Reviewer 2

Comment 1: Page 2, Introduction, paragraph 1: "The increasing number of extreme weather events is caused by climate change". Please replace with a less categorical sentence, such as "is thought to be caused", or "is associated with", or "is likely caused", etc.. Although this is not the main issue in the manuscript, and there is scientific consensus on the effects of human activity on climate, attributing such a direct causality seems beyond what the current consensus can provide.

Response to comment 1: Thanks for your suggestion to improve our manuscript. In the revised manuscript, the corresponding sentence has been removed and the introduction has been completely redrafted.

Comment 2: Page 4, paragraph 2: "Notably". This is not particularly notable. It is expected that the dynamics changes with time, given its dependence on solar wind conditions.

Response to comment 2: In the revised manuscript line 54-58, the above statement has be corrected as "The SSW effect introduces spatial and temporal variability in the ionospheric plasma density (Yamazaki, 2013, 2014; Klimenko et al., 2018). As a result, the ionosphere exhibits nonlinear dynamical behavior, characterized by high sensitivity to minor perturbations originating from the lower atmosphere that could lead to transitions between orderliness and chaotic behavior of ionosphere, rendering its state susceptible to sudden and significant changes."

Comment 3: Page 4, paragraph 3: "the European-African sector needs special attention". This claim should be justified.

Response to comment 3: In the revised manuscript, the justification why European-African sector needs a special attention during SSW has been included in manuscript line 62-68 as: "The European and Africa's geographical locations manifest pronounced ionospheric variability in response to SSW events. This phenomenon provides a unique opportunity to investigate the complex coupling mechanisms between the stratosphere and ionosphere. Specifically, it enables the study of atmospheric wave propagation and its impact on the ionosphere, which can lead to disruptions in satellite communication and navigation system in the region. Therefore, this study explores the latitudinal variations in ionosphere during 2009 and 2021 SSW."

Comment 4: Page 5, paragraph 1: "due to the emerging influence of the SSW over this sector". What is special about the influence of SSW in this sector, with respect to other zones on the planet?

Response to comment 4: the impact of SSW events might differ between the European and African sectors due to their geographical and geomagnetic features. For instance, proximity to the geomagnetic equator in Africa could lead to different responses compared to higher latitude regions in Europe. This is reason why our present study utilizes the concept of chaos theory to examine the impact of SSW on the regional ionosphere of European and Africa sector.

Comment 5: Page 6, paragraph 2: "Notably". Why is this notable?

Response to comment 5: The above statement has been redrafted in manuscript line 70-71 as: "One of the atmospheric parameters that can reveal the extent of the SSW-induced effects on the regional ionosphere is the solar quiet current, $S_q(H)$."

Comment 6: Page 7, paragraph 1: "leading to instability and divergence from its initial state". Unstable is different from chaotic. The latter implies sensitivity to initial conditions, not necessarily instability.

Response to comment 6: The corresponding sentence has been removed in the revised manuscript.

Comment 7: Page 7, paragraph 1: "due to its continuous response". I would not say that this is the only reason to consider the ionosphere as a dynamical system.

Response to comment 7: The above statement has been re-written in the revised manuscript line 54-58 as: The SSW effect introduces spatial and temporal variability in the ionospheric plasma density (Yamazaki, 2013, 2014; Klimenko et al., 2018). As a result, the ionosphere exhibits nonlinear dynamical behavior, characterized by high sensitivity to minor perturbations originating from the lower and upper atmosphere that could lead to transitions between orderliness and chaotic behavior of ionosphere, rendering its state susceptible to sudden and significant changes.

Comment 8: Page 7, paragraph 1: "disorderliness (chaotic)". Disorder is not the same as chaos. This occurs in many places along the text, and should be clarified. Both concepts are not equivalent.

Response to comment 8: In the revised manuscript, we have removed the term "disorderliness" throughout the manuscript and instead consistently used the terms "chaos" and "orderliness" to describe the observed phenomena.

Comment 9: Page 7, paragraph 1: "Therefore, it is crucial to examine. . . ". It is not clear that, due to the arguments above, it is crucial to study what the authors state in the following sentence. Thus, "therefore" is not a proper word here.

Response to comment 9: The above statement has been removed.

