



# Short Communications: Multiscale topographic complexity analysis with pyTopoComplexity

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**Abstract.** pyTopoComplexity is a Python package designed for efficient and customizable quantification of topographic complexity using four advanced methods: two-dimensional continuous wavelet transform analysis, fractal dimension estimation, Rugosity Index, and Terrain Position Index calculations. This package addresses the lack of open-source software  
10 for these advanced terrain analysis techniques essential for modern geomorphology research, enhancing data comparison and reproducibility. By assessing topographic complexity across multiple spatial scales, pyTopoComplexity allows researchers to identify characteristic morphological scales of the studied landform. The software repository also includes a Jupyter Notebook that integrates components from the surface-process modeling platform Landlab (Hobley et al., 2017), facilitating the exploration of how terrestrial processes, such as hillslope diffusion and stream power incision, drive the evolution of  
15 topographic complexity over time. When these complexity metrics are calibrated with absolute age dating, they offer a means to estimate in-situ hillslope diffusivity and fluvial erodibility, which are critical factors in determining the efficiency of landscape recovery after significant geomorphic disturbances like landslides. By integrating these features, pyTopoComplexity expands the analytical toolkit for measuring and simulating the time-dependent persistence of geomorphic signatures against environmental and geological forces.

## 20 1 Introduction and overview of the package

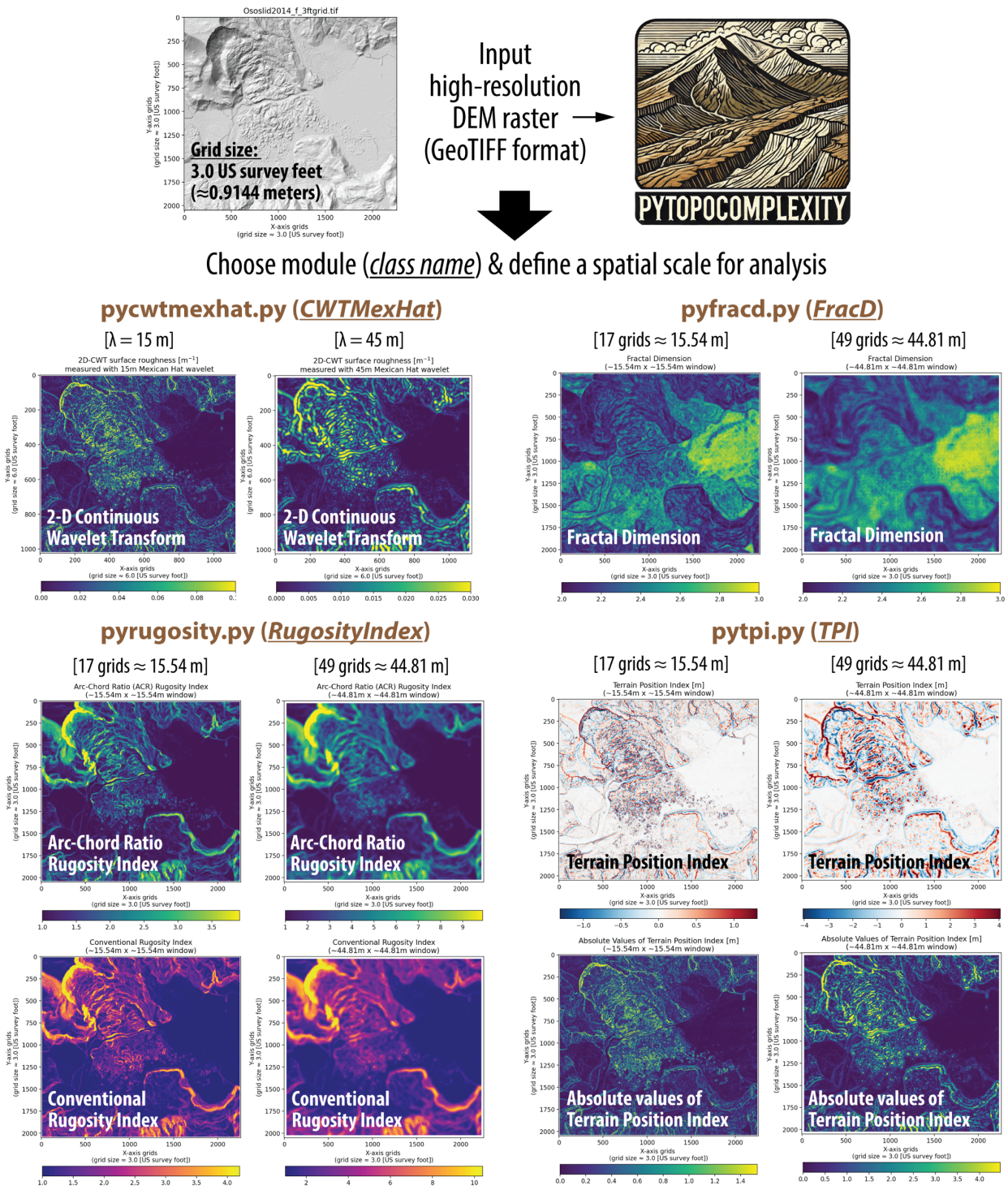
Topographic complexity, often referred to as topographic roughness or surface roughness, provides critical insights into surface processes and the interactions among the geosphere, biosphere, and hydrosphere (Dietrich and Perron, 2006). With the increasing availability, utility, and popularity of digital terrain model (DTM) data, quantifying topographic complexity has become an essential step in terrain analysis across various research fields. This necessity spans applications such as terrain  
25 classification and mapping at various spatial scales (Weiss, 2001; Robbins, 2018; Lindsay et al., 2019; Pardo-Igúzquiza and Dowd, 2022a, b), evaluating the depositional age of event sedimentation (e.g., landslides, avulsion on alluvial fans) and subsequent erosion processes (Hetz et al., 2016; Johnstone et al., 2018; Booth et al., 2017; Herzig et al., 2024; Lahusen et al., 2020; Woodard et al., 2024; Doane et al., 2024), and identifying habitats to assess ecological diversity on land and on the seafloor (Frost et al., 2005; Hetz et al., 2016; Wilson et al., 2007).



