Dear Editor,

Thank you for overseeing our manuscript. We have thoroughly revised the manuscript to address the reviewer's concerns, including improvements to the introduction section, refinement of the handling of geomagnetic indices (K_p value), enhancement of the methodology, and clarification of the results interpretation. Below is our point-by-point response to the reviewer's comments.

Reviewer One

Comment 1: The introduction covers important topics but has some key issues that reduce its clarity and impact. The main concepts, like the effects of SSW (Sudden Stratospheric Warmings) on the ionosphere and the role of nonlinear dynamics, are repeated too often, making the text feel redundant.

Response to comment 1: The introduction section of manuscript has been completely redrafted. The repetitive statements of the SSW effects on the ionosphere and the role of nonlinear dynamics have been removed.

Comment 2: The structure is also unclear, as ideas like chaos theory and its applications are introduced suddenly without enough explanation or connection to the previous points. Additionally, there are too many citations in a short space, which makes the text harder to follow. To improve, I suggest focusing on the main research question, organizing the ideas in a clearer order, and reducing repetitions. This will make the introduction more concise, engaging, and easier to understand.

Response to comment 2: Thank you for your suggestions towards the improvement of the manuscript. In the introduction section of the revised manuscript, chaos theory and its application to ionospheric current system are now better motivated, embedded and explained. The citation clarity has also been addressed. Finally, the introduction section of the revised manuscript is now concise and easier to understand.

Comment 3: Example: Lines 93-94. "*The research question of the contribution of SSW formation to the regional ionosphere across the European-African sector needs special attention*."

This statement needs justification. Why is this region of particular interest? Does it have any unique characteristics? What additional insights can we gain by analyzing the response of the Sq current during an SSW event in this specific region?

Response to comment 3: The justification why the Africa-European sector need special attention has been included in the revised manuscript line 63-69 as: "The regional ionosphere of European and Africa sectors manifest pronounced ionospheric variability in response to SSW events. For example, proximity to the geomagnetic equator in Africa could lead to different responses compared to higher latitude regions in Europe. This phenomenon provides a unique opportunity to investigate the complex coupling mechanisms between the stratosphere and ionosphere. Specifically, it enables the study of atmospheric wave propagation and its impact on the ionosphere, which can lead to disruptions in satellite communication and navigation system in the region."

Comment 4: Characterization of 2009 and 2021 Major SSW Events

Lines 162-190. It would be better to specify the source of the data used in Figures 1, 2, 3, and 4 at the point of introduction, rather than three pages later.

Response to comment 4: The source of the data has now been included in the Figures 1, 2, 3 and 4 of the revised manuscript.

Comment 5: Additionally, the handling of the data and the subsequent claims in this section are problematic. Let me elaborate:

• The Kp index is a planetary three-hour index, yet the values shown in Figures 3 and 4 appear to represent daily averages. This discrepancy should be explicitly addressed in the text, clearly stating that the reported values are likely daily averages and ensuring the associated error is indicated. This clarification is critical, as Kp values can vary significantly throughout the day due to geomagnetic disturbances.

Response to Comment 5: In our initial submission, the K_p index was plotted as daily averages. However, in the revised manuscript, we have replotted the K_p variation considering three-hourly values K_p index and excluded days with K_p indices exceeding 3 from the analysis.

Comment 6: In both selected periods, the Kp index frequently exceeds a value of 4, which makes the claim that "*During the 2009 SSW event (January–March), the planetary index (Kp) recorded values of Kp<2+*" inaccurate and misleading. The actual Kp trend for this period contradicts this statement. A similar issue arises in the second period, where the authors state that Kp remains below 3+. However, during the latter part of February, for example, Kp consistently reaches or exceeds 4.

Response to comment 6: In the revised manuscript, we have implemented corrections by excluding the days when the Kp index exceeded 3 during the periods of the 2009 SSW (January-March) and the 2021 SSW (December-February) from our analysis. Details of these corrections have been incorporated in the revised manuscript, specifically in lines 156-161 as: "During the 2009 SSW event (January-March), the planetary K_p index generally depicts quiet geomagnetic conditions ($K_p \leq 3$) for most days, as shown in Figure 3. However, there were 11 exceptions (January 3, 19, 26, February 4, 14-15, 25, 28, March 13, and 24) where K_p index values exceeded 3, indicating geomagnetic disturbance. These days were excluded from the SSW characterization to maintain a focus on quiet geomagnetic periods."

