the introduction part to set upfront what the experiments are expected to deliver (which improvements and which possible degradations). We also add these few sentences to put emphasis on the obvious results:

L.239: "

- *FREE: experiment without any assimilation, which has consistent biases in all sea ice variables due to model and forcing limitations, providing a baseline for evaluating the impact of assimilation.*
- *UNIVAR: experiment similar to the current operational system, using the previously described univariate SIC assimilation method. Assimilating SIC alone is expected to significantly reduce sea ice concentration errors but may induce unrealistic adjustments in sea ice thickness (SIT) and snow depth (SNT).*
- *MULTIVAR: experiment with the multivariate assimilation scheme described previously, assimilating SIC, RFB and SNT observations, and updating the SIC, SIV and SNV model variables. Assimilating multiple variables is anticipated to improve agreement with all assimilated observations (SIC, RFB, SNT), though possibly at the cost of reduced SIC accuracy and increased risk of numerical or dynamical imbalances, especially in a coupled ice–ocean model.***"**

And in the conclusion, we highlight the results that were not expected and highlight the results in a more positive manner:

L.719: "*Despite the heterogeneous nature and varying resolutions of the assimilated data sets, the multidata/multivariate assimilation system demonstrates robust behavior even in the absence of certain observations (summer, spatial hole), indicating a consistent and physically coherent adjustment of the sea ice state.*"

L.735, we replace the sentence "*The results for the southern hemisphere also show the strong interactions with the oceanic surface layers in the life cycle of the sea ice cover.*" **with** *"In the Southern Hemisphere, the results highlight the strong interactions between sea ice and the upper ocean layers. These interactions lead to complex impacts on polynya dynamics, which underlines the need for further investigation and the development of assimilation strategies that are better suited to these sensitive coupled environments.***".**

The case for assimilating RFB rather than SIT could be more conclusive. As long as the auxiliary data used in the RFB-to-SIT conversion are the same, there is no obvious benefit of assimilating RFB rather than a converted SIT product. The ice and snow density do not seem to take an active role in the sea ice model so belong in the observation operator. However the snow depth is a proper state variable, so I would expect that more accurate (satellite) snow depths justifies alone the assimilation of RFB. By opposition, the assimilation may lead to deteriorations of SIT in areas where the KaKu snow depths are missing, but I could not find any evidence of that in the manuscript.

Sentences in the introduction stress how the conversion RFB-SIT depends on the choice for alternative parameters such as the snow depth, and the ice and snow densities: L.66 "*The sea water, ice and snow densities and the snow depth above the ice are required for the RFB-SIT conversion, and the assumptions made on these variables result in a significant uncertainty in the sea ice volume products (Kern et al., 2015; Kwok and Cunningham, 2015). The snow layer accounts for most of the uncertainty in the calculation of SIT from RFB (Garnier et al., 2021).***"; L.79 "***Other sources of uncertainty in the RFB-SIT conversion stems from the choice of ice and snow densities.***".**

The choice of assimilating RFB rather than a SIT product comes from a desire to stay closer to the satellite measurement. Our experiment is a first step showing that direct RFB assimilation is possible, with a full high-resolution along-tracks product. No interpolation is needed, and the data is available at a sub-daily frequency since there is no post-processing of the measurements. Then, future studies will be able to build upon this framework and

implement varying snow and ice densities, and to reproduce the parameterization used in the observations for the model.

Thanks to a direct RFB assimilation, we can also constrain the snow depth separately, which profoundly affects the sea ice state. We hope, in the future, to assimilate the snow measurements along-tracks.

We did not compare these experiments to an experiment assimilating a SIT product.

Concerning the deterioration where snow KaKu data is missing, we noticed a very localized high bias in the north of Greenland, which could be due to the lack of snow observations in this area for the MULTIVAR simulation in April 2017 (Figure 3c), see L381 of the preprint. However, we did not notice any particular bias in the large snow KaKu observation gap around the North pole. We added the following sentences to clarify this finding:

L.585: "*However, the largest RFB differences between the MULTIVAR experiment and the RFB LEGOS assimilated observations are located on the north of the Canadian Archipelago and Greenland, with an especially thin RFB in our simulation locally north of Greenland. No snow observations are available in this area, and the MULTIVAR presents thicker snow values than the FREE and UNIVAR simulations. No particular RFB bias is present in the large snow KaKu observation gap around the North pole, suggesting that in the absence of snow observations, an inaccurate modelled snow depth does not affect the RFB assimilation performance on a large scale, but can result in higher RFB biases very locally.***"**

The manuscript overall structure is good, with a few exceptions noted above. However some repetitions could be avoided across different sections. The writing style is generally good and very careful, sometimes too careful to the point of becoming confusing. The statements are often too neutral, not indicating whether the results are as expected, good or bad. This hampers the reading of the paper.

We have taken care to reduce repetitions and improve clarity in the sentences that were modified or added in response to reviewers comments. In these revised sections, we made a deliberate effort to be more direct and explicit about the results.

The graphics are very clear and highlight well the main messages.

Thank you.

Overall I am very positive that the study makes an important and interesting contribution to the field and is worthy of publication in The Cryosphere, provided that the issues pointed out above are corrected in a revised version.

2 Detailed comments

- L40 Chen et al. 2024 is not in the Arctic but in an idealised square model.

Thank you. This citation was removed in L. 40: "*Experiments have used EnKF or variations of this multivariate scheme with multidata frameworks: both SIC and SIT products have been assimilated in the Arctic (e.g. Cheng et al., 2023; Williams et al., 2023)***".**

- L100 Since only RFB_mD is used in the experiments, it could be mentioned that RFB_og is only shown indicatively for users of the original product.

Thank you. The sentence has been added L. 101: "*Data using varying sea ice and snow densities are only shown in the figures indicatively for users of the original product.***"**

- L120 The section on the sea ice model does not mention the treatment of submerged sea ice. Since negative RFBs occur in the results it is necessary to know if the assimilative model will freeze submerged snow into more saline ice.