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In recent years, several advanced methods for quantifying topographic complexity have been developed, including two-dimensional continuous wavelet transform (2D-CWT) analysis (Booth et al., 2009; Berti et al., 2013), fractal dimension estimation (Taud and Parrot, 2005; Glenn et al., 2006; Robbins, 2018; Pardo-Igúzquiza and Dowd, 2020), and Rugosity Index (RI) calculation (Jenness, 2004; Du Preez, 2015). We consider these methods more effective for analysing multiscale topographic complexity and understanding the geomorphic and ecological significance compared to conventional morphological metrics like variations in local slope, relief, and surface curvature because they can be more directly connected to process-based geomorphic models. Despite their importance, comprehensive publicly available tools that incorporate these advanced methods are lacking. Common open-source geospatial analysis software, such as QGIS (Qgis Development Team, 2024), GRASS GIS (Grass Development Team, 2024), and WhiteboxTools (Lindsay, 2016), only implement basic conventional methods, limiting the reproducibility and comparability of these newer approaches. Although specialized programs for calculating the RI exist (Walbridge et al., 2018; Benham, 2022), they have been confined to marine bathymetric studies and involve various mathematical limitations, assumptions, and designs.

To address this gap, we have developed an open-source Python toolkit called pyTopoComplexity. This toolkit offers computationally efficient and easily customizable implementations for performing and visualizing the results of 2D-CWT, fractal dimension, and RI calculations (**Fig. 1**). Additionally, pyTopoComplexity includes a module for calculating the Terrain Position Index, a widely used metric in geomorphology research (Newman et al., 2018; Deumlich et al., 2010; Liu et al., 2011) and often used alongside RI in marine geological and ecological studies (Wilson et al., 2007; Walbridge et al., 2018). Each module of pyTopoComplexity includes a corresponding example Jupyter Notebook file with usage instructions. These examples utilize 3 ft (~0.9144 m) resolution lidar DTM data (Washington Geological Survey, 2023) from the 2014 Oso deep-seated landslide along the North Fork Stillaguamish River valley in Washington State, USA (Iverson et al., 2015; Wartman et al., 2016; Collins and Reid, 2019). In the software repository, we also include an additional Jupyter Notebook file *Landlab\_simulation.ipynb*, which allows researchers to simulate the change of topographic complexity over time via terrestrial processes including non-linear hillslope diffusion (Roering et al., 1999) and fluvial stream power incision (Whipple and Tucker, 1999; Braun and Willett, 2013). This is achieved by running the landscape evolution simulation in the Landlab environment (Hobley et al., 2017) and integrating it with the functionality of pyTopoComplexity.



60 **Figure 1:** Workflow of applying pyTopoComplexity and products from each module (Table 1) for topographic complexity analysis at spatial scales of ~15 m and ~45 m, using 2014 Oso landslide lidar data (Washington Geological Survey, 2023).



## 2 Methods

The pyTopoComplexity package can be installed directly from the Python Package Index (PyPI) using the Linux command `pip install pytopocomplexity`. This package includes four modules for performing two-dimensional continuous wavelet transform analysis (2D-CWT), fractal dimension estimation, Rugosity Index (RI) calculation, and Terrain Position Index (TPI) calculation (**Table 1**). Users only need to specify the directories for input and output files, as well as the spatial scale for analysis (e.g., the Fourier wavelength ( $\lambda$ ) for the Mexican hat wavelet in 2D-CWT analysis or the size of the moving window ( $\Delta$ ) for other methods). When loading a raster DTM file (acceptable in the GeoTIFF format) into pyTopoComplexity, the toolkit automatically detects grid spacing and units of the projected coordinate system (supported units include meters, U.S. survey feet, and international feet) and applies any necessary unit conversions to maintain consistency in analysis (**Fig. 1**). By default, results from nodes affected by edge effects due to no-data values are excluded. Users can specify the appropriate spatial scale for their research and select computational methods (e.g., chunk processing, faster mathematical approximations) to optimize performance.

**Table 1:** Modules contained in the pyTopoComplexity package.

Modules	Classes	Method Descriptions	References
<i>pycwtmexhat.py</i>	CWTMexHat	Quantifies the wavelet-based curvature of the land surface using two-dimensional continuous wavelet transform with a Mexican Hat wavelet	Booth et al. (2017); Booth et al. (2009)
<i>pyfracd.py</i>	FracD	Conducts fractal dimension analysis on the land surface using variogram procedures	Wen and Sinding-Larsen (1997); Pardo-Igúzquiza and Dowd (2020)
<i>pyrugostiy.py</i>	RugosityIndex	Calculates the Rugosity Index of the land surface	Jenness (2004); Du Preez (2015)
<i>pytpi.py</i>	TPI	Calculates Terrain Position Index of the land or seafloor surface	Weiss (2001); Walbridge et al. (2018)

### 2.1 Two-dimensional continuous wavelet transform (2D-CWT) analysis

The *pycwtmexhat.py* module in pyTopoComplexity implements the 2D-CWT method for terrain analysis, providing detailed information on how amplitude is distributed across spatial frequencies at each position in the data by transforming spatial data into position-frequency space. When used with the Gaussian family of wavelets, this method is particularly effective for depicting the Laplacian of topography (Torrence and Compo, 1998; Lashermes et al., 2007), revealing concave and convex regions of topography at various smoothing-length scales (Malamud and Turcotte, 2001; Struble et al., 2021; Perron et al.,



2008b), identifying deep-seated landslides (Booth et al., 2009; Berti et al., 2013), and estimating the depositional ages of landslide deposits (Booth et al., 2017; Herzig et al., 2024; Lahusen et al., 2020; Underwood, 2022).

