Dear Editor,

Thanks for handling our revised manuscript. We appreciate your contribution towards the improvement of the manuscript. Below is our point-by-point response to your comments and also to the reviewer's comments.

Response to Editor Comment

Comment 1: The authors have not included references and discussion of external (solar wind) causes of SSWs, only internal waves. Such waves have never been shown to cause SSWs, only theoretical speculation.

Response to comment 2: Thanks for handling our revised manuscript. We appreciate your contribution towards the improvement of the manuscript. As suggest, we have included the following references

1. Liu, N., Jin, Y., He, Z., Yu, J., Li, K., and Cui, J.: Simultaneous Evolutions of Inner Magnetospheric Plasmaspheric Hiss and EMIC Waves Under the Influence of a Heliospheric Plasma Sheet, Geophys. Res. Lett., 49, 1–12, https://doi.org/10.1029/2022GL098798, 2022.
2. Tsurutani, B. T., Hajra, R., Tanimori, T., Takada, A., Bhanu, R., Mannucci, A. J., Lakhina, G. S., Kozyra, J. U., Shiokawa, K., Lee, L. C., Echer, E., Reddy, R. V., and Gonzalez, W. D.: Heliospheric plasma sheet (HPS) impingement onto the magnetosphere as a cause of relativistic electron dropouts (REDs) via coherent EMIC wave scattering with possible consequences for climate change mechanisms, J. Geophys. Res. Sp. Phys., 121, 10,130-10,156, https://doi.org/10.1002/2016JA022499, 2016.
3. Salminen, A., Asikainen, T., Maliniemi, V., and Mursula, K.: Dependence of Sudden Stratospheric Warmings on Internal and External Drivers, Geophys. Res. Lett., 47, 1–9, https://doi.org/10.1029/2019GL086444, 2020.

we also include in the "INTRODUCTION SECTION" of the revised manuscript line 44-56: SSW is one of the usual meteorological events, where the stratospheric temperature increases rapidly in the winter polar region due to the rapid growth of quasi-stationary planetary waves from the troposphere (Baldwin et al., 2021; Butler et al., 2015). The connections between the troposphere and stratosphere during SSW introduces upward wave energy propagation that can reshape the plasma density variability in the ionosphere. The dominant mechanism facilitating the connection of these processes include planetary waves, atmospheric tides, and gravity waves (Goncharenko et al., 2021; Chau et al., 2012; Goncharenko et al., 2010, 2012; Liu et al., 2010). Also, other external sources that could facilitates SSWs are Heliospheric Plasma Sheet (HPS) and magnetosphere interaction. This occur when HPS impinges on the magnetosphere, compressing it and accelerates protons, which in turn generate Electromagnetic Ion Cyclotron (EMIC) waves.

These waves interact with relativistic electrons, causing their rapid loss to the atmosphere, potentially affecting climate mechanism (Tsurutani et al., 2016; Liu et al., 2022)

Finally, we also include in the "DISCUSSION SESSION" of the revised manuscript 550-555: the assertion that the external sources could contribute to the evidence of ionospheric chaos during SSW in this form: Another contributing factor to the emergence of ionospheric chaos could be the Heliospheric plasma sheet's interaction with the Earth's magnetosphere which lead to significant magnetospheric processes like Relativistic Electron Dropout (RED) and EMIC wave generation. These processes have downstream effects on the lower atmospheric activities, potentially influencing weather pattern and climate through energy deposition and ionization changes (Tsurutani et al., 2016; Salminen et al., 2020).

Comment 2: The fact that referee 1 has pointed out your error/bias on identifying low Kp values for a particular interval and now preferentially deleting high Kp days is very troubling.

Response to comment 2: In the results and figures of the revised manuscript. We have returned our findings on the days of high kp whose index value exceeds 3 to plots and shown in Figures 7-14 of the revised manuscript. Also, we acknowledge by listing those days in the revised manuscript those days whose kp index value exceed 3 in Manuscript Line 527-543 as: Interestingly, during the 2009 SSW periods, the daily geomagnetic activities were mostly characterized by a planetary index of $K_p \leq 3$. However, we notice some certain days during the SSW periods, namely SSW 2009 (January 3, 19, 26, February 4, 14-15, 25, 28, March 13, and 24) and SSW 2021 (December 10, 21, 23, January 5, 6, 11, 24, 25, 27, February 2, 6, 12, 16, 20, 22, and 23), where there was a transient increase in the K_p values exceeding 3 within the three-hour intervals of K_p index. Notably, these transient emergence of K_p index exceeding 3 within few hours during SSW periods have a tendency of inducing a mild geomagnetic disturbance during the SSW period, but may not be strong enough to be a dominant factor to influence ionospheric instability during SSW. The reason is that SSW events occur on a longer time scale, i.e. number of days compare to emergence of sudden geomagnetic disturbance within few hours. The contributing influence of SSW will be more significant than the transient disturbance of geomagnetic activities. While, we acknowledge that there is some transient geomagnetic disturbance revealed by K_p index during SSW periods. To address these possible geomagnetic disturbances indicated by the K_p index, all our daily solar quiet current derivation analysis during the SSW periods, we implement the subtraction of the geomagnetic storm index in minutes (SYM-H) from the H-component of the magnetic field to minimize the influence of geomagnetic disturbances.

Comment 3: The authors need to do a further major revision of the paper to bring it to a more objective viewpoint for the readership of NPG. Otherwise, the paper will be rejected.

Response to comment 3: We have return back to the revised manuscript, our findings on the deleted days of high kp value and explain about it in the manuscript.

Comment 4: I will consider accepting the paper even if the initial hypothesis is found to be incorrect. A negative result is as important as a positive result.

Response to comment 4: We have revised the manuscript thoroughly as suggested.

Comment 5: The authors have considered internal atmospheric waves as the source of the SSWs. They should mention that nothing special about these waves were noted during the SSWs. The authors should mention (and even test?) that there could be external sources that could cause SSWs. One such source can be found in JGRSP, 121, 2016 doi:10.1002/2016JA022499. There are others as well but their suggestions were generally related to geomagnetic activity. Please give a balanced review for the NPG readership

Response to comment 5: We have included with references in the "INTRODUCTION SECTION" and "DISCUSSION SECTION" of the revised manuscript. The external sources that could cause SSWs.