Comment 10: Page 7, paragraph 2: "a theoretically robust method". What do the authors mean with this?

Response to comment 10: The above statement has been corrected in the revised manuscript line 105-117 as: "this study utilizes a robust methodology combining two innovative techniques: the Horizontal Visibility Graph (HVG), rooted in graph theory, to preprocess solar quiet current time

series data, and Fuzzy Entropy (FuzzyEn) analysis to reveal the underlying chaotic behavior in the ionosphere during SSW. The reason why this study considers the combination of HVG and Fuzzy Entropy techniques is that Fuzzy Entropy is indeed robust to small-amplitude noise, some subtle features in the solar quiet current time series may still be obscured if we rely on FuzzyEn alone. The HVG transformation helps by emphasizing the "visibility" relations between data points—effectively highlighting structural patterns that may be drowned out in the raw time series. In addition, when FuzzyEn is computed on node-degree sequence (complex network representation), it often provides clearer differentiation of regimes or subtle changes in the system that might otherwise remain hidden. Thus, the combined HVG and FuzzyEn approach can yield features more robust to measurement noise and more sensitive to underlying structural variations in the solar quiet current."

Comment 11: Page 14, paragraph 2: "the Sq(H) current can be regarded as an observational". Being an "observational time series" should depend only on being observed and being a time series. No relationship to changing dynamical behavior.

Response to comment 11: The subtitle has been revised to "Derivation of Solar quiet Current Sq(H) Time Series" for clarity. Additionally, the statement describing the changing dynamical behavior has been removed to ensure accuracy. These corrections have been implemented in the revised manuscript, specifically in lines 215-224 as: "Derivation of Solar quiet Current $S_q(H)$ time Series

To derive the day-to-day $S_q(H)$ current time series, magnetic field data from various magnetometer stations across European and Africa sector were archived. We focus on acquiring magnetic field data from the magnetometer stations that are situated within the geographical longitude (26°-40°). Some of the acquired magnetic data, especially the stations in Europe sector are provided in Cartesian (X, Y, Z) coordinate system, and was converted to geomagnetic (H, D, Z) coordinate system using Rotation Matrix method (Barton and Tarlowski, 1991). We applied a magnetic field model (CHAOS-8.1) with the acquired magnetic field data to obtain the ionospheric field. The H-component of the magnetic field model (CHAOS) was subtracted from the H-component of the acquired magnetic data."

Comment 12: Page 14, before Eq. (2): "the average value between 24:00 and 1:00". It is not clear that (2) represents a useful average, as it takes two particular hours within the day.

Response to comment 12: In the revised manuscript, the Eq.(2) has been explained to address clarity. In revised manuscript line 230-232, the Eq. (2) has been adjusted to Eq. (3) and explained as: "the average nighttime values (in minutes) of the H-component between 24:00 and 1:00 local time (LT) for a particular day refers as the Baseline Value (BLV)."

Comment 13: Page 15, paragraph 2: “the inherent characteristics of the transformed time series ”. Some inherent characteristics, rather. This largely depends on what one is interested in studying. It is an abstraction, so it cannot keep all features of the time series.

Response to comment 13: The above statement has been removed and redrafted in the revised manuscript.

Comment 14: Page 15, paragraph 2: “preserving topological information”. Not all topological information is preserved. This depends on the questions one is interested in asking.

Response to comment 14: The above comment has been removed and re-written in clarify form.

Comment 15: Page 16, paragraph 1: “the HorizontalVG class, which represents one of the types of visibility graphs, namely the “Horizontal Visibility Graph”. This is already implied by what has been said before.

Response to comment 15: In the revised manuscript, we have removed the repetitive statement (by removing the first mention of HVG).

Comment 16: Page 16, paragraph 1: “a network where each point in the series becomes a node, and edges are formed based on the visibility criteria between points”. This has already been said.

Response to comment 16: In the revised manuscript, we have removed the repetitive statement (by removing the first description of HVG).

Comment 17: Page 16, paragraph 2: “entropy indicates a more chaotic structure”. Entropy is not the same as chaos. This occurs in many places along the text, and should be clarified. Both concepts are not equivalent.