85 The 2D-CWT is computed by convolving the elevation data  $z$  with a wavelet family  $\psi$ , using a wavelet scale parameter  $s$  at every location  $(x, y)$ :

$$C(s, x, y) = \delta^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z(x, y) \psi(x, y) dx dy \quad (1)$$

, where the resultant wavelet coefficient  $C(s, x, y)$  provides a measure of how well the wavelet  $\psi$  matches the elevation ( $z$ ) at each grid point (Torrence and Compo, 1998). Unlike other techniques that use moving windows to detect regional topographic  
90 complexity, the 2D-CWT method stands out by isolating specific landform features at the scale of the designated wavelength while filtering out noise from terrain variations at longer or shorter wavelengths. When  $s$  is large,  $\psi$  is spread out, capturing long-wavelength features of  $z$ ; when  $s$  is small,  $\psi$  becomes more localized, making it sensitive to fine-scale features of  $z$ . In this implementation, we use the 2D Ricker or Marr wavelet (i.e., Mexican hat wavelet) function to define  $\psi$  (Ricker, 1943):

$$\psi = -\frac{1}{\pi(s\delta)^4} \left(1 - \frac{x^2+y^2}{2s^2}\right) e^{\left(-\frac{x^2+y^2}{2s^2}\right)} \quad (2)$$

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The wavelet function  $\psi$  is scaled based on the wavelet scale parameter  $s$  and the grid spacing  $\delta$ , ensuring that the resultant wavelet coefficient  $C$  represents curvature (i.e., the Laplacian of elevation), which depicts concave and convex landforms according to the wavelet scale  $s$ . The Mexican hat wavelet, as defined above, corresponds to the second derivative of a Gaussian envelope. Its Fourier wavelength ( $\lambda$ ) depends on the chosen wavelet scale ( $s$ ) and the grid spacing ( $\delta$ ) of the input DTM raster:

$$100 \quad \lambda = \frac{2\pi s}{\sqrt{5/2}} \delta \quad (3)$$

Users can specify the wavelength  $\lambda$  in meters as the target spatial scale for landform roughness analysis (**Fig. 1**). The `pycwtmexhat.py` module will automatically compute the wavelet scale  $s$  based on the grid spacing ( $\delta$ ) of the input raster file. In this module, users can choose to perform convolution either in the original domain (i.e., direct convolution) or in the  
105 frequency domain (i.e., using Fast Fourier Transform) with the ‘convolve2d’ and ‘fftconvolve’ functions from the SciPy package, respectively (Virtanen et al., 2020). By default, the module uses ‘fftconvolve’ for greater computational efficiency.

We note that the equations for  $C$  and  $\psi$  presented here are mathematical approaches adopted in recent publications (Herzig et al., 2024; Lahusen et al., 2020; Woodard et al., 2024) on landslide mapping and age dating studies. There are minor differences  
110 in the proportionality constant used to define  $\psi$  and the conventions used to present the magnitude of the wavelet coefficients  $C$ , compared to earlier similar research (Booth et al. (2009)). Specifically, Booth et al. (2009) used a different proportionality



constant in order to calculate a power spectrum for a range of spatial frequencies, rather than curvature, while Booth et al., (2017) reported the mean of  $C^2$  rather than mean of its absolute value. These differences in mathematical approach will by definition result in different units and order of magnitude (e.g.,  $C^2$  values around  $10^{-3}$  to  $10^{-4}$   $\text{m}^{-2}$  in Booth et al. (2017) and 115 prior studies;  $C$  values around  $10^{-2}$  to  $10^{-3}$   $\text{m}^{-1}$  in Lahusen et al. (2020) thereafter). Despite this discrepancy, the complexity measures yielded from these approaches that only differ in their proportionality constant are linearly scaled and interconvertible (Fig. S1), and they both reflect identical spatiotemporal patterns of topographic complexity (i.e., surface roughness).

## 2.2 Fractal dimension analysis

The *pyfracd.py* module in *pyTopoComplexity* calculates the fractal dimension, which measures the fractal characteristics of 120 natural features (Mandelbrot and Wheeler, 1983). This method provides insights into the self-similarity of landscapes, helping quantify their irregularity and fragmentation, which is crucial for studying the surface processes that shape Earth and planetary surfaces (Xu et al., 1993).

In this module, we adapt the variogram method to estimate the local fractal dimension within a moving window centered at 125 each cell of the DTM (Taud and Parrot, 2005; Pardo-Igúzquiza and Dowd, 2020, 2022b, a). This approach simplifies the problem to estimating the fractal dimension of one-dimensional topographic profiles (Dubuc et al., 1989) within a two-dimensional moving window. For a one-dimensional profile of length  $R$  ( $r = 1, \dots, R$ ), the variogram  $\gamma_1(p)$  can be estimated at the  $P$  lag distances ( $p = 1, \dots, P$ ) by:

$$\gamma_1(p) = \frac{1}{2(R-p)} \sum_{r=1}^{R-p} [z(i) - z(i+r)]^2 \quad (4)$$

130 , where  $z(i)$  is the elevation at location  $i$  along the profile. The local fractal dimension (FD) is estimated from one-dimensional profiles in principal directions (i.e., horizontal, vertical, and diagonal) within a square moving window. Assuming that fractional Brownian motion is an appropriate stochastic model for natural surfaces, its variogram follows a power-law model with respect to  $p$  (Wen and Sinding-Larsen, 1997):

$$\gamma_1(p) = \alpha r^\beta, \quad \alpha \geq 0; \quad 0 \leq \beta < 2 \quad (5)$$

135 , and its exponent  $\beta$  is related to the local fractal dimension (FD) by:

$$\text{FD} = \text{TD} + 1 - \frac{\beta}{2} \quad (6)$$

, where TD is the topological dimension in the Euclidean space of the fractional Brownian motion. For one-dimensional fractional Brownian motion,  $\text{TD} = 1$ . The fractal dimension of the two-dimensional surface  $(\text{FD})_2^*$  can be estimated as the average fractal dimension of the one-dimensional profiles  $(\text{FD})_1^*$ :

$$140 \quad (\text{FD})_2^* = 1 + (\text{FD})_1^* \quad (7)$$



Users can specify the size (number of grids along each edge) of the moving window to study fractal characteristics at desired spatial scales (**Fig. 1**). In addition to calculating the fractal dimension, the *pyfracd.py* module also computes reliability parameters such as standard error and the coefficient of determination ( $R^2$ ) to assess the robustness of the analysis.