Response to Reviewer One

Comment 1: The introduction covers important topics but has some key issues that reduce its clarity and impact. The main concepts, like the effects of SSW (Sudden Stratospheric Warmings) on the ionosphere and the role of nonlinear dynamics, are repeated too often, making the text feel redundant.

Response to comment 1: The introduction section of manuscript has been completely redrafted. The repetitive statements of the SSW effects on the ionosphere and the role of nonlinear dynamics have been removed.

Comment 2: The structure is also unclear, as ideas like chaos theory and its applications are introduced suddenly without enough explanation or connection to the previous points. Additionally, there are too many citations in a short space, which makes the text harder to follow. To improve, I suggest focusing on the main research question, organizing the ideas in a clearer order, and reducing repetitions. This will make the introduction more concise, engaging, and easier to understand.

Response to comment 2: Thank you for your suggestions towards the improvement of the manuscript. In the introduction section of the revised manuscript, chaos theory and its application to ionospheric current system are now better motivated, embedded and explained. The citation clarity has also been addressed. Finally, the introduction section of the revised manuscript is now concise and easier to understand.

Comment 3: Example: Lines 93-94. "*The research question of the contribution of SSW formation to the regional ionosphere across the European-African sector needs special attention*."

This statement needs justification. Why is this region of particular interest? Does it have any unique characteristics? What additional insights can we gain by analyzing the response of the Sq current during an SSW event in this specific region?

Response to comment 3: The justification why the Africa-European sector need special attention has been included in the revised manuscript line 63-69 as: "The regional ionosphere of European and Africa sectors manifest pronounced ionospheric variability in response to SSW events. For example, proximity to the geomagnetic equator in Africa could lead to different responses compared to higher latitude regions in Europe. This phenomenon provides a unique opportunity to investigate the complex coupling mechanisms between the stratosphere and ionosphere. Specifically, it enables the study of atmospheric wave propagation and its impact on the ionosphere, which can lead to disruptions in satellite communication and navigation system in the region."

Comment 4: Characterization of 2009 and 2021 Major SSW Events

Lines 162-190. It would be better to specify the source of the data used in Figures 1, 2, 3, and 4 at the point of introduction, rather than three pages later.

Response to comment 4: The source of the data has now been included in the Figures 1, 2, 3 and 4 of the revised manuscript.

Comment 5: Additionally, the handling of the data and the subsequent claims in this section are problematic. Let me elaborate:

• The Kp index is a planetary three-hour index, yet the values shown in Figures 3 and 4 appear to represent daily averages. This discrepancy should be explicitly addressed in the text, clearly stating that the reported values are likely daily averages and ensuring the associated error is indicated. This clarification is critical, as Kp values can vary significantly throughout the day due to geomagnetic disturbances.

Response to Comment 5: In our initial submission, the K_p index was plotted as daily averages. However, in the revised manuscript, we have replotted the K_p variation considering three-hourly values K_p index and acknowledging those days with K_p indices exceeding 3. For instance, we notice that some days in SSW 2009 (January 3, 19, 26, February 4, 14-15, 25, 28, March 13, and 24) and SSW 2021 (Dec 10, 21, 23, Jan 5, 6, 11, 24, 25, 27, Feb 2, 6, 12, 16, 20, 22, 23), have a sharp increase in K_p values exceeding 3 within few three-hourly interval.

Comment 6: In both selected periods, the Kp index frequently exceeds a value of 4, which makes the claim that "*During the 2009 SSW event (January–March), the planetary index (Kp) recorded values of Kp<2+*" inaccurate and misleading. The actual Kp trend for this period contradicts this statement. A similar issue arises in the second period, where the authors state that Kp remains below 3+. However, during the latter part of February, for example, Kp consistently reaches or exceeds 4.

Response to comment 6: In the revised manuscript, specifically lines 156-189, we acknowledge certain days during the SSW periods, namely SSW 2009 (January 3, 19, 26, February 4, 14-15, 25, 28, March 13, and 24) and SSW 2021 (December 10, 21, 23, January 5, 6, 11, 24, 25, 27, February 2, 6, 12, 16, 20, 22, and 23), where there was a transient increase in the K_p values exceeding 3 within the three-hour intervals. In lines 166-170 of the revised manuscript, it is stated: "During the 2021 SSW event (December 2020 to February 2021), the planetary $K_p$index, illustrated in Figure 4, generally showed quiet geomagnetic conditions ($K_p \leq 3$) for most days, with solar flux activity around F_10.7~100. However, on 17 specific days (December 10, 21, 23, January 5, 6, 11, 24, 25, 27, February 2, 6, 12, 16, 20, 22, 23), elevated K_p index values (> 3) were observed. Notably, these transient emergence of K_p index exceeding 3 within few hours during SSW periods have a tendency of inducing a mild geomagnetic disturbance during the SSW period, but may not be strong enough to be a dominant factor to influence ionosphere instability during SSW. The reason is that SSW events occur on a longer time scale, i.e. number of days compare to emergence of sudden geomagnetic disturbance within few hours. The contributing influence of SSW will be more significant than the transient disturbance of geomagnetic activities. While, we acknowledge that there is some transient geomagnetic disturbance revealed by K_p index during SSW periods.

To address these possible geomagnetic disturbances indicated by the K_p index, all our daily solar quiet current derivation analysis during the SSW periods, we implement the subtraction of the geomagnetic storm index in minutes (SYM-H) from the H-component of the magnetic field to minimize the influence of geomagnetic disturbances. This observation is explained in the discussion section of the revised manuscript line 525-543

Comment 7: This inconsistency is significant, as it undermines the claim that both periods were characterized by particularly low geomagnetic activity (e.g., Kp<2+). Moreover, the assertion that there was a meaningful difference in geomagnetic activity between the two periods is not supported by the actual Kp trends. This issue must be addressed to ensure the accuracy and reliability of the study's conclusions.

Response to comment 7: This misunderstanding has been clarified in the revised manuscript line 525-543. We acknowledge and listed those days in the revised manuscript, where the K_p index exceeding 3 was noticed. As a result of this, all our derivation of solar quiet current, we implement the subtraction of the geomagnetic storm index in minutes (SYM-H) from the H-component of the magnetic field to minimize the influence of geomagnetic disturbances.