A sentence has been added L.132 to explain the process in the model: "*Snow exclusively comes from the solid precipitations of the atmospheric forcing and disappears either by melting processes or by snow-ice conversion when the snow base gets below the sea level. The model accounts for snow-ice formation when snow is deep enough to depress the* *snow-ice interface below the sea level. Then seawater infiltrates and refreezes into the snow, creating a new ice layer whose thickness depends on the ice and snow densities (Fichefet and Maqueda, 1997; Vancoppenolle et al., 2023).***"**

Added reference:

Fichefet, T. and Maqueda, M. A. M.: Sensitivity of a global sea ice model to the treatment of ice thermodynamics and dynamics, J. Geophys. Res. Oceans, 102, 12609–12646, https://doi.org/10.1029/97JC00480, 1997.

A sentence is included in the article to explain this negative RFB phenomenon L.214: "*Radar freeboard values can be negative because of the term accounting for the radar speed reduction in the snow layer: it is not a real physical distance contrarily to ice freeboard.***". We change it to: "***Radar freeboard measurements depend on the radar speed reduction in the snow layer and are consequently not physical measurements. The ice/snow interface is therefore not necessarily underwater when the RFB is negative.***". When a radar freeboard is negative, it is mostly due to a thick snow layer and a thin ice layer.**

- L135 The data assimilation scheme does not mention that the ensemble is static and indirectly indicates a "long simulation without assimilation". The - strong - assumption that the long simulation is representative of the model errors during the assimilation period should be stated explicitly, including an indication of the years of the long simulation.
The manuscript has been corrected L.146 to explain that "*The static anomalies are computed from a long simulation (2010-2020) without assimilation, using the same model configuration and parameters with respect to a 7-day running mean. This approach is based on statistical ensembles in which the ensemble of these anomalies is representative of the error covariances (Lellouche et al., 2013).***".**

- L148 Why the reduced grid for the ocean? Maybe this point is unimportant for the paper and should be omitted to simplify this section.
We agree to remove this precision. L.146 the sentence "*Anomalies are computed on a reduced grid for the ocean (1 out of 2 points) and on a full grid for the sea ice.***" has been removed from the manuscript.**

- L162 Indicate that assimilating the OSTIA freezing point temperature is intended to make the sea ice and ocean assimilation consistent with each other. If the freezing point temperatures in OSTIA and NEMO differ due to sea surface salinity, can the difference induce sea ice melt or freeze?
We didn't assess the impact alone of such assimilation of OSTIA. In the model physics, the SST is set to the freezing point to maintain thermodynamic equilibrium with the ice above. If SST is above (resp. under) the freezing point, a heat flux is estimated to melt (resp. create) ice at the bottom.

- L178 Why a Gamma distribution? What are its parameters?
Several results have shown a gamma distribution of ice thicknesses in the observations. We have applied it to the model distribution with the parameters k=2.0 and theta=0.4 for the gamma law, see the figure below. We have modified the text L.178: "*Then, the total ice concentration is redistributed into each existing thickness category using a Gamma-type distribution commonly found in observed measurements (Toppaladoddi et al., 2023; Petty et al., 2020). This chosen distribution (with parameters k=2.0 and theta=0.4) adds most of the increment to the middle and smallest thickness categories and less to the extreme categories."*

The following references have been added to the reference list:

Toppaladoddi, S., Moon, W., & Wettlaufer, J. S. (2023). Seasonal evolution of the Arctic sea ice thickness distribution. *Journal of Geophysical Research: Oceans*, 128, e2022JC019540. https://doi.org/10.1029/2022JC019540

Petty, A. A., Kurtz, N. T., Kwok, R., Markus, T., & Neumann, T. A. (2020). Winter Arctic sea ice thickness from ICESat-2 freeboards. *Journal of Geophysical Research: Oceans*, 125, e2019JC015764. https://doi.org/10.1029/2019JC015764

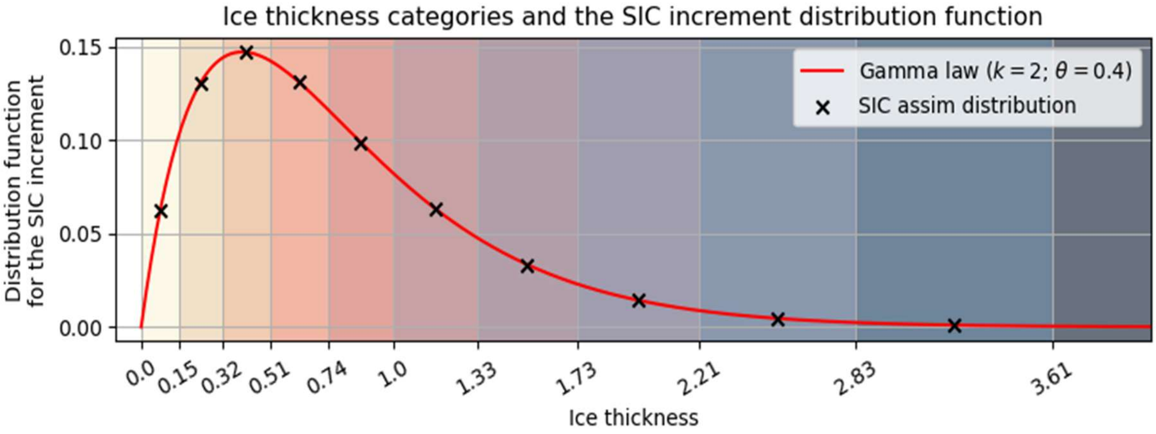

Figure RC2fig2: Sea ice categories, defined by the values [0., 0.15, 0.32, 0.51, 0.74, 1.00, 1.33, 1.73, 2.21, 2.83, 3.61, 99.00]; and the redistribution function for the SIC increment in the different ice categories.