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### 2.3 Rugosity Index (RI) calculation

The *pyrugosity.py* module in *pyTopoComplexity* measures the Rugosity Index (RI) of the land surface, which is widely used to assess structural complexity of topography and has been applied in classifying seafloor types by marine geologists and geomorphologists, understanding small-scale hydrodynamics by oceanographers, and studying available habitats in the landscape by ecologists and coral biologists (Lundblad et al., 2006; Wilson et al., 2007).

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The RI is determined as the ratio of the contoured area  $A_c$  (i.e., true geometric surface area) to the planimetric area  $A_p$  within the square moving window, highlighting smaller-scale variations in surface height:

$$RI = \frac{A_c}{A_p} \quad (8)$$

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This module adapts the Triangulated Irregular Networks method from Jenness (2004) to approximate the contoured area as the sum of eight truncated-triangle areas. These triangles connect the central grid point, four corner grid points, and four grid points at the middle points of the surrounding edges within the moving window. If no local slope correction is applied, the planimetric area is considered to be the horizontal planar area of the moving window, as described in Jenness (2004). Another approach considers slope correction where the planimetric area is projected onto a plane of the local gradient (Du Preez, 2015).

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By definition, the RI has a minimum value of one (completely flat surface). Typical values of the conventional RI (without slope correction) range from one to three although larger values are possible in very steep terrains. The slope-corrected RI, also called Arc-Chord Ratio Rugosity Index (ACR-RI), could provide a better representation of local surface complexity because it is not biased by slope (**Fig. 1**).

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### 2.4 Terrain Position Index (TPI) calculation

The *pytpi.py* module in *pyTopoComplexity* calculates the Terrain Position Index (TPI) of the land surface. The TPI, also known as the Topographic Position Index in terrestrial studies (Weiss, 2001), measures the relative topographic elevation of a point compared to those of its surrounding landforms. This metric highlights regions that are relatively higher or lower than

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their surroundings, which is useful for distinguishing landscape features such as hilltops, valleys, flat plains, and slopes. In oceanography, an equivalent metric is the Bathymetric Position Index (BPI), which applies the TPI algorithm to bathymetric data to evaluate seafloor complexity.

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TPI is widely applicable for various purposes, including determining surface ruggedness (Newman et al., 2018), classifying terrain (Zwoliński and Stefańska, 2015), assessing local soil formation and hydrodynamics (Deumlich et al., 2010; Liu et al., 2011), and identifying habitat hotspots (Wilson et al., 2007). It is calculated by comparing the elevation of a grid cell ( $z$ ) to the mean elevation of its surrounding grid cells ( $\bar{z}_{nb}$ ) within a specified neighborhood:

180 
$$TPI = z - \bar{z}_{nb} \quad (9)$$

In this module, the TPI is calculated for the central grid within a square moving window. Users can specify the size of the window (i.e., the number of grids along each edge) to evaluate topographic positions at various spatial scales. Positive TPI values indicate generally convex, elevated features (e.g., ridges), while negative values represent concave depressions (e.g., valleys, saddles). Values close to zero denote relatively flat surface or area with near spatially constant slope (**Fig. 1**). The *pytpi.py* module also returns the absolute values of the TPI, which only indicate the magnitude of the vertical position at each grid point relative to its neighbors.

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## 2.5 Integrating Landlab with pyTopoComplexity

190 The Jupyter Notebook file *Landlab\_simulation.ipynb* in the pyTopoComplexity repository offers a sophisticated tool for simulating time-dependent changes in topographic complexity driven by hillslope and fluvial processes. This tool runs the landscape evolution modeling in the Landlab environment (version  $\geq 2.7$ ) (Hobley et al., 2017) by employing two components: (1) the ‘TaylorNonLinearDiffuser’ component from the terrainBento package (Barnhart et al., 2019), which simulates topographic smoothing over time through nonlinear hillslope diffusion processes caused by near-surface soil disturbance and downslope soil creeping; and (2) the core ‘StreamPowerEroder’ component from Landlab v1.0 that simulates topographic dissection through fluvial incision over time.

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The ‘TaylorNonLinearDiffuser’ component applies the nonlinear diffusion model to predict changes in surface elevation  $z$  over time  $t$  (i.e., erosion rate  $E = \partial z / \partial t$ ) on a land surface:

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$$E = -\nabla \cdot \vec{q}_s \quad (10)$$

Here,  $\vec{q}_s$  represents 2D vector of the sediment flux per unit slope width at the surface. This sediment flux vector is further defined by a nonlinear flux law (Roering et al., 1999) that is approximated using a Taylor series expansion (Ganti et al., 2012):

$$\vec{q}_s = D\vec{S}_{hd} \left[ 1 + \sum_{i=1}^N \left( \frac{S_{hd}}{S_c} \right)^{2i} \right] \quad (11)$$





205 , where  $\vec{S}_{hd} = -\nabla z$  represents the vector of topographic downslope gradient at each grid point (calculated using the elevation  
of that grid cell and its surrounding grid cells), and  $S_{hd}$  is its magnitude.  $S_c$  is magnitude of the critical slope representing  
the asymptotic maximum hillslope gradient. The parameter  $D$  is a diffusion-like transport coefficient with dimensions of length  
squared per time.  $N$  denotes the number of terms in the Taylor expansion, while  $i$  specifies the number of additional terms  
included. If  $N = 0$ , the expression simplifies to linear diffusion (Culling, 1963). By default,  $N$  is set to 2, that gives the  
behavior described in Ganti et al. (2012).