Comment 8: 2. Data Acquisition and Method of Analysis

Table 1 presents the magnetic stations used in this study, including both geographic and magnetic coordinates for each station. It is important to note that magnetic coordinates are not fixed over time; they depend on the position of the magnetic pole. Therefore, when reporting them in a table, the reference year should always be specified.

Response to comment 8: In the revised manuscript line 179-180, the geomagnetic reference year has been included as **"The geomagnetic coordinate reference year for the stations listed in Table 1 is 2009."**

Comment 9: This implies that the stations selected in 2009 and 2021 have different magnetic coordinates. Furthermore, when studying the Sq current, the correct coordinates to use are the magnetic ones, not the geographic coordinates. As shown in Figure 5, the stations are not aligned with respect to magnetic coordinates. This discrepancy should be taken into account for accurate analysis.

Response to comment 9: In the revised manuscript, the map of the study presented in Figure 5 has been replotted using magnetic coordinates.

Comment 10: Lines 236-243: This paragraph describes the source of the data used and introduced earlier. The paragraph should be revised to include information about the data source at the point where the data is first presented and described.

Response of comment 10: The above suggestion has been incorporated in the revised manuscript.

Comment 11: 2.1 Ionospheric Solar Quiet Current Sq(H) as Observational Time Series

Lines 242-243: The authors state that they use the H component of the magnetic field. However, there is an issue: some of the observatories used, such as KIV and SOD–possibly others, although I have not verified–provide data in the X, Y, Z reference system. This suggests that either the authors performed a data rotation, transforming the coordinates from the (X, Y, Z) system to the (H, D, Z) system, a process which they did not mention, or they simply calculated the modulus of the H component as √(x2 + y2). In this case, they are not working with the H component itself, but rather its intensity. This procedure needs to be explicitly explained. Additionally, where the H component is directly provided, the specific steps taken in processing the data should be clarified.

Response to comment 11: In our previous manuscript, we calculate the H-component by simply estimating the modulus as $\sqrt{X^2 + Y^2}$. Following the suggestion of the reviewer, the estimation of the H-component was re-analyzed by transforming the coordinates of the magnetic data provided in X, Y, Z reference system to the (H, D, Z) geomagnetic system using Rotation Matrix method. We added an explanation in the revised manuscript line 206-209.

Comment 12: Lines 245-268: • Honestly, I am not convinced by the method used to derive the Sq variation from measurements of the horizontal component of the magnetic field. A more standard approach would be to use for example the CHAOS model, which enables the reconstruction of the magnetic field at a specific point in space over time, such as at the location of the observatory. One of the latest version of the CHAOS model can accurately simulate all components of the magnetic field, including secular variation, the crustal field, induced fields, and fields generated by magnetospheric currents. The only component it fails to model is the ionospheric field. Therefore, by simply subtracting the modeled value of H from the real value, the ionospheric field can be obtained, from which the Sq component can then be derived.

Response to Comment 12: In the revised manuscript, the derivation of the solar quiet current was re-analyzed. Following the suggestion, we implemented the CHAOS-8.1 magnetic field model by subtracting the modeled H-component values from the observed H-component magnetic data, thereby obtaining the ionospheric field. The solar quiet current was then derived from this ionospheric field.

At the end of implementing the CHAOS-8.1 and obtaining the solar quiet current. We present a comparison of the 2009 SSW phases result:

1. Our previous results, which did not incorporate a magnetic field model in deriving the solar quiet current time series, see Figure (A) below.

2. New results obtained by implementing the CHAOS magnetic field model to derive the solar quiet current time series, see Figure (B) below.

This comparison allows us to evaluate the advantage of incorporating a magnetic field model on the derived solar quiet current time series and its implications for understanding SSW phases. We noticed that both Figure (A & B) looks similar.

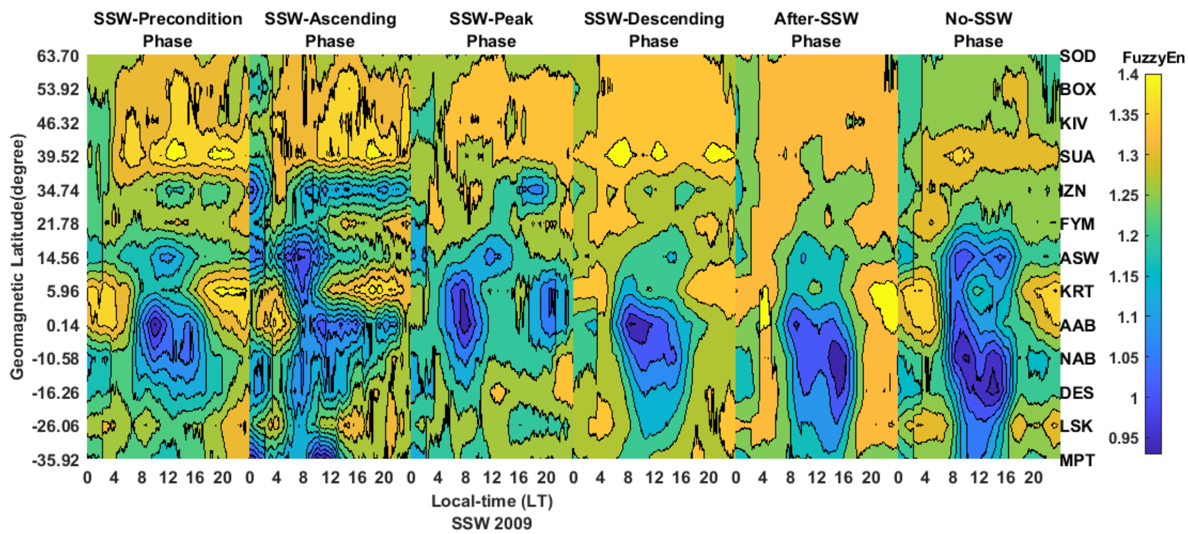


Figure (a): SSW Phases of 2009 of previous result (without considering magnetic field model)