- L183 Are there any cases when the SIT exceeds the bounds of its thickness category? How is that handled by the model?
In the assimilation algorithm, in the case of a positive SIV increment, the increment is added first to the thinnest ice category, increasing its thickness until it reaches the upper limit. Then, the remainder of the increment is added to the next category, and the algorithm keeps doing so until the whole SIV increment is used. Similarly, in the case of a negative increment, the algorithm begins with removing ice from the thinnest ice category, and when there is no more ice in said category, it moves up a thicker category to continue removing ice until the increment has been entirely used up. A sentence describing this process is added L.183: "*If the change of thickness of a category exceeds its bounds, any excess or deficit in volume is transferred to the next thicker category, and this redistribution continues until the entire SIV increment is applied.***"**

- L189 This threshold should be explained since I did not expect it to become important later. This is especially mysterious since it reduces snow depths that are otherwise allowed in the forward model.
The discussion about the snow threshold has been removed in the new version of the manuscript since you rightly said that it was unclear and since a new experiment showed no impact of this threshold on the simulation's results.

- L235 I am not sure what this statement refers to. By construction the SNV depends directly on SIC and on SNT so the Kalman filter should estimate it well, unless the long free simulation has a pathological behaviour.
Yes, that is true, we chose to remove the sentence L.235: "*It is important to note that the snow volume increment depends on all the assimilated data and reflects how well the volume correlates with them.***".**

- Tables 1 and 2 are very welcome to summarise the complex information, but Table 2 could include a little more information like the seasons and latitudes at which the different observations are available, which would prepare the reader to what happens when and where RFB or snow are not assimilated.
A new table (see *Table 2* below) has been added to the manuscript. The different resolutions and data gaps of the 3 assimilated products are described in this new table.

Observations	*SIC SSMIS*	*RFB-LEGOS*	*SNOW-KaKu*
Producer	*EUMETSAT OSI-SAF*	*LEGOS*	*LEGOS*
Temporal resolution	*Daily*	*20 Hz*	*Monthly → weekly (linear interpolation)*
Temporal coverage	*All-time*	*Winter: November to April in the Arctic; May to October in the Antarctic.*	

Spatial resolution	*40 km (effective resolution); 25 km (grid resolution).*	*Along-tracks*	*12.5 km (grid resolution).*
Spatial gaps	*None (reprocessed).*	*Central Arctic (latitude > 88°N); in-between satellite tracks.*	*Central Arctic (latitude > 81.5°N) ; coastal areas.*
nsel	*400*	*4000*	*400*

Table 2: Assimilated observation products and their specificities.

- L255 Ivanova et al. compared 11 algorithms. Coming down to 2 is not really quantifying the uncertainty. There are weaknesses of PMW observations in the summer and different choices of tie points made at OSI-SAF and NSIDC to cope with them. Maybe they provide a lower and upper bound rather than an uncertainty estimate.

The lower and upper bound part might be true, but I did not check whether or not these products actually estimate the maximum and minimum values. We modify the sentence to describe it as "*showcase the variety of estimates in the different observation products***".**

- L270 The SSMIS data is assimilated, not the NSIDC, so it should be expected that the results agree better with SSMIS.

It is the free simulation that compares favourably with NSIDC, perhaps by chance. It also shows that observations can have very different estimates.

- L307 Between 0.04 and 0.13 is rather vague, can you provide the number?

The exact RMSE mean value of 0.08 has been added in the manuscript.

L.307: *"During the other months, the RMSE of 0.08 for the MULTIVAR simulation is lower, falling between the mean RMSEs of the UNIVAR and FREE simulations, which are 0.04 and 0.13, respectively."*

- L318 "Unobserved polynyas" This sounds like the blame is on observations, unless you mean "too large polynyas"? An oceanographic discussion of the ice-ocean conditions for polynyas should precede this sentence but is only coming near the end of the paper.

Here, the word "unobserved" was meant to point out that even if the polynyas do not appear in the observations, they could still exist (because the observations may be too low resolutions or may have missed the detection of the open waters) or they could be an artifact of the model (this often happens, see for instance Mohrmann et al. (2021)). To be clearer, the sentence was modified to "*In the Antarctic, the two assimilated experiments generate a large number of polynyas which are not detected bythe satellite observations, with the MULTIVAR experiment showing them more frequently and broadly across the region (Figure S2). While some smaller polynyas may go undetected in the observational data, the modelled polynyas are likely overestimated.***".**

- L324 Shouldn't it be expected that the assimilated run matches the assimilated data?

We mitigate this statement L.324: "*The MULTIVAR snow distribution is logically very close to the Arctic SNOW-KaKu during winter…*"

- Comparing Figure 3 to Figure 2, I see that the deterioration of SIC in the summer happens outside of the KaKu snow data coverage, so it appears to be caused by the assimilation of RFB. Since the free model underestimates the RFB, I expect that the positive innovation of RFB turns into a negative increment of SIC. Is it so that the RFB is somehow negatively correlated to the SIC in the summer covariances, and is that model-based correlation trustworthy?

The SIC deterioration area in the Arctic summer for the MULTIVAR experiment is actually larger than the orbital gap of the snow KaKu observation (see figure below). We did not analyze the correlations between the different variables in our simulations, so we are not able to be conclusive on this question. The RFB indeed seems negatively correlated to the SIC in the Arctic summer. The model based-covariances are however biased since they are based on a long (2010-2020) free simulation. This long simulation has issues maintaining the ice cover in the northern hemisphere in summer, with a sea ice extent that falls below the one of the SSMIS OSISAF observations. Hence, the model-based correlations in the summer may not be reliable.

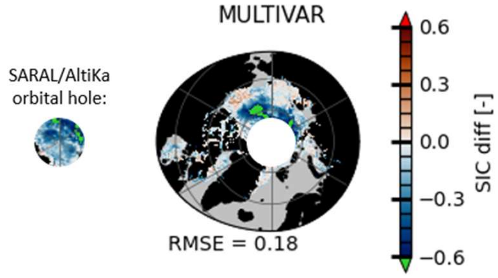

Figure RC2fig3: Same as Figure 2 of the paper, only keeping the MULTIVAR experiment's result, highlighting which areas are covered by the KaKu snow depth measurements (right) and which are not (left).

- L355-360 If the FREE run has excessive sea ice cover, does it also receive snow that should have fallen in open water? Does that explain why UNIVAR does better?

We assume that this comment refers to excess snow in the FREE experiment and not ice cover. This is indeed the case, as the FREE experiment has a larger surface area covered by ice than the UNIVAR experiment, so it intercepts more snow precipitation. We have not quantified this effect.