Also, in the revised manuscript line 166-170: "The planetary K_p index during the 2021 SSW event (December 2020 to February 2021), as shown in Figure 4, generally indicated quiet geomagnetic conditions ($K_p \leq 3$) for most days and the solar flux activity was within $F_{10.7}$~100. However, elevated K_p index values (> 3) were observed on 17 specific days (Dec 10, 21, 23, Jan 5, 6, 11, 24, 25, 27, Feb 2, 6, 12, 16, 20, 22, 23), which were excluded from the analysis to focus on geomagnetically quiet periods."

Comment 7: This inconsistency is significant, as it undermines the claim that both periods were characterized by particularly low geomagnetic activity (e.g., Kp<2+). Moreover, the assertion that there was a meaningful difference in geomagnetic activity between the two periods is not supported by the actual Kp trends. This issue must be addressed to ensure the accuracy and reliability of the study's conclusions.

Response to comment 7: This discrepancy has been now addressed/corrected in the revised manuscript

Comment 8: 2. Data Acquisition and Method of Analysis

Table 1 presents the magnetic stations used in this study, including both geographic and magnetic coordinates for each station. It is important to note that magnetic coordinates are not fixed over time; they depend on the position of the magnetic pole. Therefore, when reporting them in a table, the reference year should always be specified.

Response to comment 8: In the revised manuscript line 179-180, the geomagnetic reference year has been included as **"The geomagnetic coordinate reference year for the stations listed in Table 1 is 2009."**

Comment 9: This implies that the stations selected in 2009 and 2021 have different magnetic coordinates. Furthermore, when studying the Sq current, the correct coordinates to use are the magnetic ones, not the geographic coordinates. As shown in Figure 5, the stations are not aligned with respect to magnetic coordinates. This discrepancy should be taken into account for accurate analysis.

Response to comment 9: In the revised manuscript, the map of the study presented in Figure 5 has been replotted using magnetic coordinates.

Comment 10: Lines 236-243: This paragraph describes the source of the data used and introduced earlier. The paragraph should be revised to include information about the data source at the point where the data is first presented and described.

Response of comment 10: The above suggestion has been incorporated in the revised manuscript.

Comment 11: 2.1 Ionospheric Solar Quiet Current Sq(H) as Observational Time Series

Lines 242-243: The authors state that they use the H component of the magnetic field. However, there is an issue: some of the observatories used, such as KIV and SOD–possibly others, although I have not verified–provide data in the X, Y, Z reference system. This suggests that either the authors performed a data rotation, transforming the coordinates from the (X, Y, Z) system to the (H, D, Z) system, a process which they did not mention, or they simply calculated the modulus of the H component as √(x2 + y2). In this case, they are not working with the H component itself, but rather its intensity. This procedure needs to be explicitly explained. Additionally, where the H component is directly provided, the specific steps taken in processing the data should be clarified.

Response to comment 11: In our previous manuscript, we calculate the H-component by simply estimating the modulus as $\sqrt{X^2 + Y^2}$. Following the suggestion of the reviewer, the estimation of the H-component was re-analyzed by transforming the coordinates of the magnetic data provided in X, Y, Z reference system to the (H, D, Z) geomagnetic system using Rotation Matrix method. We added an explanation in the revised manuscript line 206-209.

Comment 12: Lines 245-268: • Honestly, I am not convinced by the method used to derive the Sq variation from measurements of the horizontal component of the magnetic field. A more standard approach would be to use for example the CHAOS model, which enables the reconstruction of the magnetic field at a specific point in space over time, such as at the location of the observatory. One of the latest version of the CHAOS model can accurately simulate all components of the magnetic field, including secular variation, the crustal field, induced fields, and fields generated by magnetospheric currents. The only component it fails to model is the ionospheric field. Therefore, by simply subtracting the modeled value of H from the real value, the ionospheric field can be obtained, from which the Sq component can then be derived.

Response to Comment 12: In the revised manuscript, the derivation of the solar quiet current was re-analyzed. Following the suggestion, we implemented the CHAOS-8.1 magnetic field model by subtracting the modeled H-component values from the observed H-component magnetic data, thereby obtaining the ionospheric field. The solar quiet current was then derived from this ionospheric field.