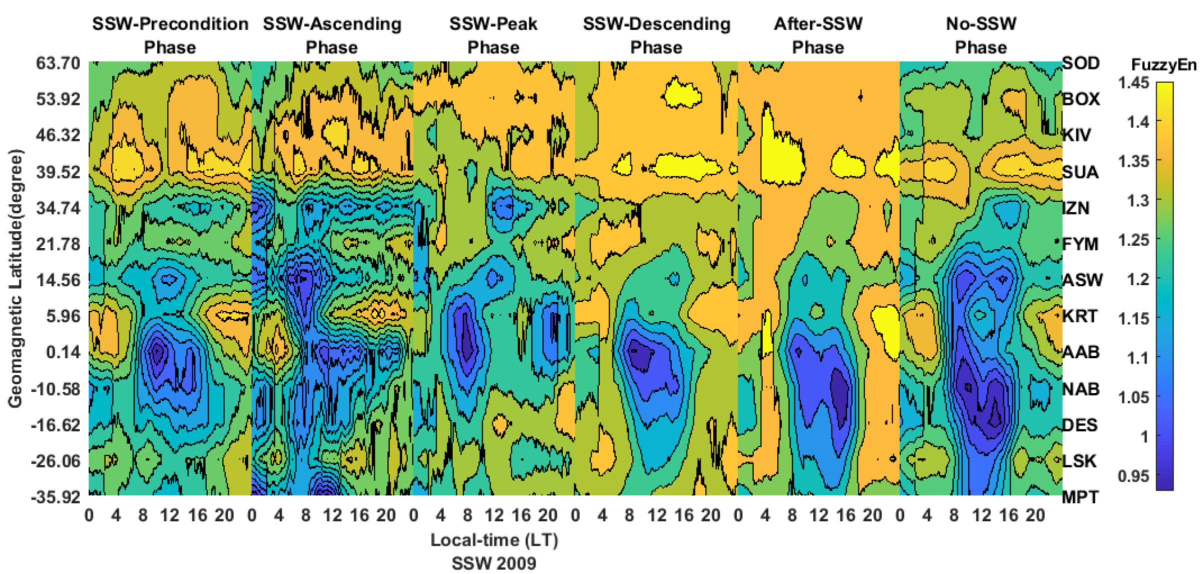


Figure (b): SSW Phases of 2009 re-analyzed result (With CHAOS magnetic field model)

Comment 13: • In the method used by the authors, however, it is unclear what the value of BLV represents, how it is calculated, and over which days. If the goal is to remove the background field (i.e., the main field), then it would be more appropriate to work with variational data, which are free from the main field, rather than using absolute magnetic measurements. Additionally, since this step forms the basis for the entire subsequent analysis, I would expect to see at least three or four days with varying Kp values, showing the ionospheric magnetic field component at all observatories. This would help ensure that the resulting structure aligns with expectations and provides meaningful insight.

Response to comment 13: The estimation of the Baseline Value (BLV) is crucial in calculating the solar quiet current. It is recognized that the average of nighttime values of the Sq variation is more physically meaningful, as the source currents in the ionosphere effectively vanish at nighttime. Various methods exist for estimating the BLV. In our study, we employed the approach of Bolaji et al. (2015a) and Siddiqui et al. (2015a) [doi:10.1002/2014JA020728; doi:10.5194/angeo-33-235-2015]. Specifically, the BLV is calculated by averaging the nighttime values (in minutes) of the H-component between 24:00 and 01:00 local time (LT) for a given day. This estimation of BLV is now thoroughly explained in the revised manuscript line 230-240 as:

"To estimate the solar quiet current $S_q(H)$ time series, the average nighttime values (in minutes) of the H-component between 24:00 and 1:00 local time (LT) for a particular day refers as Baseline Value (BLV) was estimated using equation (3).

$$BLV = \frac{\Delta H_{24} + \Delta H_{01}}{2} \tag{3}$$

The notation ΔH_{24} and ΔH_{01} are the 60 minutes values of H component at 24:00 and 01:00 LT respectively. Where BLV represent the Baseline line value. The residual value after subtracting the baseline value from the H-component gives rise to the solar quiet current time series.

$$S_q(H) = \Delta H_{local} - BLV \tag{4}$$"

Where $S_q(H)$ is the solar quiet current considered in minutes. The analysis of the $S_q(H)$ was deduced for all the day-to-day activities of the 2009 SSW (January-March) and 2021 SSW (December 2020-Februay) periods for all stations under investigation.

Comment 15: • Finally, a crucial point is that, even assuming this procedure is correct, the authors claim that the resulting H component of the magnetic field represents the solar quiet daily variation. In reality, not only does this component primarily exist around noon, but not all the selected observatories are capable of recording this contribution to the magnetic field. Stations near the magnetic equator, such as AAB and KRT, are more likely to be influenced by the equatorial electrojet rather than by the ionospheric current system that generates the Sq variation.

Response to comment 15: The magnetic data utilized in our study have been successfully employed in previous investigations of solar quiet currents, such as those by Bolaji et al. (2016, doi.org/10.1002/2016JA022857) and Bolaji et al. (2015, doi:10.1002/2014JA020728), which leveraged MAGDAS magnetic data to estimate solar quiet currents. Also, the methodology for deriving solar quiet currents differs, from that used to estimate the Equatorial Electrojet (EEJ), as described by Yamazaki and Maute (2017, DOI 10.1007/s11214-016-0282-z).

Comment 16: The analysis proposed by the authors aims to demonstrate how the properties of the quiet ionosphere (where the perturbation caused by the current system responsible for Sq is observable) tend to change under the influence of Sudden Stratospheric Warming (SSW) events. To support this, the authors introduce the calculation of Fuzzy Entropy applied to time series transformed into a complex network representation using the Horizontal Visibility Graph.

It is generally understood that where stable current systems flow in the ionosphere, these regions will exhibit greater stability, which is reflected in lower entropy values. This is evident in the results presented in Figure 6, where, near the peak variation of the H component—which corresponds not so much to Sq but rather to the presence of the equatorial electrojet (since the station is near the magnetic equator)—entropy decreases. This decrease is due to the current itself stabilizing the ionospheric system.