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Inspired by (Braun and Willett, 2013), the ‘StreamPowerEroder’ component adapts the stream power erosion model that  
predicts the erosion rate ( $E$ ) according to the following equation:

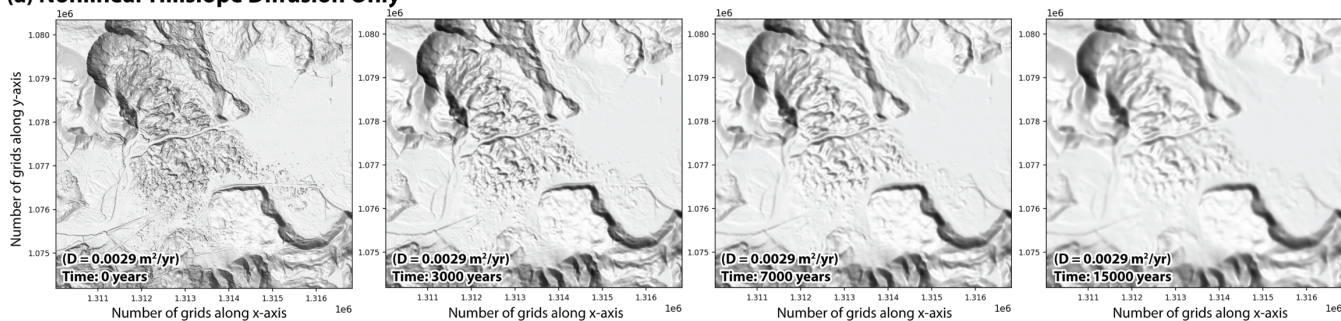
$$E = KA^m S_{sp}^n - \Omega \quad \text{when } KA^m S_{sp}^n > \Omega \quad (12)$$

215 , where  $A$  represents the drainage area.  $S_{sp}$  denotes the magnitude of channel slope (positive in the downslope direction)  
calculated from the steepest descent from each grid cell to its surrounding grid cells (using the D8 method from Fairfield and  
Leymarie (1991) by default).  $K$  is a positive constant representing the erodibility coefficient, which correlates positively with  
climate wetness or storminess and negatively with rock strength.  $m$  and  $n$  are positive exponents, typically assumed to have a  
ratio  $m/n \approx 0.5$  (Whipple and Tucker, 1999).  $\Omega$  represents the erosion threshold in the stream power equation. When the  
stream power does not exceed the erosion threshold ( $KA^m S_{sp}^n \leq \Omega$ ), the model considers the erosion rate to be zero ( $E = 0$ ).  
220 When  $\Omega$  is set to zero, the model represents a purely detachment-limited erosion system (i.e., without substantial fluvial  
sediment effects). In this case, fluvial erosion acts at every point on the surface, following the conventional stream power law  
(Howard, 1994).

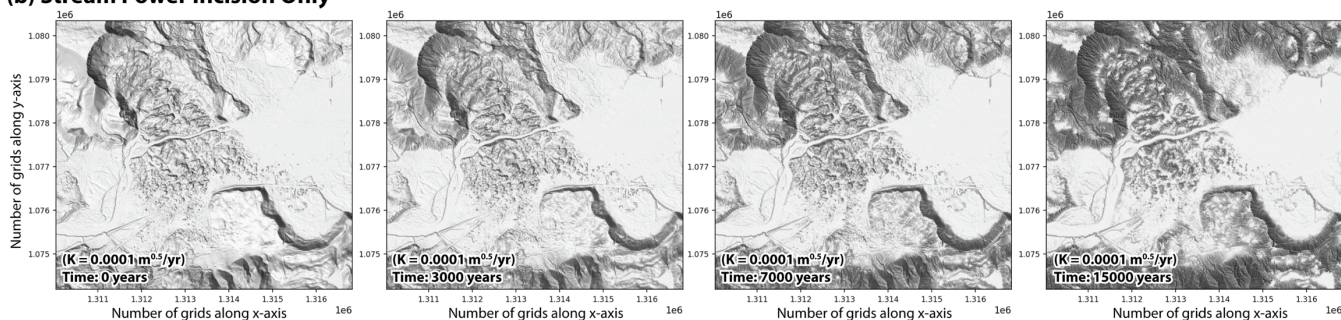
This notebook offers a comprehensive workflow that guides users through setting up Landlab components, importing raster  
225 DTM files, running simulations, and analyzing topographic complexity on simulated landforms using pyTopoComplexity.  
Because Landlab primarily processes DTM data in ESRI ASCII format, the notebook converts the raster DTM files between  
GeoTIFF and ESRI ASCII to meet the required data formats for each toolkit. For landscape evolution modeling, users must  
specify values for key parameters  $S_c$ ,  $D$ ,  $K$ , and the duration of the total simulation time and each time step (in years). The  
examples provided in the notebook use the 2014 Oso landslide lidar DTM data (Washington Geological Survey, 2023) to run  
230 simulations of landscape evolution over 15,000 years, activating either or both hillslope diffusion and fluvial incision  
components (**Fig. 2**). By setting  $D = 0.0029$  ( $\text{m}^2 \text{yr}^{-1}$ ) and  $S_c = 1.25$ , the hillslope diffusion-only simulation successfully  
reproduces results from Booth et al. (2017) (**Fig. 2a**). Upon completing the Landlab simulation, users can apply the notebook’s  
customized functions to measure topographic complexity on the simulated landscape at each time step using  
pyTopoComplexity, and the resulting GeoTIFF rasters can then be utilized for further geospatial analysis.