- L365 Conversely, is the missing snow falling into too large simulated polynyas?

Yes, for the same amount of snowfall, MULTIVAR should intercept less snow because of the presence of open waters. A regional study would be needed to quantify this effect.

- L370 With local data assimilation, one would expect that the spatial distribution is respected.

Yes, the sentence L.370 has been modified from : "*The assimilation of SNT is also able to rapidly modify the snow spatial distribution in accordance with the SNOW KaKu observations distribution.***" to "***While a localized assimilation scheme is expected to modify the spatial distribution of the variable to match the observations, it is noteworthy that the assimilation of SNT leads to rapid corrections, with most spatial biases already reduced within the first month of the simulation (Figure S3).***".**

- L376 The small bias should be expected for the assimilated observation.

Yes, this statement has been mitigated to account for that remark L.376 "*The MULTIVAR simulation logically exhibits a very small bias of -0.5 cm in the assimilated region and a RMSE of 2.2 cm.***"**

- L380 The observation errors should be given earlier in Section 2.2.2.

The observation errors are not constant. We have added this precision in the section 2.2.1, 2.2.2 and 2.2.3 as follows:
L..206:" We *use the daily- and spatially-varying "standard_error" provided with the dataset to construct the observation error for the assimilation …***"**
L. 217:" We *use the uncertainty provided for each track as the observation error, constraining it to a range of 0.01 m to 5 m.***"**
L. 232:" The *observation error used in the analysis comes from the monthly varying uncertainty supplied with the data, constrained to an arbitrary range of 0.01 m to 5 m.***"**
The values given L.380 refer to the mean values of the discussed month, namely April 2017. The sentence "*The SARin data are assimilated with higher observation errors compared to SAR data, with mean values of 19.2 cm and 9.2 cm, respectively.***" refers to the monthly mean values, averaged over the northern hemisphere ice-covered area. The term "typically" is added to emphasize that the error varies for each measurement: "***The SARin data are provided with higher observation errors compared to SAR data.***"**

- L383 More positively skewed. Explain why the values of a sea ice variable like RFB should be skewed.

The positive skewness of the RFB distribution arises because sea ice freeboard typically has a long tail toward thicker ice, particularly in regions where thick, deformed ice (e.g., ridges) persists. The sea ice thickness is also positively skewed (Toppaladoddi and Wettlaufer, 2023), and this property of the sea ice thickness is reflected in the RFB measurements. This results in a positively skewed probability density function, which is more pronounced in the MULTIVAR experiment due to its improved representation of spatial variability and persistence of thick ice structures even in the absence of assimilated RFB observations in summer.
The sentence is modified as follows, L.383: "*In summer, when no RFB observations are assimilated, the probability density function of the MULTIVAR RFB values remains more positively skewed than in other simulations, reflecting the persistence of localized thick ice areas among generally thinner ice.***"**

- L385 Biases cancel off if you average enough of them. I don't think that biases can be compared to observation error standard deviations.
Yes, we decided to remove the comment, L.385 "*still below the mean observation error***".**

- L392 Is the ice submerged when the RFB is negative?
Not necessarily, see the answer above.

- L410 The comparison to ICESat-2 is interesting but could be better introduced. For example, is the ICESat-2 total freeboard physically the same as the LEGOS (AltiKa) total freeboard? Indicate upfront that ICESat-2 is also available in the summer.
Physically, the total freeboard measured by IceSat-2 is the same as the one measured by the SARAL/AltiKa satellite: their beams reflect on the snow-air interface. However, the measurement technique differs: IceSat-2 uses laser altimetry while SARAL/AltiKa uses radar altimetry.
The beginning of paragraph 4.1 is modified L.411: "*Both ICESat-2 (Ice, Cloud and Land Elevation Satellite) and SARAL/Altika satellites measure total freeboard but the first one with a laser altimeter (Markus et al., 2017), the second one with a radar altimeter. However, the ICESat-2 product presents a smaller orbital hole (88° latitudinal limit) and a full-year availability, starting from the 14th of October 2018. The monthly ICESat-2 NSIDC ATL-20 gridded along-tracks product (Petty et al., 2023) is used on Figure 5, as a scatterplot between its total freeboard values and the total freeboard collocated in time and space for the LEGOS data and the FREE, UNIVAR and MULTIVAR experiments in the Arctic."*
Reference to add: Markus, T.; Neumann, T.; Martino, A.; Abdalati, W.; Brunt, K.; Csatho, B.; Farrell, S.; Fricker, H.; Gardner, A.; Harding, D.; et al. The Ice, Cloud, and land Elevation Satellite-2 (ICESat-2): Science requirements, concept, and implementation. ***Remote Sens. Environ.*** **2017,** ***190*****, 260–273.**

- L410 Can the validation of ICESat-2 separate cases of bare ice from snow-covered ice? Could this validate the RFB alone instead of the more ambiguous RFB+snow?
We are not aware of the existence of such a product derived from ICESat-2 measurements, does it mean a use of a different snow product to isolate bare ice and snow-covered ice?

- L414 "the constant densities": do you mean the model densities or other constants?
We mean the model constant densities. We added this in the manuscript to avoid confusion L414: "*The LEGOS total freeboard is made using LEGOS RFB and SNOW-KaKu data, and the constant water, ice and snow densities of the model.*"**.**

- L416, 418, 422: should we expect that MULTIVAR yields better statistics?
This is expected when MULTIVAR results and LEGOS data are compared but not necessarily when MULTIVAR and ICESat-2 are compared. So, we mitigate the following sentences: L. 415: "*The MULTIVAR simulation and LEGOS data present anticipated similar linear correlation statistics (slopes and r-values), MULTIVAR has then logically better statistics than the FREE and UNIVAR experiments, MULTIVAR simulation and the LEGOS data have similar mean RMSE compared to ICESat-2 data (6.7 cm and 7.2 cm respectively) and the MULTIVAR simulation and LEGOS data also display comparable mean total freeboard in January-February 2019, with values of 22.2 cm and 22.0 cm respectively, slightly thinner than the ICESat-2 estimate of 23.7 cm.***"**

- L420 Why does UNIVAR have lower freeboard than the free run?

