Consider the shallow ice approximation on a flat bed with zero accumulation, which can be written as

$$\frac{\partial H}{\partial t} = \nabla \cdot \left[\frac{2A(\rho g)^n}{n+2} H^{n+2} |\nabla H|^{n-1} \nabla H \right], \tag{1}$$

where $H(x, y, t)$ is the ice thickness, A and n are the flow-law coefficient and exponent, ρ is the ice density, and g is gravitational acceleration. The corresponding horizontal velocity field $\mathbf{u}(x, y, z, t)$ and vertical velocity $w(x, y, z, t)$ are given by

$$\mathbf{u} = -\frac{2A(\rho g)^n}{n+1} |\nabla H|^{n-1} \nabla H \left(H^{n+1} - (H-z)^{n+1} \right), \qquad w = -\int_0^z \nabla \cdot \mathbf{u} \, \mathrm{d}z. \tag{2}$$

Equation (1) has an axisymmetric similarity solution given by

$$H(r,t) = H_0 \left(\frac{t}{t_0} \right)^{-\frac{2}{5n+3}} \left[1 - \left(\left(\frac{t}{t_0} \right)^{-\frac{1}{5n+3}} \frac{r}{R_0} \right)^{\frac{n+1}{n}} \right]^{\frac{n}{2n+1}}, \tag{3}$$

in which H_0 and R_0 are the height and radius at time t_0, which are related by

$$t_0 = \frac{1}{5n+3} \left(\frac{2n+1}{n+1} \right)^n \frac{n+2}{2A(\rho g)^n} \frac{R_0^{n+1}}{H_0^{2n+1}}. \tag{4}$$

This is what is referred to as Halfar's solution. The corresponding velocity field can be calculated from (2), with the horizontal velocity $\mathbf{u} = u\mathbf{e}_r$ being in the radial direction. To simplify the algebra, it is helpful to scale the variables, by writing

$$t = t_0 \hat{t}, \quad r = R_0 \hat{r}, \quad H = H_0 \hat{H}, \quad z = H_0 \hat{z}, \quad u = u_0 \hat{u}, \quad w = w_0 \hat{w}, \tag{5}$$

where $u_0 = 2A(\rho g)^n H_0^{2n+1}/R_0^n$ and $w_0 = (H_0/R_0)u_0$. In the dimensionless (hatted) variables, the similarity solution (3) becomes

$$\hat{H} = \frac{1}{\hat{t}^{\frac{2}{5n+3}}} \left[1 - \left(\frac{\hat{r}}{\hat{t}^{\frac{1}{5n+3}}} \right)^{\frac{n+1}{n}} \right]^{\frac{n}{2n+1}}, \tag{6}$$

and (2) becomes

$$\hat{u} = \frac{1}{n+1} \hat{S}^n \left(\hat{H}^{n+1} - (\hat{H} - \hat{z})^{n+1} \right), \qquad \hat{w} = -\int_0^{\hat{z}} \left(\frac{\partial \hat{u}}{\partial \hat{r}} + \frac{\hat{u}}{\hat{r}} \right) \mathrm{d}\hat{z}, \tag{7}$$

where we have written the surface slope as

$$\hat{S} = |\nabla \hat{H}| = -\frac{\partial \hat{H}}{\partial \hat{r}}. \tag{8}$$

From differentiating (6), this slope is given by

$$\hat{S} = \frac{n+1}{2n+1} \frac{\hat{r}^{\frac{1}{n}}}{\hat{t}^{\frac{(3n+1)}{n(5n+3)}}} \left[1 - \left(\frac{\hat{r}}{\hat{t}^{\frac{1}{5n+3}}} \right)^{\frac{n+1}{n}} \right]^{-\frac{n+1}{2n+1}} = \frac{n+1}{2n+1} \frac{\hat{r}^{\frac{1}{n}}}{\hat{t}^{\frac{1}{n}}} \frac{1}{\hat{H}^{\frac{n+1}{n}}}. \tag{9}$$

We can then calculate form (7), after a little algebra, that

$$\frac{\partial \hat{u}}{\partial \hat{r}} + \frac{\hat{u}}{\hat{r}} = \left(\frac{2}{n+1} \frac{\hat{S}^n}{\hat{r}} + \frac{\hat{S}^{n+1}}{\hat{H}} \right) \left(\hat{H}^{n+1} - (\hat{H} - \hat{z})^{n+1} \right) - \hat{S}^{n+1} \left(\hat{H}^n - (\hat{H} - \hat{z})^n \right), \tag{10}$$

and, after performing the integral, that

$$\hat{w} = -\left(\frac{2}{n+1} \frac{\hat{S}^n}{\hat{r}} + \frac{\hat{S}^{n+1}}{\hat{H}} \right) \left[\hat{H}^{n+1} \hat{z} - \frac{\hat{H}^{n+2} - (\hat{H} - \hat{z})^{n+2}}{n+2} \right] - \hat{S}^{n+1} \left[\hat{H}^n z - \frac{\hat{H}^{n+1} - (\hat{H} - \hat{z})^{n+1}}{n+1} \right]. \tag{11}$$

The dimensional velocities are obtained by multiplying $\hat{u}$ and $\hat{w}$ by the scales u_0 and w_0 respectively.