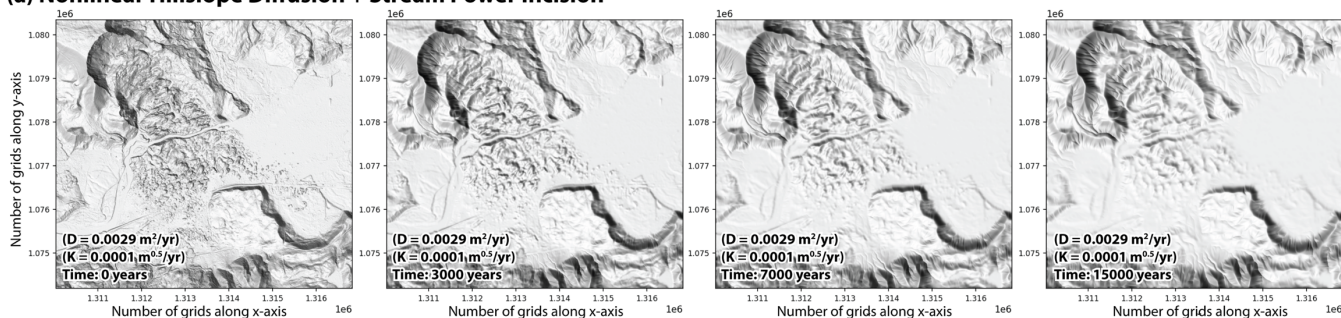
**(a) Nonlinear Hillslope Diffusion Only**



**(b) Stream Power Incision Only**



**(a) Nonlinear Hillslope Diffusion + Stream Power Incision**



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**Figure 2: Hillshade maps of the 2014 Oso landslide and landscape evolution over 15,000 years predicted by models of nonlinear hillslope diffusion and stream power incision in the *Landlab\_simulation.ipynb*.**

**3 Case study of multiscale surface-complexity based age estimation for landslide deposits**

To validate and assess the effectiveness of different metrics for quantifying topographic complexity, we applied each method from pyTopoComplexity to a published landslide inventory of the North Fork Stillaguamish River (NFSR) valley (Fig. 3a). Complexity measurements were conducted across multiple spatial scales, ranging from ~3 m to ~75 m, using lidar DTM data (Washington Geological Survey, 2023) for seven mapped landslides with radiocarbon age constraints from Booth et al. (2017). Areas with extreme linear depressions (e.g., gullies), flat water surfaces (e.g., ponds), and artificial modifications (e.g., roads, power lines) were excluded from the analysis. To evaluate the quality and predictability of the age-complexity relationship, we identified the best-fit exponential decay functions between the dated landslide ages and the measured complexity metrics

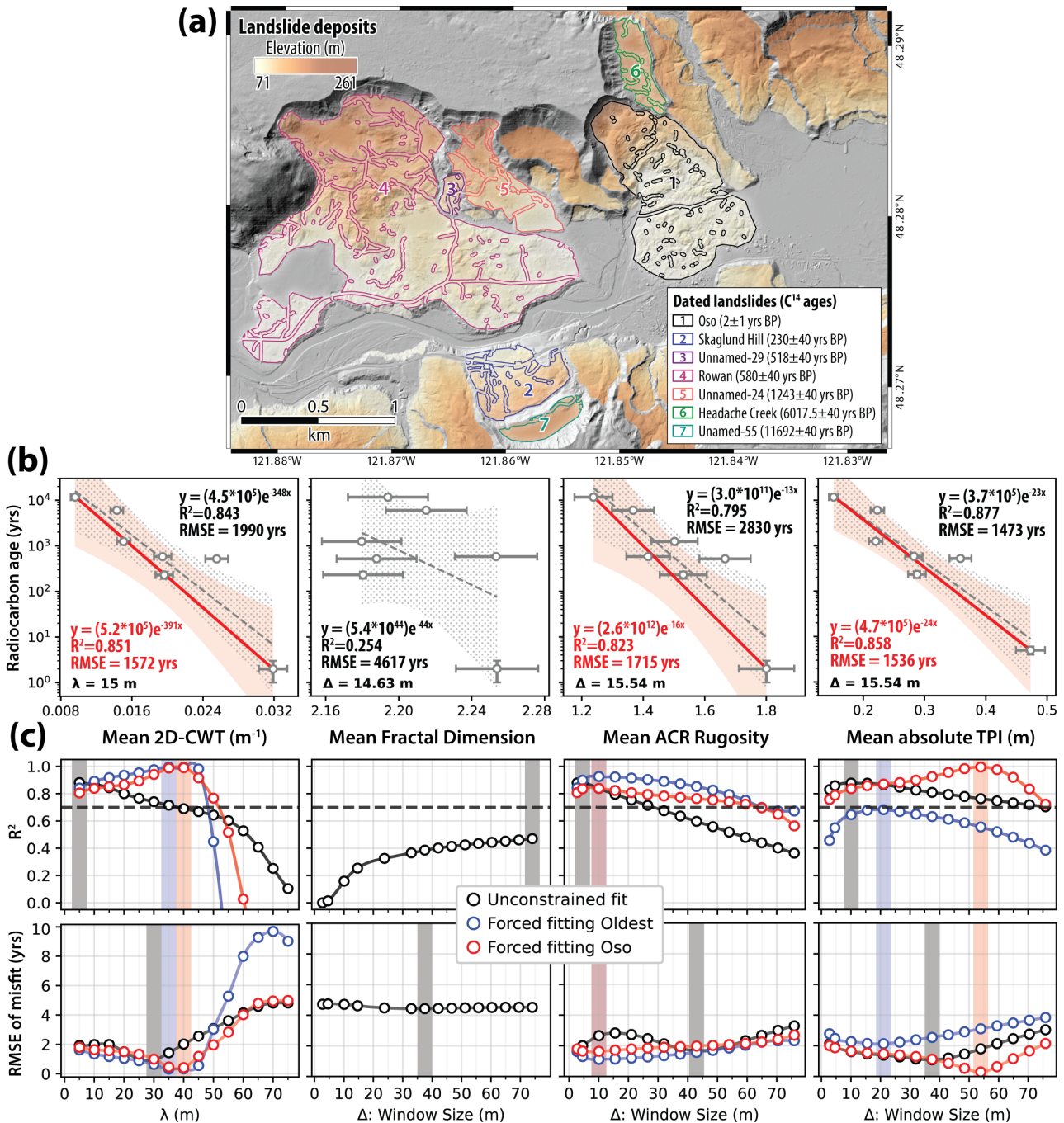
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at each scale. We calculated the coefficient of determination ( $R^2$ ) and root-mean-square error (RMSE) between the log-transformed radiocarbon ages and the function's predictions (**Fig. S2** to **Fig. S7**). In exploring these best-fit functions, we conducted standard linear regression on the log-normal scale of the mean of the seven data points and also constrained the functions to pass through either the youngest (2014 Oso landslide) or the oldest (unnamed-55 landslide) data points. In our  
250 assessment, only the best-fit functions with  $R^2 > 0.7$  and low RMSE values were considered suitable for predicting the age of landslide deposits based on the evolution of surface complexity. The fitting with the highest  $R^2$  value and/or the lowest RMSE indicates the optimal spatial scale for age-complexity correlation, likely representing the characteristic scale of geomorphic features influenced by substrate geology, landsliding mode, dominant weathering processes, and environmental forces (Booth et al., 2009; Booth et al., 2017; Herzig et al., 2024; Lahusen et al., 2020). Best-fit curves with negative  $R^2$  values were excluded  
255 from our discussion, as they suggest an exponential decay model does not adequately explain the data trend.