The lower total freeboard in the UNIVAR experiment compared to the FREE run during January–February 2019 in the Arctic is due to both reduced snow depth and reduced sea ice thickness in the UNIVAR simulation. As shown in the scatterplots below, total freeboard is calculated from the modeled snow and ice volumes per unit area, each weighted by its respective coefficient. In these plots, the individual contributions of snow and ice are shown in red and green, while their sum (the total freeboard) is shown in black. For nearly all cases, both the snow and ice components are thinner in UNIVAR than in FREE, leading to the observed reduction in total freeboard.

We added the precision L.420 "*due to thinner sea ice and snow cover in the UNIVAR experiment***" in the sentence.**

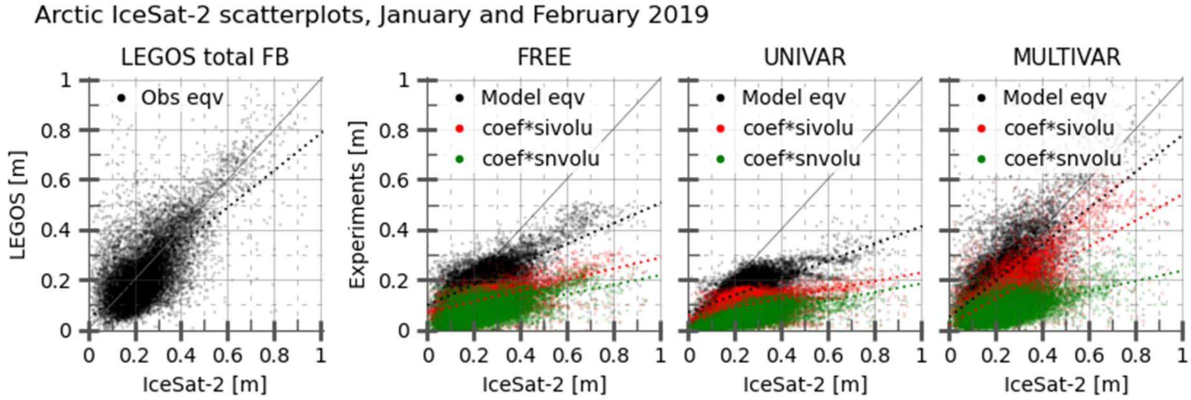

Figure RC2fig4: Scatterplots of the monthly Arctic ICESat-2 total freeboard against FREE, UNIVAR, and MULTIVAR experiments for January and February 2019. The total freeboard model equivalents are computed following the equation:

$$\boldsymbol{Model\ eqv} = [1 - \frac{\rho_{ice}}{\rho_{water}}] * \boldsymbol{sivolu} + [1 - \frac{\rho_{snow}}{\rho_{water}}] * \boldsymbol{snvolu} \qquad \textbf{(Equation RC2eq1)}$$

Each member of the equation is shown on the scatterplot: black dots represent the $\boldsymbol{Model\ eqv}$; red dots the quantity $[1 - \frac{\rho_{ice}}{\rho_{water}}] * \boldsymbol{sivolu}$; and green dots the quantity $[1 - \frac{\rho_{snow}}{\rho_{water}}] * \boldsymbol{snvolu}$. The x=y line (full grey line) and linear regressions (dashed lines) are shown for each quantity in their corresponding color.

- L425 Do you have insights whether this is thanks to the update of SIV or to the snow that insulates the ice?

From the figure shown above, updates of snow are rather similar in both UNIVAR and MULTIVAR; the update of SIV in MULTIVAR is the major component of the total freeboard change compared to UNIVAR.

A sentence is added in the article: "*The change in the total freeboard modelled by the MULTIVAR experiment is mainly due to a larger SIV, thanks to the assimilation update, compared to the UNIVAR experiment.***"**

- L427 I don't have an impression of how good a spatial correlation of 0.6 is. Maybe remove this statement.

We agree and removed the statement L.427: "*All the experiments exhibit correlations higher than 0.6 reflecting a general consistency with ICESat-2 total freeboard in terms of spatial distributions.*"

However, we decide to emphasize the better statistics of MULTIVAR experiment's slopes in both hemispheres and for all represented periods by changing the sentence L.441 to "*The MULTIVAR shows a favourable systematic increase of the slopes in winter as in summer.***", and place it in the L.444 to apply it to both hemispheres.**

- L438 "unobserved ice-free zones" again implies observations are wrong, while the model obviously has excessive ice.

The FREE simulation has a large deficit of sea ice cover during summer in Antarctica, we changed the sentence L. 438: "*The FREE experiment again has large unrealistic ice-free zones with total freeboard values at 0 cm.*"

- Figures 5 and 6: Should we expect that the summer freeboard is lower than the winter?

Yes, it is expected that the total freeboard is lower in summer than in winter. As shown in Equation RC2eq1, total freeboard is a linear combination of sea ice thickness and snow thickness, each weighted by coefficients dependent on

their respective densities. During summer, both sea ice and snow melt, leading to a reduction in their thicknesses. Snow cover may also disappear entirely in some regions. Since both components contribute positively to the total freeboard, a seasonal decrease in either the ice or the snow results in a lower total freeboard in summer.
However, the figures 5 and 6 emphasize that the total freeboard is more reduced in the summer in our experiments than in the ICESat-2 measurements. See for instance the sentences L.426: "*However, compared to ICESat-2, MULTIVAR still underestimates the thickness of the total freeboard at the end of Arctic summer.*" **and L.437:** "*The melting season (January-February 2019) highlights the excessive thinning of the total freeboard in the simulations compared to the ICESat-2 data.*".
We add the sentence L.423: "*In summer, total freeboard has decreased during the melting season; however, the thinning is more pronounced in our simulations than in the ICESat-2 observations which does not seem to show a reduction in the mean freeboard compared with winter.***" to better describe these phenomenons.**

- L493 Please clarify that the LEGOS and CS2SMOS data are not completely independent due to the use of CryoSat-2.
We changed the next sentence L. 494: "Based on CS2 measurements, the LEGOS_og logically displays a consistent sea ice thickness spatial distribution compared to the CS2SMOS product …"