At the end of implementing the CHAOS-8.1 and obtaining the solar quiet current. We present a comparison of the 2009 SSW phases result:

1. Our previous results, which did not incorporate a magnetic field model in deriving the solar quiet current time series, see Figure (A) below.

2. New results obtained by implementing the CHAOS magnetic field model to derive the solar quiet current time series, see Figure (B) below.

This comparison allows us to evaluate the advantage of incorporating a magnetic field model on the derived solar quiet current time series and its implications for understanding SSW phases. We noticed that both Figure (A & B) looks similar.

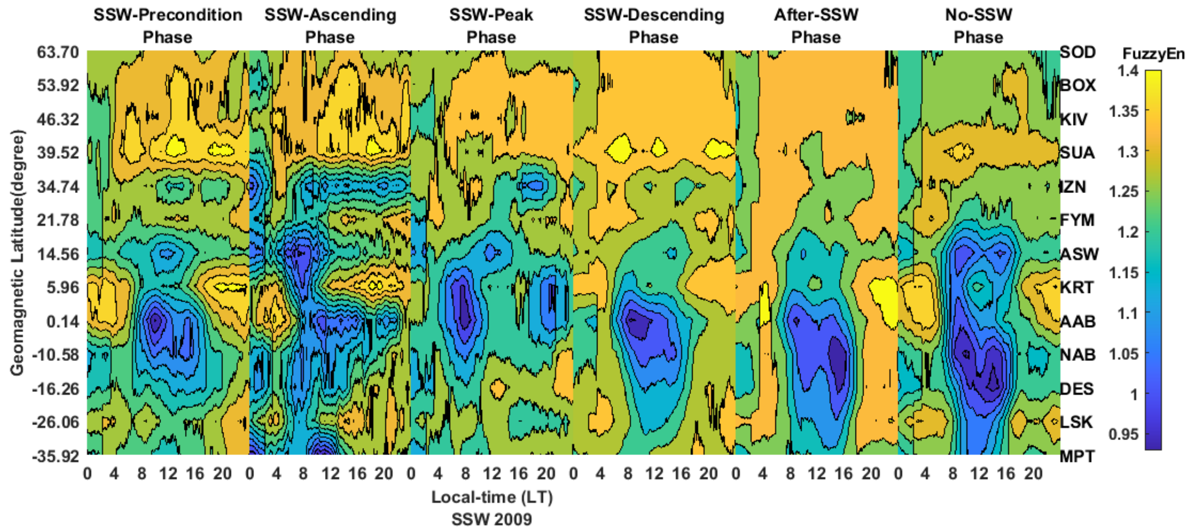

Figure (a): SSW Phases of 2009 of previous result (without considering magnetic field model)