The results indicate that 2D-CWT and TPI measurements provide the most accurate predictions for the NFSR valley data, followed by the RI, and then the fractal dimension (**Fig. 3b-c**). Curve fitting models constrained to pass through either the youngest (2014 Oso landslide) or oldest (unnamed-55 landslide) data point generally offer better predictability for landslide  
260 ages compared to unconstrained regressions. While previous studies have employed unconstrained regression (e.g., Lahusen et al., 2016; Herzig et al., 2024; Underwood, 2022) or regression constrained to the oldest dated landslide (e.g., Lahusen et al., 2020; Woodard et al., 2024), our findings demonstrate that regressions passing through the youngest landslide data point yield the highest  $R^2$  values and the lowest RMSE for most complexity metrics (**Fig. S2** to **Fig. S9**). Among the RI types (mean of ACR-RI and conventional RI) and TPI measures (standard deviation of TPI and mean of absolute TPI values), similar  
265 predictability is observed, though the mean of ACR-RI and absolute TPI show slightly better performance (**Fig. S4** to **Fig. S7**). Notably, only the mean of the absolute value of 2D-CWT (called "mean 2D-CWT" hereafter) and absolute values of TPI achieve best-fit functions with  $R^2 > 0.98$  and  $RMSE < 500$  years, highlighting them as the most effective surface complexity metrics for estimating landslide ages. This is likely because they focus on quantifying the extent and variability of surface concavity and convexity, which effectively represents the hummocky topography formed by displaced blocks and closed  
270 depressions. Conversely, the fractal dimension fails to establish a satisfactory relationship with  $R^2 > 0.7$ . Although this method can detect scale-invariant complexity features (Huang and Turcotte, 1990; Mark and Aronson, 1984), it may not be suitable for explaining the scale-dependent smoothing trends observed in landslide-prone landscapes.





275 **Figure 3: Evaluation of surface complexity metrics in pyTopoComplexity for estimating landslide deposit ages. (a) Seven landslides with radiocarbon ages before present (BP) from Booth et al. (2017) used in the evaluation. (b) Example of the complexity-age relationship at  $\sim 15$  m spatial scale, showing best-fit exponential decay functions and their 95% confidence intervals. (c) Coefficient of determination ( $R^2$ ) and root-mean-square error (RMSE) values between radiocarbon ages and predicted functions across spatial scales from  $\sim 3$  m to  $\sim 75$  m. Best-fit curves with negative  $R^2$  values are excluded. Shaded bars highlight the optimal spatial scales for age estimation for each method. See complete plots in Fig. S2 to Fig. S7 and further details in the main text.**

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We also applied the same NFSR valley dataset to comparable analyses using other traditional topographic complexity metrics available through native and GRASS plugins in QGIS software (Qgis Development Team, 2024; Grass Development Team, 2024). The evaluated metrics include standard deviations of slope, roughness index, total curvature, maximum curvature, absolute minimum curvature, curvedness index, unsphericity, profile curvature, plan curvature, tangential curvature, difference  
285 curvature, mean curvature (Shary, 1995; Florinsky, 2017), as well as the Terrain Ruggedness Index (TRI) (Riley et al., 1999). Of these, only QGIS's TRI tool allows for multiscale assessment of surface complexity. The results indicate that most traditional metrics, except the standard deviation of plan curvature, provide acceptable predictability for the complexity-age regression (**Fig. S8** and **Fig. S9**). The standard deviation of slope stood out as the best conventional metric for predicting  
290 landslide ages when the regression is constrained to pass through the oldest landslide, consistent with the findings in Lahusen et al. (2016). Nevertheless, the 2D-CWT and TPI methods from pyTopoComplexity still outperform all tested traditional metrics in terms of regression quality, showing higher  $R^2$  and lower RMSE values.

From the multiscale analysis using the two most effective metrics—mean 2D-CWT and absolute values of TPI—we identified the optimal spatial scales for achieving the best-fit regression as  $\sim 40$  m and  $\sim 54$  m, respectively (**Fig. 3c**). These characteristic  
295 scales are notably larger than those suggested in previous studies for similar analyses in the NFSR valley ( $\sim 15$  m), Seattle fault zone ( $\sim 15$  m), and Oregon Coast Range ( $\sim 20$  m) (Booth et al., 2017; Herzig et al., 2024; Lahusen et al., 2020; Lahusen et al., 2016). This discrepancy may stem from differences in regression approaches (e.g., unconstrained regression or those constrained to pass through the oldest landslide data point therein) and the fact that earlier studies only examined spatial scales  
300 between 10 m and 30 m. We also emphasize the unique capability of spectral analysis methods, such as 2D-CWT, in isolating landform signals at specific spatial scales (Perron et al., 2008b; Booth et al., 2009). This indicates that the  $\sim 40$  m optimal scale derived from the 2D-CWT method is likely more representative of the characteristic size of landform features associated with landslide deposits in the NFSR valley, rather than the  $\sim 54$  m suggested by the TPI method. The exponential-decay linear relationship between landslide age and the mean 2D-CWT measure deteriorates when the spatial scale ( $\lambda$ ) exceeds 50 m, indicating that the lengths of hummocks and depressions in these landslide deposits, influenced by substrate geology and  
305 failure mode, are generally smaller than 50 m.