- Figure 8: That the MULTIVAR is performing best in the polar hole is interesting. But I am confused by the notion of "LEGOS zone", is it the zone where RFB data or KaKu data is available?
The "LEGOS zone" refers to the region where both the RFB and KaKu snow depth products are available. The spatial extent of this zone is primarily constrained by the KaKu product, which is limited to 81.5°N due to the orbital inclination of SARAL/AltiKa and the coverage of CryoSat-2's SAR mode (excluding SARin mode). In contrast, the RFB product covers a broader region, extending up to 89°N and closer to coastlines, thanks to the use of CryoSat-2's SARin mode. Therefore, the KaKu product defines the common overlap area used in this study.
The precision is added to complete the L.460: "*Figure 7 also presents the experiments collocated within the spatial coverage of the assimilated observations, which excludes the central Arctic orbital gap and limited coverage of marginal seas (solid lines).*"**, we added: "***This area, where both the RFB and KaKu data are available, is hereafter referred to as the 'LEGOS zone' or the 'LEGOS observations domain'.***". We also added in the caption for Figures 8 and 9: "**The LEGOS zone corresponds to areas where the KaKu snow depth is available.**".**

- L520 Clarify that LEGOS and SMOS data are independent.
We added the sentence L.518: "*In Antarctica, the SMOS product (Tian-Kunze and Kaleschke, 2021) detects ice thinner than 1m using brightness temperature measurements, hence the data is completely independent from the LEGOS altimetric data assimilated in the MULTIVAR experiment.***".**
Added reference:
Tian-Kunze, X. and Kaleschke, L.: SMOS-derived sea ice thickness in the Antarctic from 2010 to 2020, https://doi.org/10.1594/PANGAEA.934732, 2021.

- Figure 9 has masked the areas where the SMOS data is saturated. So, it is difficult to see where the "LEGOS zone" stops. Is it at 80S latitude?
The LEGOS zone includes the areas where both the RFB and the KaKu snow depth are available. In Antarctica, this corresponds to the SAR mode mask only (see Figure RC2fig1), which excludes the SARin mask defined over a wide band along the Antarctic coast.

- L545 The section is inconclusive because the authors have not indicated which of LEGOS or SMOS data is more realistic. In the Antarctic, the SMOS thickness increases gradually into the ice, which seems more realistic to me, but I would appreciate the authors' view on this.
We thank the reviewer for this thoughtful comment. Indeed, this section shows the current uncertainty in the realism of available sea ice thickness products in Antarctica. The SMOS product may appear more physically consistent, because of the gradual increase of thickness into the ice pack. However, this product is relatively recent (described as a 'preliminary product'), and its validation over the Antarctic remains limited due to the scarcity of independent

reference data (Kaleschke et al., 2024). The LEGOS products show some agreement with ICESat-2 observations, but both rely on altimetry-based retrievals and may share similar biases.
L.630, we add a sentence to explain better this point: "*While the assimilation improves the agreement between the assimilated products, the contrasting patterns seen in LEGOS and SMOS sea ice thickness highlight the current observational uncertainty in Antarctica, making it difficult to assert which product more accurately represents the true state of the sea ice.*"

- L552-557 This passage could be less contorted if the pros and cons of a threshold was stated upfront (see comment above).
Following a previous comment, all sentences regarding the snow threshold in the assimilation algorithm has been removed from the manuscript.

- L559 Even if the snow melts away completely in the summer, the timing of the snow melt also affects the melting of the sea ice below, which can hold a longer memory than the snow itself.
We agree that, in principle, the timing of snow melt can influence the evolution of sea ice through its effect on surface albedo and insulation. However, in our experiments, we do not find direct or conclusive evidence linking changes in the timing of snow melt with a corresponding shift in sea ice evolution. Moreover, the assimilation of snow observations does not appear to significantly alter the timing of snow melt in our simulations. We have revised the manuscript to reflect this nuance more clearly.
In the manuscript, the sentence L.559: "*The snow cover completely melts in summer in each hemisphere and shows no long-term effect of the winter snow assimilation.*" **is modified to "***The snow cover completely melts in summer in both hemispheres, and while the timing of melt should influence the sea ice evolution, our results do not indicate a persistent or clearly attributable long-term impact of the winter snow assimilation.*".

- L566 Is it unrealistic that FREE and UNIVAR have submerged sea ice?
A negative radar freeboard does not necessarily imply that the sea ice is physically submerged. It reflects the relative distribution of snow and ice thicknesses under hydrostatic equilibrium, and the slower velocity of the radar wave when penetrating into the snow, meaning that it is not a real distance (see previous answers). The concern in our simulations is not the negative sign per se, but the fact that the radar freeboard is significantly more negative than observed values, suggesting an imbalance in the modeled snow–ice system. Diagnostic analyses indicate that this underestimation stems from a combination of an overlying thick snow cover (particularly in the FREE experiment) and a too thin sea ice, both of which contribute to lower modeled radar freeboard compared to observations.
The revised sentence is the following, L. 566: "*In the Antarctic, the RFB is significantly underestimated in the FREE and UNIVAR simulations, reflecting an imbalance between snow and ice thicknesses: the snow cover is too thick and the sea ice too thin, resulting in radar freeboard values that are more negative than observed.***".**

- L586 This is the first time that the authors mention what happens in the absence of KaKu snow observations, and in indirect wording. See previous recommendation to lay the special cases in plain sight earlier in the description of experiments.
We modified the manuscript regarding this issue thanks to the previous comments.

- L587 The snow and RFB are indeed related both by Equation 1 and by the model dynamics, it would be interesting to see what this relationship becomes in the long model simulation.
We have not explicitly studied these relationships, neither in free mode nor in assimilated mode, to understand the impact of the observations on these physical relationships. This is a study that really needs to be undertaken.