Response to comment 17: Thank you for your observation. This revision avoids the direct conflation of entropy with chaos while maintaining the intended meaning. The entropy is a measure of complexity and helps to quantify unpredictability in a time series. Applying it to the HVG-transformed data provides a way to evaluate how the graph topology (derived from the original time series) evolves. When FuzzyEn is low, it typically indicates more regular (less complex) dynamics. A high FuzzyEn value may indicate higher complexity or a greater degree of irregularity (chaotic) dynamics. As an example, we can refer to the results of the study:

Conejero, J.A.; Velichko, A.; Garibo-i-Orts, Ò.; Izotov, Y.; Pham, V.-T. *Exploring the Entropy-Based Classification of Time Series Using Visibility Graphs from Chaotic Maps. Mathematics* **2024**, *12*, 938. https://doi.org/10.3390/math12070938.

Figure A1 illustrates the bifurcation diagram (a) for the chaotic sine map and the corresponding Fuzzy Entropy (FuzzyEn) values (b) before and after applying the Horizontal Visibility Graph (HVG) transformation. In the bifurcation diagram, the chaotic regimes for a given parameter (r) correspond to the densely filled regions.

The lower plot (b) presents the average Fuzzy Entropy (FuzzyEn_AV) as a function of *r*. The red curve represents the original FuzzyEn values for the sine map, while the green curve shows the values after applying the HVG transformation. It can be observed that the HVG transformation enhances the contrast in the entropy distribution, making chaotic regions more distinguishable. Furthermore, the Fuzzy Entropy values tend to increase as the level of chaos intensifies.

Appendix A

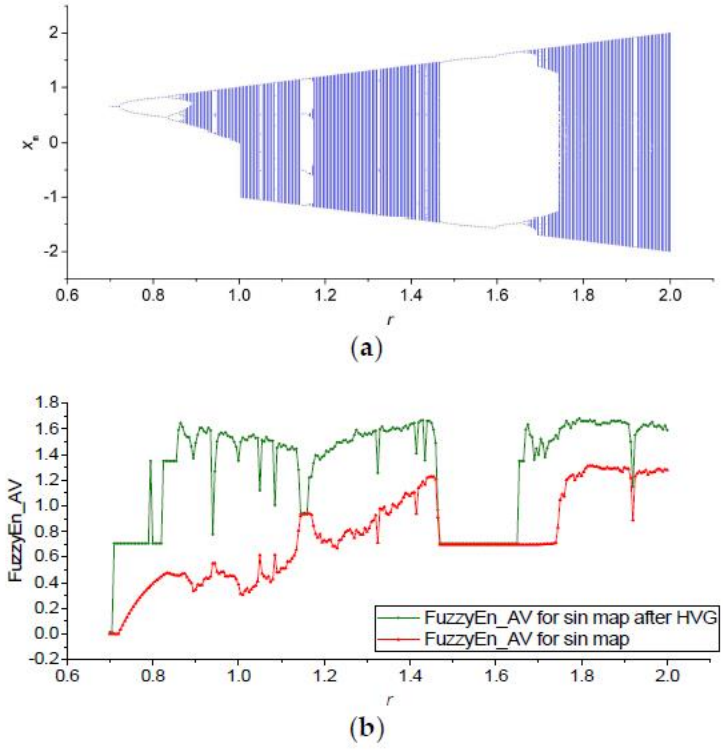

Figure A1. *Cont.*

Also, in the revised manuscript line 251, we have rewritten "entropy indicates a more chaotic structure" as: "a more irregularity (chaotic) dynamics."

Comment 18: Page 18, paragraph 2: "after applying the Horizontal Visibility Graph (HVG)". Does this mean that the fuzzy entropy is calculated for the graph, not for the time series? This should be clarified.

Response to comment 18: In the revised manuscript (lines 281–286), we clarify that: First, the solar quiet current time series during the SSW periods were transformed into a complex network using the HVG approach. Then, Fuzzy Entropy was calculated for the node degree distribution of this network representation.

Comment 19: Page 19, paragraph 1: “panel (b) is the detrended time series of solar quiet current transformed through Horizontal Visibility Graph (HVG)”. This should be explained. HVG yields a graph, not a time series.