However, the authors fail to consider that Sq is not symmetric about the magnetic equator and is strongly influenced by seasonal variations. Since Sq depends mainly on solar radiation, it is more intense during the summer months compared to winter. This means that when analyzing data from December, January, and February, Sq will naturally be stronger in the Southern Hemisphere (e.g., Africa) than in the Northern Hemisphere (e.g., Europe). Thus, the presence of a blue zone around local noon in the Southern Hemisphere in Figures 7 and 8 is entirely expected, as it reflects the higher intensity of the current system.

Response to comment 16: While the author appreciates your suggestion regarding the interpretation of results, it is important to clarify that the depiction of orderliness behavior (i.e., low entropy values) observed in the African sector is not related to the equatorial electrojet (EEJ).

The EEJ typically manifests within ±3° of magnetic dip. The presence of the blue zone, indicating orderliness behavior, extends beyond the boundaries of the magnetic dip equator. Therefore, the observed orderliness behavior in Africa cannot be attributed to the EEJ. Rather, the features observed, including the suppression of orderliness behavior and the consistency of this behavior in the Africa sector's ionosphere during the phases of SSW, reflect a modulation of the Equatorial Ionization Anomaly (EIA) structure due to the forcing effect of SSW. This is now thoroughly explained in the revised manuscript line 530-539.

Comment 17: Similarly, the symmetry observed near the equator in March (Figure 9) can be attributed to the equinoctial period, during which the current systems in the two hemispheres become more similar in intensity. A similar trend might have been observed in Figures 10, 11, and 12, but, inexplicably, the authors chose to show only data for the Northern Hemisphere (i.e., Europe).

Response to comment 17: The observed feature of orderliness behavior is not due to equinoctial effect because. 1. SSW events typically occur in the winter of polar regions, whereas the equinoctial effects occur around the equinoxes (March and September), regardless of hemisphere. Also, the temporal occurrence of SSW is dictated by specific atmospheric conditions in the stratosphere, often unrelated to the timing of equinoxes.

2. SSW events induce changes via atmospheric wave interactions and thermal structure shifts, affecting the ionosphere indirectly. The equinoctial effect, however, results from direct solar influence due to the equinox alignment, leading to increases in geomagnetic disturbance and ionospheric current systems.

In conclusion, while both SSW and equinoctial conditions can influence the ionosphere, their mechanisms, timing, and resulting impacts are distinct. Therefore, the modulation caused by SSW is not related to the equinoctial effect.

Finally, we chose to only show data for the Norther Hemisphere (i, e. Europe) due to the unavailability of magnetic data in the African sector during the 2021 SSW event, our analysis is limited to the European sector for this specific event.

Comment 18: Additionally, there are days when the distribution of entropy values does not seem to correspond to the current systems responsible for variations in H. This could be probably due to the fact that, contrary to the authors claims, there are days with a K_p value of 4. Given this, the subsequent analysis attempting to relate these observations to SSW events is unconvincing.

Response to Comment 18: In response to this comment, we re-analyzed the solar quiet current in the revised manuscript. To ensure the accuracy of our results, we excluded days with a K_p index exceeding 3 from the solar quiet current analysis. We provide the comparison of the result of obtained when days whose K_p index exceed 3 were excluded from the SSW analysis in **Response to Comment 12 (Figure A & B)** and our previous results.

Comment 19: Before establishing any potential connection between entropy variations and SSW events, it is essential to disentangle the seasonal effects and those linked to geomagnetic activity from the entropy variations. Only after accounting for these factors would it be reasonable to explore any potential relationship with SSW events.

Response to comment 19: The above comments have been addressed in the revised manuscript. For instance, in the re-analysis of the solar quiet current, we ensure that, we exclude the days whose K_p index exceed 3 from the analysis. In addition, we implement the CHAOS model to obtained the derivation of the solar quiet current. Finally, the observed features obtained from our analysis during SSW are not seasonal effect (i.e. not equinoctial) because: SSW events induce changes via atmospheric wave interactions and thermal structure shifts, affecting the ionosphere indirectly. The equinoctial effect, however, results from direct solar influence due to the equinox alignment, leading to increases in geomagnetic disturbance and ionospheric current systems.

Response to Reviewer Two

Comment 1: Page 2, Introduction, paragraph 1: "The increasing number of extreme weather events is caused by climate change". Please replace with a less categorical sentence, such as "is thought to be caused", or "is associated with", or "is likely caused", etc.. Although this is not the main issue in the manuscript, and there is scientific consensus on the effects of human activity on climate, attributing such a direct causality seems beyond what the current consensus can provide.

Response to comment 1: Thanks for your suggestion to improve our manuscript. In the revised manuscript, the corresponding sentence has been removed and the introduction has been completely redrafted.

Comment 2: Page 4, paragraph 2: "Notably". This is not particularly notable. It is expected that the dynamics changes with time, given its dependence on solar wind conditions.

Response to comment 2: In the revised manuscript line 54-58, the above statement has be corrected as "The SSW effect introduces spatial and temporal variability in the ionospheric plasma density (Yamazaki, 2013, 2014; Klimenko et al., 2018). As a result, the ionosphere exhibits nonlinear dynamical behavior, characterized by high sensitivity to minor perturbations originating from the lower atmosphere that could lead to transitions between orderliness and chaotic behavior of ionosphere, rendering its state susceptible to sudden and significant changes."

Comment 3: Page 4, paragraph 3: "the European-African sector needs special attention". This claim should be justified.

Response to comment 3: In the revised manuscript, the justification why European-African sector needs a special attention during SSW has been included in manuscript line 62-68 as: "The European and Africa's geographical locations manifest pronounced ionospheric variability in response to SSW events. This phenomenon provides a unique opportunity to investigate the complex coupling mechanisms between the stratosphere and ionosphere. Specifically, it enables the study of atmospheric wave propagation and its impact on the ionosphere, which can lead to disruptions in satellite communication and navigation system in the region. Therefore, this study explores the latitudinal variations in ionosphere during 2009 and 2021 SSW."

Comment 4: Page 5, paragraph 1: "due to the emerging influence of the SSW over this sector". What is special about the influence of SSW in this sector, with respect to other zones on the planet?