- L622 Why not assimilate SMOS data as well?
Thank you for the suggestion. The primary reason SMOS data were not assimilated is our aim to apply a consistent assimilation system across both hemispheres. While SMOS and CryoSat-2 sea ice thickness products are reasonably consistent in the Arctic and have been validated over several years, their agreement is much weaker in the Antarctic, where discrepancies between the two measurements remain significant. Assimilating both products without a clear

understanding of their relative reliability in the Southern Ocean could introduce inconsistencies or reinforce biased signals. That said, assimilating complementary data sources such as SMOS for thin ice and CryoSat-2 for thick ice is indeed a promising direction; once further validation and cross-comparison efforts improve our confidence in the Antarctic observational products.

Added sentence L.630: "*In the future, the system could also assimilate both CryoSat-2 (for thick ice) and SMOS (for thin ice) products in both hemispheres, provided that Antarctic sea ice thickness estimates have greater consistency and agreement.*".

- L650 "mostly the MYI" should be "only the MYI", right?

Thank you for this observation. While it is true that the model uses a constant ice density equivalent to that of FYI in the observational dataset, the observed ice density varies spatially as a function of the proportion of FYI and MYI within each grid cell. Therefore, the impact of using a constant model density is most significant in regions dominated by MYI, but not exclusively limited to them — since even MYI-rich cells generally contain some fraction of FYI, which influences the cell-averaged density. For this reason, we chose to describe the affected regions as "mostly the MYI regions."

Modified sentence L.650: "*Hence, assimilating radar freeboard and snow with the model constant ice density primarily affects regions dominated by MYI in the Arctic*".

- L655 Is 895 kg/m3 the model or observation density?

This is the observation's mean ice density. The model density is 917 kg/m3.

The sentence is changed to L.655: "*The model's ice density (constant 917 kg/m3) exceeds that of the LEGOS_og observations (895 kg/m3 in average), with a particularly significant difference in October.***".**

- L666 the "physical accuracy" is misleading since the densities are only used for calculating the RFB in the output.

While it is true that the densities are used diagnostically for calculating the RFB model equivalent in the observation operator, they also play a more active role in the physical model itself. For example, the ice and snow densities are involved in parameterizations such as snow–ice formation processes, which are especially relevant in the Antarctic. Therefore, implementing seasonally evolving densities could improve not just the consistency of the assimilation system, but also the realism of physical processes in the model.

We have modified the sentence L.666: "*Moreover, implementing seasonally evolving ice and snow densities in the model could improve the realism of key physical processes such as snow–ice formation, particularly in the Antarctic.*"

- L675 is another instance when multivariate covariances are discretely mentioned. This sentence could imply for example that the SIV and SNT are incorrect in summer and then feedback negatively to the SIC, but the authors should clarify if this is the case.

We indeed already mentioned the impact of multivariate covariance on SIC during summer in the modified text L.171.

In MULTIVAR, the SIC observations are used not only to update SIC but also to generate increments in SIV and SNV via model-derived covariances. The resulting SIV and SNV increments (though unverified because there are no RFB and snow observations in summer) appear to degrade SIC, suggesting that the covariances used to propagate SIC information to other variables may be inaccurate or less reliable during summer. Alternatively, the structure of the increments or the dynamical model's sensitivity could also contribute to the degradation. Without direct observations of SIV or SNV in summer, we cannot isolate the exact cause, but the degradation of SIC performance in MULTIVAR highlights the need to improve or seasonally adapt these multivariate relationships.

Modified sentence L.673: "*The degradation of modelled SIC in summer in the MULTIVAR configuration, while UNIVAR uses the same SIC observations, suggests that the multivariate assimilation may introduce erroneous corrections through model covariances between SIC, SIV, and SNV. These propagated increments, applied in the absence of direct summer observations of SIV or SNV, appear to deteriorate SIC consistency, highlighting the need to reassess or seasonally adapt the covariances used in the assimilation.***".**

- L682 Indicate whether these open waters are correct or artificial.
The open water features present in the multivariate experiment are not detected in the assimilated SIC SSMIS observations and appear too large and persistent to be considered realistic. While we cannot confirm their presence in the high-resolution RFB data (as we have only examined averaged fields) we note that this product is capable of detecting polynyas that are missed by the SSMIS SIC product. Therefore, it is possible that some of these features are consistent with RFB data, although we cannot assert this definitively. Since the RFB product is assimilated in the MULTIVAR experiment, its influence may also contribute to the emergence of these unrealistic polynyas in the model.
Modified sentence L.682: "*The multivariate experiment shows an even higher presence of open waters than the UNIVAR experiment during the peak boreal summer. The scale and duration of these features are not supported by the assimilated SIC SSMIS observations and are likely artificial. The assimilated along-tracks RFB data is capable of detecting finer-scale polynyas that are not visible in the coarser SIC SSMIS product, but the direct link with the modelled polynya has not yet been established.***"**

- L687 Indicate the inversion of ocean temperatures below the ice.
Yes, thank you for this input. We change the sentence to L.687: "*As none of these openings occur in the FREE experiment, the thick snow and ice layer likely insulates the ocean from the atmosphere, maintaining the temperature inversion beneath the ice and limiting oceanic heat flux toward the ice base.***".**

- L699-705 These assimilation settings are repeated from earlier methods section. It would be more logical to indicate first there that these settings are different from the default in order to mitigate the appearance of polynyas.
The description of the changes in SST and in situ assimilation has been updated accordingly to explain why we modified the original set up of the oceanic assimilation system. We then modified L.161 in paragraph 2.1.2: "*The ocean observations are not assimilated under the sea ice in the original operational system. Following experiments to set up the new ice assimilation system, instabilities in the water column appeared in the Southern Ocean. To reduce these static instabilities, we activated the OSTIA SST assimilation under the ice to maintain the ocean temperature at the freezing point. We also stopped assimilating in situ data to the south of 60°S, regardless of the season, because the surface thermohaline properties were being durably modified on large spatial scales, despite the few profiles present. Assimilating these in situ data modified ocean stratification, causing upwellings of warm water at the surface and creating unrealistic open water areas within the sea ice cover.*"
We removed the paragraph L.698-704: "*Changes have been implemented in the assimilation system to mitigate the occurrence of these simulated polynyas (see paragraphs 2.1.2 and 2.2.1). The SST assimilation under the ice has been activated to keep the surface waters close to the freezing point. Very few in situ profiles are available in the Southern Ocean, and some of them were radically changing the thermohaline properties of the ocean in a large area and over a long period of time, thus we did not activate the in-situ profile assimilation poleward 60°S to keep the modelled ocean stratification. We increased the maximum SIC observation error to 40% to moderate the intensity of sea ice assimilation in the Southern hemisphere***".**
We then modified the sentence L.704: "*The modifications in the assimilation scheme of the SST and in situ profiles described in section 2.1.2 have reduced the likelihood of triggering polynyas in both UNIVAR and MULTIVAR simulations but has not been able to prevent their occurrence.***".**