Response to comment 19: We have clarified in the revised manuscript (lines 302–308) that while the Horizontal Visibility Graph (HVG) step indeed yields a graph (i.e., a complex network) of the solar quiet current time series, we subsequently derive a new time series from this network (e.g., by taking the node degrees). Panel (b) thus presents the detrended time series after HVG transformation, effectively reflecting the node-degree sequence derived from the original data, which preserves the same length as the original time series.

Comment 20: Page 19, paragraph 1: “solar quiet current transformed through HVG”. Same as above. HVG yields a network, not a time series.

Response to comment 20: In the revised manuscript line 310-311. The comment has been rewritten as: “The result in panel (d) depicts the Fuzzy Entropy for the node degree distribution of this network representation for solar quiet current.”

Comment 21: Page 19, paragraph 1: “These distinct features of entropy changes obtained in Fuzzy Entropy after HVG transformation of the solar quiet time series was not obvious in the results of Fuzzy Entropy obtained without HVG transformation method.” This could say that the Fuzzy Entropy is not a good metric for this phenomenon. Then, why should one trust a further abstraction such as the HVG, applied to a first abstraction which does not yield clear results?

It would be different if the HVG were directly applied to observed data.

Response to comment 21: While Fuzzy Entropy (FuzzyEn) is indeed robust to small-amplitude noise, some subtle features in the solar quiet current time series may still be obscured if we rely on FuzzyEn alone. The HVG transformation helps by emphasizing the “visibility” relations between data points—effectively highlighting structural patterns that may be drowned out in the raw time series. In this way, HVG is not merely an additional abstraction on top of an inconclusive result, but rather a transformation that captures a different aspect of the data’s dynamics.

Moreover, we do apply HVG directly to the observed data. From the resulting graph, we derive a node-degree time series that retains the length and fundamental characteristics of the original data but filters out short-lived fluctuations by focusing on dominant peaks. As a result, when FuzzyEn is computed on this node-degree sequence, it often provides clearer differentiation of regimes or subtle changes in the system that might otherwise remain hidden. Thus, the combined HVG and FuzzyEn approach can yield features more robust to measurement noise and more sensitive to underlying structural variations in the solar quiet current.

Comment 22: Page 19, paragraph 1: "indicates that the HVG transformation method captures the dynamical characteristics". At most, it suggests something, but the evidence of the usefulness of the HVG for this issue, so far, is inconclusive.

Response to comment 22: We have included supporting explanation in the revised manuscript (lines 317–325) that the HVG transformation can highlight distinct entropy changes in the solar quiet current time series:

"By transforming the solar quiet current time series into a complex network using HVG, we observe transient changes in Fuzzy Entropy (FuzzyEn) that may not be as evident when applying FuzzyEn directly to the raw data. The HVG approach highlights peaks and troughs in the data through horizontal visibility, thereby unveiling subtle fluctuations in the dynamical behavior of the system. When FuzzyEn is low, it typically suggests more orderly (less chaotic) behavior, whereas higher FuzzyEn values are associated with greater complexity or chaos. Through this lens, the HVG and FuzzyEn combination appears to reveal dynamical features of the solar quiet current, including the potential emergence of chaotic behavior during the 2009 (January–March) and 2021 (December 2020–February 2021) SSW events."

Comment 23: Page 19, paragraph 2: "across Europe and Africa". How are these values, representing a region on the Earth's surface, obtained from single point measurements at specific locations?

Response to comment 23: They are the values of Fuzzy entropy for the complex network representation for each station investigated in Europe and Africa sector. From this network, we derive a new node-degree time series (maintaining the same length as the original). We then apply a sliding-window approach to calculate the Fuzzy Entropy on this new node-degree time series. Thus, while the HVG step converts the solar quiet current time series into a graph, the FuzzyEn measure is ultimately computed on the node-degree time series derived from that graph. Then, the day-to-day latitudinal distribution of entropy is plotted in contour plot. The selected location in latitude variation cut across the African and European sector within geographical longitudes that ranges between (26^0-40^0).

Comment 24: Page 19, paragraph 2: "The contour map depicts". What is the meaning of the countour if one axis is time and the other is space?

Response to comment 24: The above statement has been corrected to "The result depicts" in the revised manuscript line 327.