Response to comment 4: the impact of SSW events might differ between the European and African sectors due to their geographical and geomagnetic features. For instance, proximity to the geomagnetic equator in Africa could lead to different responses compared to higher latitude regions in Europe. This is reason why our present study utilizes the concept of chaos theory to examine the impact of SSW on the regional ionosphere of European and Africa sector.

Comment 5: Page 6, paragraph 2: “Notably”. Why is this notable?

Response to comment 5: The above statement has been redrafted in manuscript line 70-71 as: “One of the atmospheric parameters that can reveal the extent of the SSW-induced effects on the regional ionosphere is the solar quiet current, $S_q(H)$.”

Comment 6: Page 7, paragraph 1: “leading to instability and divergence from its initial state” .Unstable is different from chaotic. The latter implies sensitivity to initial conditions, not necessarily instability.

Response to comment 6: The corresponding sentence has been removed in the revised manuscript.

Comment 7: Page 7, paragraph 1: “due to its continuous response”. I would not say that this is the only reason to consider the ionosphere as a dynamical system.

Response to comment 7: The above statement has been re-written in the revised manuscript line 54-58 as: The SSW effect introduces spatial and temporal variability in the ionospheric plasma density (Yamazaki, 2013, 2014; Klimenko et al., 2018). As a result, the ionosphere exhibits nonlinear dynamical behavior, characterized by high sensitivity to minor perturbations originating from the lower and upper atmosphere that could lead to transitions between orderliness and chaotic behavior of ionosphere, rendering its state susceptible to sudden and significant changes.

Comment 8: Page 7, paragraph 1: “disorderliness (chaotic)”. Disorder is not the same as chaos. This occurs in many places along the text, and should be clarified. Both concepts are not equivalent.

Response to comment 8: In the revised manuscript, we have removed the term "disorderliness" throughout the manuscript and instead consistently used the terms "chaos" and "orderliness" to describe the observed phenomena.

Comment 9: Page 7, paragraph 1: “Therefore, it is crucial to examine. . . ”. It is not clear that, due to the arguments above, it is crucial to study what the authors state in the following sentence. Thus, “therefore” is not a proper word here.

Response to comment 9: The above statement has been removed.

Comment 10: Page 7, paragraph 2: “a theoretically robust method”. What do the authors mean with this?

Response to comment 10: The above statement has been corrected in the revised manuscript line 105-117 as: “this study utilizes a robust methodology combining two innovative techniques: the Horizontal Visibility Graph (HVG), rooted in graph theory, to preprocess solar quiet current time series data, and Fuzzy Entropy (FuzzyEn) analysis to reveal the underlying chaotic behavior in the ionosphere during SSW. The reason why this study considers the combination of HVG and Fuzzy Entropy techniques is that Fuzzy Entropy is indeed robust to small-amplitude noise, some subtle features in the solar quiet current time series may still be obscured if we rely on FuzzyEn alone. The HVG transformation helps by emphasizing the "visibility" relations between data points—effectively highlighting structural patterns that may be drowned out in the raw time series. In addition, when FuzzyEn is computed on node-degree sequence (complex network representation), it often provides clearer differentiation of regimes or subtle changes in the system that might otherwise remain hidden. Thus, the combined HVG and FuzzyEn approach can yield features more robust to measurement noise and more sensitive to underlying structural variations in the solar quiet current."

Comment 11: Page 14, paragraph 2: "the Sq(H) current can be regarded as an observational". Being an "observational time series" should depend only on being observed and being a time series. No relationship to changing dynamical behavior.

Response to comment 11: The subtitle has been revised to "Derivation of Solar quiet Current Sq(H) Time Series" for clarity. Additionally, the statement describing the changing dynamical behavior has been removed to ensure accuracy. These corrections have been implemented in the revised manuscript, specifically in lines 215-224 as: "Derivation of Solar quiet Current $S_q(H)$ time Series

To derive the day-to-day $S_q(H)$ current time series, magnetic field data from various magnetometer stations across European and Africa sector were archived. We focus on acquiring magnetic field data from the magnetometer stations that are situated within the geographical longitude (26°-40°). Some of the acquired magnetic data, especially the stations in Europe sector are provided in Cartesian (X, Y, Z) coordinate system, and was converted to geomagnetic (H, D, Z) coordinate system using Rotation Matrix method (Barton and Tarlowski, 1991). We applied a magnetic field model (CHAOS-8.1) with the acquired magnetic field data to obtain the ionospheric field. The H-component of the magnetic field model (CHAOS) was subtracted from the H-component of the acquired magnetic data."

Comment 12: Page 14, before Eq. (2): "the average value between 24:00 and 1:00". It is not clear that (2) represents a useful average, as it takes two particular hours within the day.

Response to comment 12: In the revised manuscript, the Eq.(2) has been explained to address clarity. In revised manuscript line 230-232, the Eq. (2) has been adjusted to Eq. (3) and explained as: "the average nighttime values (in minutes) of the H-component between 24:00 and 1:00 local time (LT) for a particular day refers as the Baseline Value (BLV)."

Comment 13: Page 15, paragraph 2: "the inherent characteristics of the transformed time series". Some inherent characteristics, rather. This largely depends on what one is interested in studying. It is an abstraction, so it cannot keep all features of the time series.

Response to comment 13: The above statement has been removed and redrafted in the revised manuscript.

Comment 14: Page 15, paragraph 2: "preserving topological information". Not all topological information is preserved. This depends on the questions one is interested in asking.

Response to comment 14: The above comment has been removed and re-written in clarify form.

Comment 15: Page 16, paragraph 1: "the HorizontalVG class, which represents one of the types of visibility graphs, namely the "Horizontal Visibility Graph". This is already implied by what has been said before.

Response to comment 15: In the revised manuscript, we have removed the repetitive statement (by removing the first mention of HVG).

Comment 16: Page 16, paragraph 1: "a network where each point in the series becomes a node, and edges are formed based on the visibility criteria between points". This has already been said.

Response to comment 16: In the revised manuscript, we have removed the repetitive statement (by removing the first description of HVG).

Comment 17: Page 16, paragraph 2: "entropy indicates a more chaotic structure". Entropy is not the same as chaos. This occurs in many places along the text, and should be clarified. Both concepts are not equivalent.