- L744 CIMR should also include SMOS-like measurements of thin ice thickness.
A sentence has been added in the conclusion to convey this information: "*CIMR will also provide thin ice estimates from L-band radiometry, similar to SMOS.***".**

- L796 Have the moorings data been used to calibrate the LEGOS product or are they strictly independent?
To my knowledge, the BGEP moorings measurements have not been used to calibrate the LEGOS products.

- L811 Is an ice draft of 0 m synonymous of open water?

Yes.

3 Typos and grammar

The English can be improved by paying attention that verbs, adjectives and nouns are chosen carefully for clarity.

- L21-23: This sentence is too contorted, avoid the "seem" for something that should be certain and rephrase the last clause.
The sentence has been modified to L.21-23: "*The multivariate system performs better in the Arctic than in Antarctica where the ice and ocean separate analyses are not designed to handle properly the strong interactions between upper oceanic layers and sea ice cover in the Southern Ocean.*"

- L38 "The EnKF".
We modified it in the newer version of the manuscript.

- L62: Incomplete sentence, or remove "that".
"that" is removed.

- L79: Replace "stems" by stem (according to "sources") and remove the bold 's'.
Yes, thank you.

- L84: Times disagreement between "stated" and "recommend".
We kept the past tense with "*recommended***" to stay coherent with the rest of the verbs.**

- L315 "higher presence of leads" -> "larger lead area"
Yes, thank you.

- L376 biase -> bias
Yes, thank you.

- L436 "mostly" here is misleading, I believe you mean "UNIVAR is underestimating [...] freeboard values the most".
Yes, thank you.

- Figure 5 caption has swapped the dotted and dashed lines.
Yes thank you.

- Figure 7 caption: use SIV instead of SIVOLU.
Yes, thank you.

- Figure 7 caption: SIC "either from the supplier or from OSI-SAF", for which product?
For the AWI-CS2SMOS product, there is a SIC variable provided with each file and we use it in Figure 7. The LEGOS files do not provide a SIC variable, hence we use the OSISAF SSMIS SIC to compute sea ice volume. The sentence is modified to "*SIV is computed using either SIC data provided by the supplier (CS2SMOS SIV) or the SIC OSISAF SSMIS data (LEGOS SIV).***".**

- L495 "coherent" is slightly misleading, do you mean "consistent" or "similar"?
Yes. Changed to "*consistent***".**

- L509 "fewer" should be "lower".

Yes, thank you.

- L552: "aims at keeping" -> "keeps"?
Yes, thank you.

- L581 "demarcation" is unclear: do you mean the edge of the observations domain?
Yes, thank you.

- L655 "a particularly pronounced discrepancy", is it much higher or much lower?
The LEGOS ice density is much lower than the model ice density. See Figure RC2fig5 below: the model's ice density is n917 kg/m3; and the LEGOS ice density is 875 kg/m3 in October.
The sentence is modified to: "*The model's ice density (constant 917 kg/m*^{3}*) exceeds that of the LEGOS_og observations (895 kg/m*^{3} *on average), with a particularly significant difference in October (LEGOS value: 875 kg/m*^{3}*).***".**

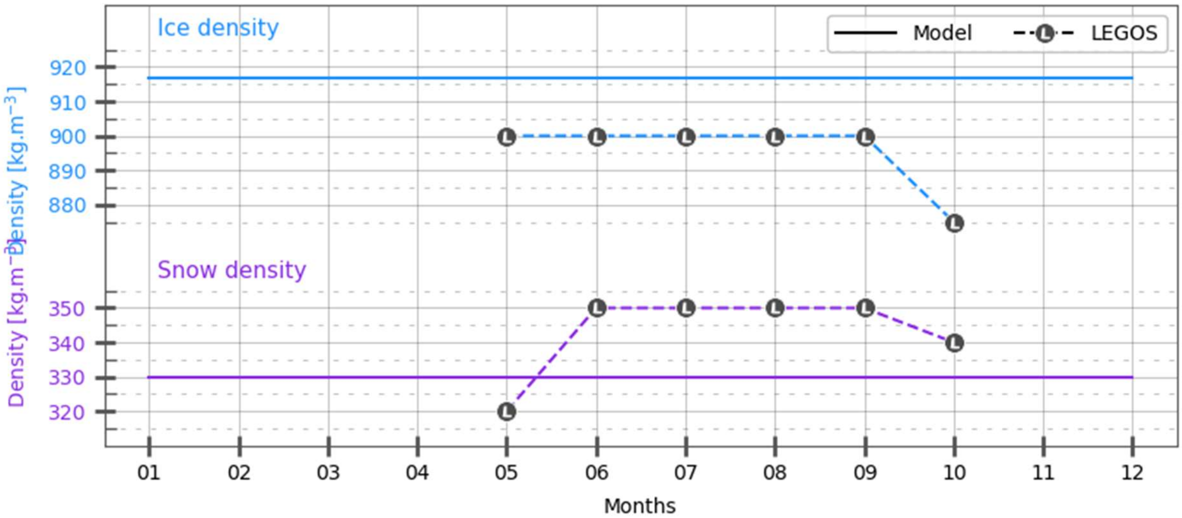

Figure RC2fig5: Sea ice and snow densities in Antarctica for the LEGOS products (circles grey Ls and dashed colored line) and for the model (horizontal colored lines).

- L697 ... in places where the equilibrium of the model ...
Yes, thank you.

- L699, 700 Enumerate "First, ... Second, ..." for clarity.
This paragraph has been removed following a previous comment.

- L718 Iceat-2. Also capitalise ICESat-2 consistently throughout the paper.
Yes, thank you.