Comment 25: "A consistent low entropy values": Consistent low entropy values

Response to comment 25: The above suggestion has been addressed in the entire manuscript.

Comment 26: "was found": were found

Response to comment 26: The above suggestion has been addressed in the entire manuscript

Comment 27: "described by an atmospheric phenomenon" : described as

Response to comment 27: The above statement has been removed.

Comment 28: "force that drive" : drives

Response to comment 28: The corresponding sentence has been corrected.

Comment 29: "can propagate forcing that can reshape the plasma density variability" : please Rephrase

Response to comment 29: The corresponding sentence has been rephased in the revised manuscript line 49-50 as: The connections between the troposphere and stratosphere during SSW introduces upward wave energy propagation that can reshape the plasma density variability in the ionosphere.

Comment 30: "These reshape" : This reshaping (?)

Response to comment 30: The corresponding sentence has been revised

Comment 31: "this influences" : these

Response to comment 31: The corresponding sentence has been revised and corrected.

Comment 32: "The main mechanism responsible for the connections" : Please rephrase

Response to comment 32: The corresponding sentence has been rephrased in the revise manuscript line 50-51 as: The dominant mechanism facilitating the connection of this processes includes planetary waves, atmospheric tides, and gravity waves

Comment 33: "SSW can infer" : induce?

Response to comment 33: The corresponding phrase has been removed in the revised manuscript.

Comment 34: "imaging system" : imaging of what?

Response to comment 34: The above phrase has been removed in the revised manuscript.

Comment 35: "They exhibit" : It exhibits? Does it refer to " the dynamics" ?, then it is singular.

Response to comment 35: The above phrase has been removed in the revised manuscript.

Comment 36: "from the aspect of chaos theory" : perspective?

Response to comment 36: The corresponding sentence has been rewritten in the revised manuscript.

Comment 37: "Implementing the concept of nonlinear dynamics, informed by information theory and graph theory": Please rephrase.

Response to comment 37: The above phrase has been removed in the revised manuscript.

Comment 38: "INVESTGATED":

Response to comment 38: The above comment has been corrected in the revised manuscript line 181.

Comment 39: "L": L

Response to comment 39: The above suggestion has been addressed.

Comment 40: "Sq(H)t is the solar quiet current considered in minutes.": This has just been said before Eq. (3).

Response to comment 40: In the revised manuscript, the corresponding sentence has been corrected in line 238-240 as: "Where $S_q(H)$ is the solar quiet current considered in minutes. The analysis of the $S_q(H)$ was deduced for all the day-to-day activities of the 2009 SSW (January-March) and 2021 SSW (December 2020-Februay) periods for all stations under investigation."

Comment 41: "Given a time series Xi, Eq. (4)": This line break should not exist.

Response to comment 41: The above suggestion has been addressed in the revised manuscript line 255-256.

Comment 42: "using the fuzzy function.": colon instead of period.

Response to comment 42: The above comment has been addressed in the revised manuscript line 261.

Comment 43: "n and r": n and r

Response to comment 43: The suggestion has been addressed

Comment 44: "1.2 ~ 0.8": It is better to write the lower number first.

Response to comment 44: The suggestion has been addressed in the revised manuscript line 315.

Comment 45: "0.8 ~ 0.6": Lower number first.

Response to comment 45: The suggestion has been addressed in the revised manuscript line 317.

Comment 46: "most of the station": stations

Response to comment 46: The suggestion has been addressed.

Comment 47: “changes in entropy reveals”: reveal

Response to comment 47: The suggestion has been addressed.

Comment 48: “during the phases of 2009 SSW. The phases of SSW are categorized into six namely: precondition phase, ascending phase, peak phase, descending phase, after SSW phase and no SSW phase”: This was said before.

Response to comment 48: The corresponding statement has been removed in the revised manuscript.

Comment 49: “most of the station”: stations

Response to comment 49: The suggestion has been addressed.

Comment 50: “Figure 7”: This plot, and similar plots after this one, can barely be understood. Vertical axes cannot be read clearly, the labels D1, D2, etc. are almost invisible, and the meaning of each of the 31 frames is not clear.

Response to comment 50: The figure 7 has been adjusted.