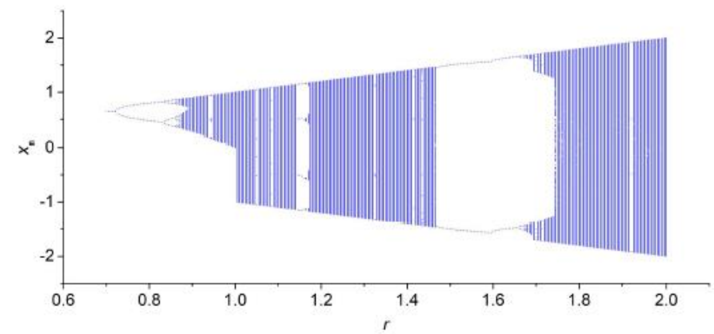
Response to comment 17: Thank you for your observation. This revision avoids the direct conflation of entropy with chaos while maintaining the intended meaning. The entropy is a measure of complexity and helps to quantify unpredictability in a time series. Applying it to the HVG-transformed data provides a way to evaluate how the graph topology (derived from the original time series) evolves. When FuzzyEn is low, it typically indicates more regular (less complex) dynamics. A high FuzzyEn value may indicate higher complexity or a greater degree of irregularity (chaotic) dynamics. As an example, we can refer to the results of the study:

Conejero, J.A.; Velichko, A.; Garibo-i-Orts, Ò.; Izotov, Y.; Pham, V.-T. *Exploring the Entropy-Based Classification of Time Series Using Visibility Graphs from Chaotic Maps. Mathematics* **2024**, *12*, 938. https://doi.org/10.3390/math12070938.

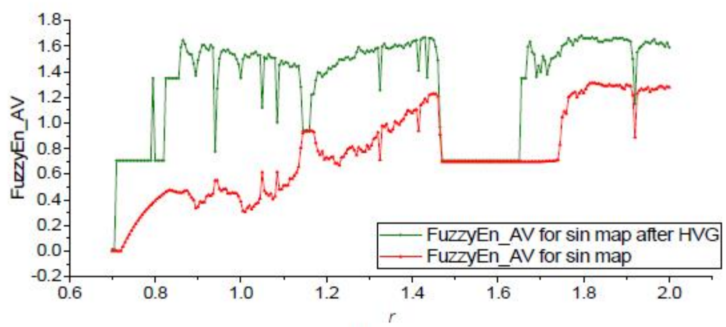
Figure A1 illustrates the bifurcation diagram (a) for the chaotic sine map and the corresponding Fuzzy Entropy (FuzzyEn) values (b) before and after applying the Horizontal Visibility Graph (HVG) transformation. In the bifurcation diagram, the chaotic regimes for a given parameter (r) correspond to the densely filled regions.

The lower plot (b) presents the average Fuzzy Entropy (FuzzyEn_AV) as a function of *r*. The red curve represents the original FuzzyEn values for the sine map, while the green curve shows the values after applying the HVG transformation. It can be observed that the HVG transformation enhances the contrast in the entropy distribution, making chaotic regions more distinguishable. Furthermore, the Fuzzy Entropy values tend to increase as the level of chaos intensifies.

Appendix A



(a)



(b)

Figure A1. *Cont.*

Also, in the revised manuscript line 251, we have rewritten "entropy indicates a more chaotic structure" as: "a more irregularity (chaotic) dynamics."

Comment 18: Page 18, paragraph 2: "after applying the Horizontal Visibility Graph (HVG)". Does this mean that the fuzzy entropy is calculated for the graph, not for the time series? This should be clarified.

Response to comment 18: In the revised manuscript (lines 281–286), we clarify that: First, the solar quiet current time series during the SSW periods were transformed into a complex network using the HVG approach. Then, Fuzzy Entropy was calculated for the node degree distribution of this network representation.

Comment 19: Page 19, paragraph 1: "panel (b) is the detrended time series of solar quiet current transformed through Horizontal Visibility Graph (HVG)". This should be explained. HVG yields a graph, not a time series.

Response to comment 19: We have clarified in the revised manuscript (lines 302–308) that while the Horizontal Visibility Graph (HVG) step indeed yields a graph (i.e., a complex network) of the solar quiet current time series, we subsequently derive a new time series from this network (e.g., by taking the node degrees). Panel (b) thus presents the detrended time series after HVG transformation, effectively reflecting the node-degree sequence derived from the original data, which preserves the same length as the original time series.

Comment 20: Page 19, paragraph 1: "solar quiet current transformed through HVG". Same as above. HVG yields a network, not a time series.

Response to comment 20: In the revised manuscript line 310-311. The comment has been rewritten as: "The result in panel (d) depicts the Fuzzy Entropy for the node degree distribution of this network representation for solar quiet current."

Comment 21: Page 19, paragraph 1: "These distinct features of entropy changes obtained in Fuzzy Entropy after HVG transformation of the solar quiet time series was not obvious in the results of Fuzzy Entropy obtained without HVG transformation method." This could say that the Fuzzy Entropy is not a good metric for this phenomenon. Then, why should one trust a further abstraction such as the HVG, applied to a first abstraction which does not yield clear results?

It would be different if the HVG were directly applied to observed data.

Response to comment 21: While Fuzzy Entropy (FuzzyEn) is indeed robust to small-amplitude noise, some subtle features in the solar quiet current time series may still be obscured if we rely on FuzzyEn alone. The HVG transformation helps by emphasizing the "visibility" relations between data points—effectively highlighting structural patterns that may be drowned out in the raw time series. In this way, HVG is not merely an additional abstraction on top of an inconclusive result, but rather a transformation that captures a different aspect of the data's dynamics.

Moreover, we do apply HVG directly to the observed data. From the resulting graph, we derive a node-degree time series that retains the length and fundamental characteristics of the original data but filters out short-lived fluctuations by focusing on dominant peaks. As a result, when FuzzyEn is computed on this node-degree sequence, it often provides clearer differentiation of regimes or subtle changes in the system that might otherwise remain hidden. Thus, the combined HVG and FuzzyEn approach can yield features more robust to measurement noise and more sensitive to underlying structural variations in the solar quiet current.