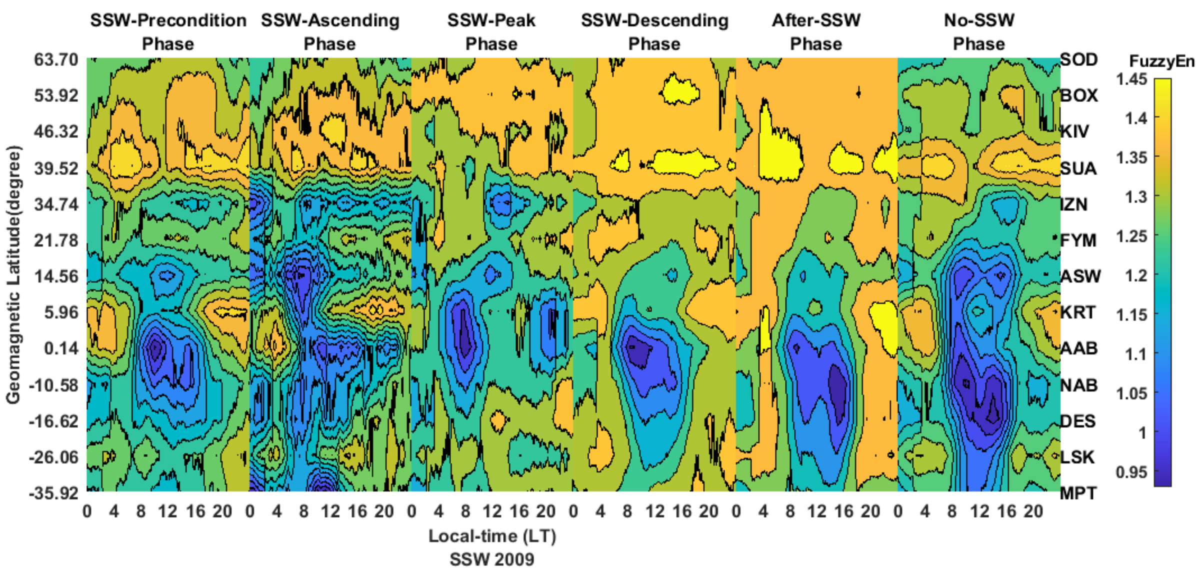

Figure (b): SSW Phases of 2009 re-analyzed result (With CHAOS magnetic field model)

Comment 13: • In the method used by the authors, however, it is unclear what the value of BLV represents, how it is calculated, and over which days. If the goal is to remove the background field (i.e., the main field), then it would be more appropriate to work with variational data, which are free from the main field, rather than using absolute magnetic measurements. Additionally, since this step forms the basis for the entire subsequent analysis, I would expect to see at least three or four days with varying Kp values, showing the ionospheric magnetic field component at all observatories. This would help ensure that the resulting structure aligns with expectations and provides meaningful insight.

Response to comment 13: The estimation of the Baseline Value (BLV) is crucial in calculating the solar quiet current. It is recognized that the average of nighttime values of the Sq variation is more physically meaningful, as the source currents in the ionosphere effectively vanish at nighttime. Various methods exist for estimating the BLV. In our study, we employed the approach of Bolaji et al. (2015a) and Siddiqui et al. (2015a) [doi:10.1002/2014JA020728; doi:10.5194/angeo-33-235-2015]. Specifically, the BLV is calculated by averaging the nighttime values (in minutes) of the H-component between 24:00 and 01:00 local time (LT) for a given day. This estimation of BLV is now thoroughly explained in the revised manuscript line 230-240 as:

"To estimate the solar quiet current $S_q(H)$ time series, the average nighttime values (in minutes) of the H-component between 24:00 and 1:00 local time (LT) for a particular day refers as Baseline Value (BLV) was estimated using equation (3).

$$BLV = \frac{\Delta H_{24} + \Delta H_{01}}{2} \quad (3)$$

The notation ΔH_{24} and ΔH_{01} are the 60 minutes values of H component at 24:00 and 01:00 LT respectively. Where BLV represent the Baseline line value. The residual value after subtracting the baseline value from the H-component gives rise to the solar quiet current time series.

$$S_q(H) = \Delta H_{local} - BLV \quad (4)$$"

Where $S_q(H)$ is the solar quiet current considered in minutes. The analysis of the $S_q(H)$ was deduced for all the day-to-day activities of the 2009 SSW (January-March) and 2021 SSW (December 2020-Februay) periods for all stations under investigation.

Comment 15: • Finally, a crucial point is that, even assuming this procedure is correct, the authors claim that the resulting H component of the magnetic field represents the solar quiet daily variation. In reality, not only does this component primarily exist around noon, but not all the selected observatories are capable of recording this contribution to the magnetic field. Stations near the magnetic equator, such as AAB and KRT, are more likely to be influenced by the equatorial electrojet rather than by the ionospheric current system that generates the Sq variation.

Response to comment 15: The magnetic data utilized in our study have been successfully employed in previous investigations of solar quiet currents, such as those by Bolaji et al. (2016, doi.org/10.1002/2016JA022857) and Bolaji et al. (2015, doi:10.1002/2014JA020728), which leveraged MAGDAS magnetic data to estimate solar quiet currents. Also, the methodology for deriving solar quiet currents differs, from that used to estimate the Equatorial Electrojet (EEJ), as described by Yamazaki and Maute (2017, DOI 10.1007/s11214-016-0282-z).

Comment 16: The analysis proposed by the authors aims to demonstrate how the properties of the quiet ionosphere (where the perturbation caused by the current system responsible for Sq is observable) tend to change under the influence of Sudden Stratospheric Warming (SSW) events. To support this, the authors introduce the calculation of Fuzzy Entropy applied to time series transformed into a complex network representation using the Horizontal Visibility Graph.

It is generally understood that where stable current systems flow in the ionosphere, these regions will exhibit greater stability, which is reflected in lower entropy values. This is evident in the results presented in Figure 6, where, near the peak variation of the H component—which corresponds not so much to Sq but rather to the presence of the equatorial electrojet (since the station is near the magnetic equator)—entropy decreases. This decrease is due to the current itself stabilizing the ionospheric system.