Comment 22: Page 19, paragraph 1: “indicates that the HVG transformation method captures the dynamical characteristics”. At most, it suggests something, but the evidence of the usefulness of the HVG for this issue, so far, is inconclusive.

Response to comment 22: We have included supporting explanation in the revised manuscript (lines 317–325) that the HVG transformation can highlight distinct entropy changes in the solar quiet current time series:

“By transforming the solar quiet current time series into a complex network using HVG, we observe transient changes in Fuzzy Entropy (FuzzyEn) that may not be as evident when applying FuzzyEn directly to the raw data. The HVG approach highlights peaks and troughs in the data through horizontal visibility, thereby unveiling subtle fluctuations in the dynamical behavior of the system. When FuzzyEn is low, it typically suggests more orderly (less chaotic) behavior, whereas higher FuzzyEn values are associated with greater complexity or chaos. Through this lens, the HVG and FuzzyEn combination appears to reveal dynamical features of the solar quiet current, including the potential emergence of chaotic behavior during the 2009 (January–March) and 2021 (December 2020–February 2021) SSW events.”

Comment 23: Page 19, paragraph 2: “across Europe and Africa”. How are these values, representing a region on the Earth's surface, obtained from single point measurements at specific locations?

Response to comment 23: They are the values of Fuzzy entropy for the complex network representation for each station investigated in Europe and Africa sector. From this network, we derive a new node-degree time series (maintaining the same length as the original). We then apply a sliding-window approach to calculate the Fuzzy Entropy on this new node-degree time series. Thus, while the HVG step converts the solar quiet current time series into a graph, the FuzzyEn measure is ultimately computed on the node-degree time series derived from that graph. Then, the day-to-day latitudinal distribution of entropy is plotted in contour plot. The selected location in latitude variation cut across the African and European sector within geographical longitudes that ranges between (26^0-40^0).

Comment 24: Page 19, paragraph 2: “The contour map depicts”. What is the meaning of the countour if one axis is time and the other is space?

Response to comment 24: The above statement has been corrected to “The result depicts” in the revised manuscript line 327.

Comment 25: “A consistent low entropy values”: Consistent low entropy values

Response to comment 25: The above suggestion has been addressed in the entire manuscript.

Comment 26: “was found”: were found

Response to comment 26: The above suggestion has been addressed in the entire manuscript

Comment 27: "described by an atmospheric phenomenon" : described as

Response to comment 27: The above statement has been removed.

Comment 28: "force that drive" : drives

Response to comment 28: The corresponding sentence has been corrected.

Comment 29: "can propagate forcing that can reshape the plasma density variability" : please

Rephrase

Response to comment 29: The corresponding sentence has been rephased in the revised manuscript line 49-50 as: The connections between the troposphere and stratosphere during SSW introduces upward wave energy propagation that can reshape the plasma density variability in the ionosphere.

Comment 30: "These reshape" : This reshaping (?)

Response to comment 30: The corresponding sentence has been revised

Comment 31: "this influences" : these

Response to comment 31: The corresponding sentence has been revised and corrected.

Comment 32: "The main mechanism responsible for the connections" : Please rephrase

Response to comment 32: The corresponding sentence has been rephrased in the revise manuscript line 50-51 as: The dominant mechanism facilitating the connection of this processes includes planetary waves, atmospheric tides, and gravity waves

Comment 33: "SSW can infer" : induce?

Response to comment 33: The corresponding phrase has been removed in the revised manuscript.

Comment 34: "imaging system" : imaging of what?

Response to comment 34: The above phrase has been removed in the revised manuscript.

Comment 35: "They exhibit" : It exhibits? Does it refer to " the dynamics" ?, then it is singular.

Response to comment 35: The above phrase has been removed in the revised manuscript.

Comment 36: "from the aspect of chaos theory" : perspective?

Response to comment 36: The corresponding sentence has been rewritten in the revised manuscript.

Comment 37: “Implementing the concept of nonlinear dynamics, informed by information theory and graph theory”: Please rephrase.

Response to comment 37: The above phrase has been removed in the revised manuscript.

Comment 38: “INVESTGATED”:

Response to comment 38: The above comment has been corrected in the revised manuscript line 181.

Comment 39: “L”: L

Response to comment 39: The above suggestion has been addressed.

Comment 40: “Sq(H)t is the solar quiet current considered in minutes.”: This has just been said before Eq. (3).

Response to comment 40: In the revised manuscript, the corresponding sentence has been corrected in line 238-240 as: “Where $S_q(H)$ is the solar quiet current considered in minutes. The analysis of the $S_q(H)$ was deduced for all the day-to-day activities of the 2009 SSW (January-March) and 2021 SSW (December 2020-Februay) periods for all stations under investigation.”

Comment 41: “Given a time series Xi, Eq. (4)”: This line break should not exist.

Response to comment 41: The above suggestion has been addressed in the revised manuscript line 255-256.

Comment 42: “using the fuzzy function.”: colon instead of period.

Response to comment 42: The above comment has been addressed in the revised manuscript line 261.

Comment 43: “n and r”: n and r

Response to comment 43: The suggestion has been addressed

Comment 44: “1.2 ~ 0.8”: It is better to write the lower number first.

Response to comment 44: The suggestion has been addressed in the revised manuscript line 315.

Comment 45: “0.8 ~ 0.6”: Lower number first.

Response to comment 45: The suggestion has been addressed in the revised manuscript line 317.

Comment 46: “most of the station”: stations

Response to comment 46: The suggestion has been addressed.

Comment 47: “changes in entropy reveals”: reveal

Response to comment 47: The suggestion has been addressed.

Comment 48: “during the phases of 2009 SSW. The phases of SSW are categorized into six namely: precondition phase, ascending phase, peak phase, descending phase, after SSW phase and no SSW phase”: This was said before.

Response to comment 48: The corresponding statement has been removed in the revised manuscript.

Comment 49: “most of the station”: stations

Response to comment 49: The suggestion has been addressed.

Comment 50: “Figure 7”: This plot, and similar plots after this one, can barely be understood. Vertical axes cannot be read clearly, the labels D1, D2, etc. are almost invisible, and the meaning of each of the 31 frames is not clear.

Response to comment 50: The figure 7 has been adjusted.