However, the authors fail to consider that Sq is not symmetric about the magnetic equator and is strongly influenced by seasonal variations. Since Sq depends mainly on solar radiation, it is more intense during the summer months compared to winter. This means that when analyzing data from December, January, and February, Sq will naturally be stronger in the Southern Hemisphere (e.g., Africa) than in the Northern Hemisphere (e.g., Europe). Thus, the presence of a blue zone around local noon in the Southern Hemisphere in Figures 7 and 8 is entirely expected, as it reflects the higher intensity of the current system.

Response to comment 16: While the author appreciates your suggestion regarding the interpretation of results, it is important to clarify that the depiction of orderliness behavior (i.e., low entropy values) observed in the African sector is not related to the equatorial electrojet (EEJ). The EEJ typically manifests within ±3° of magnetic dip. The presence of the blue zone, indicating orderliness behavior, extends beyond the boundaries of the magnetic dip equator. Therefore, the observed orderliness behavior in Africa cannot be attributed to the EEJ. Rather, the features observed, including the suppression of orderliness behavior and the consistency of this behavior in the Africa sector's ionosphere during the phases of SSW, reflect a modulation of the Equatorial Ionization Anomaly (EIA) structure due to the forcing effect of SSW. This is now thoroughly explained in the revised manuscript line 530-539.

Comment 17: Similarly, the symmetry observed near the equator in March (Figure 9) can be attributed to the equinoctial period, during which the current systems in the two hemispheres become more similar in intensity. A similar trend might have been observed in Figures 10, 11, and 12, but, inexplicably, the authors chose to show only data for the Northern Hemisphere (i.e., Europe).

Response to comment 17: The observed feature of orderliness behavior is not due to equinoctial effect because. 1. SSW events typically occur in the winter of polar regions, whereas the equinoctial effects occur around the equinoxes (March and September), regardless of hemisphere. Also, the temporal occurrence of SSW is dictated by specific atmospheric conditions in the stratosphere, often unrelated to the timing of equinoxes.

2. SSW events induce changes via atmospheric wave interactions and thermal structure shifts, affecting the ionosphere indirectly. The equinoctial effect, however, results from direct solar influence due to the equinox alignment, leading to increases in geomagnetic disturbance and ionospheric current systems.

In conclusion, while both SSW and equinoctial conditions can influence the ionosphere, their mechanisms, timing, and resulting impacts are distinct. Therefore, the modulation caused by SSW is not related to the equinoctial effect.

Finally, we chose to only show data for the Norther Hemisphere (i, e. Europe) due to the unavailability of magnetic data in the African sector during the 2021 SSW event, our analysis is limited to the European sector for this specific event.

Comment 18: Additionally, there are days when the distribution of entropy values does not seem to correspond to the current systems responsible for variations in H. This could be probably due to the fact that, contrary to the authors claims, there are days with a K_p value of 4. Given this, the subsequent analysis attempting to relate these observations to SSW events is unconvincing.

Response to Comment 18: In response to this comment, we re-analyzed the solar quiet current in the revised manuscript. To ensure the accuracy of our results, we excluded days with a K_p index exceeding 3 from the solar quiet current analysis. We provide the comparison of the result of obtained when days whose K_p index exceed 3 were excluded from the SSW analysis in **Response to Comment 12 (Figure A & B)** and our previous results.

Comment 19: Before establishing any potential connection between entropy variations and SSW events, it is essential to disentangle the seasonal effects and those linked to geomagnetic activity from the entropy variations. Only after accounting for these factors would it be reasonable to explore any potential relationship with SSW events.

Response to comment 19: The above comments have been addressed in the revised manuscript. For instance, in the re-analysis of the solar quiet current, we ensure that, we exclude the days whose K_p index exceed 3 from the analysis. In addition, we implement the CHAOS model to obtained the derivation of the solar quiet current. Finally, the observed features obtained from our analysis during SSW are not seasonal effect (i.e. not equinoctial) because: SSW events induce changes via atmospheric wave interactions and thermal structure shifts, affecting the ionosphere indirectly. The equinoctial effect, however, results from direct solar influence due to the equinox alignment, leading to increases in geomagnetic disturbance and ionospheric current systems.