Although using a 40 m spatial scale for 2D-CWT analysis improves the predictability of the complexity-age regression compared to the previously used 15 m scale (Booth et al., 2017), we note that larger spatial scales can introduce greater uncertainties for smaller landslide deposits. For example, the 'unnamed-29' landslide, dated to  $\sim 518$  years before present, has  
310 a relatively small preserved deposit in the modern landscape (**Fig. 3a**). When larger-scale complexity measures are applied, results are more likely influenced by surrounding landform features. This effect is evident in our multiscale analyses, as the unnamed-29 landslide data point progressively becomes an outlier as the spatial scale increases (e.g., **Fig. S2**). To reduce the influence of surrounding landforms, a buffer that increases in width with the spatial scale of analysis could be imposed inside the mapped landslide deposit. However, larger buffers would result in smaller fractions of the central part of the landslide



315 deposit being used to determine its average complexity, introducing bias or possibly eliminating the landslide from the analysis completely. Since pyTopoComplexity's multiscale analysis provides a range of acceptable spatial scales for complexity-age regression, researchers can select the most appropriate scale based on the quality and nature of their dataset.

#### 4 Applications in exploring erosivity parameters in a landslide-prone landform

320 Landscape erosivity parameters, such as hillslope diffusivity  $D$  (Eq. 11) and channel erodibility  $K$  (Eq. 12), are critical in process geomorphology studies for understanding a landscape's susceptibility to erosion under specific environmental and geological conditions. These parameters are also essential for linking real-world observations to numerical modeling. Constraining  $D$  and  $K$  typically requires a quasi-equilibrium landscape that has undergone continuous erosion over a long period under known, stable climatic and tectonic conditions. In such cases,  $D$  can be estimated from the steady-state curvature of hillslopes or hilltops (e.g., Roering et al., 2001; Hurst et al., 2012), while  $K$  can be derived by inverting the longitudinal  
325 channel profile using the stream power law (e.g., Stock and Montgomery, 1999). However, in complex and unstable landscapes like the NFSR valley, which are prone to frequent landslides, estimating  $D$  and  $K$  becomes challenging. Although it is possible to approximate  $D$  based on its correlation with mean annual precipitation (Richardson et al., 2019), constraining  $K$  in a non-steady-state landscape remains impractical. To address this, we present an approach that combines landslide inventory data, modeling results, and surface complexity analysis, providing a potential solution to the problem.

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We set up three modeling scenarios to explore the parameter space of  $D$  and  $K$  in the NFSR valley: (1) hillslope diffusion-only scenario; (2) stream power incision-only scenario; and (3) coupled hillslope diffusion and stream power incision scenario. In each scenario, we independently vary  $D$  and  $K$  and run the simulation for 15,000 years on the 2014 Oso landslide (e.g., Fig. 2). For hillslope diffusion, we set  $S_c = 1.25$  consistent with Booth et al. (2017) and examine a range of  $D$  values from 0.00082  
335  $\text{m}^2 \text{yr}^{-1}$  to  $0.023 \text{m}^2 \text{yr}^{-1}$ , generally in line with measured values from the Pacific Northwest (Roering et al., 1999; Hurst et al., 2012). For stream power incision, we assume  $m = 0.5$ ,  $n = 1$ , and  $\Omega = 0$  as convention (Whipple and Tucker, 1999) and test common  $K$  values ranging from  $0.000001 \text{m}^{0.5} \text{yr}^{-1}$  to  $0.001 \text{m}^{0.5} \text{yr}^{-1}$  (Gasparini and Brandon, 2011). In the coupled scenario, we explore the parameter space by either fixing  $D = 0.0029 \text{m}^2 \text{yr}^{-1}$  or  $K = 0.0001 \text{m}^{0.5} \text{yr}^{-1}$ , while varying the other. After completing the simulations, we applied 2D-CWT analysis with pyTopoComplexity to the simulated landscapes at each time  
340 step, extracting surface complexity signals across various spatial scales and monitoring their evolution over time. Finally, we compared these simulation trends with the real-world NFSR valley data.

The results show that, as expected, hillslope diffusion smooths the landscape over time at all spatial scales, while fluvial incision increases complexity by dissecting the terrain (Fig. 4). In the hillslope diffusion-only scenario, higher  $D$  values  
345 accelerate the reduction of surface complexity, with smaller-scale (<15 m) complexity diminishing more quickly and showing greater sensitivity to changes in  $D$ . In the stream power incision-only scenario, channelization significantly increases surface

complexity, particularly at smaller scales (<30 m). Regardless of the value of  $K$ , there exists a consistent maximum complexity value at every spatial scale. Simulations employing higher  $K$  values attain this maximum value more rapidly in time. Once this time threshold is exceeded, surface complexity declines rapidly as the entire topography of the landslide deposit is progressively eroded by the retreating headwaters and drainage systems.

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By comparing real-world NFSR landslide data to these scenarios running with separate models, we assure that fluvial incision process alone cannot produce the natural smoothing trend of the post-landslide landscape recovery. While the hillslope diffusion-only scenario can generate an increasingly smoothed landscape over time, no single run using a constant  $D$  value can replicate the observed exponential-decay relationship between surface complexity and landslide age across the entire real-world dataset. For a given  $D$ , the non-linear diffusion model tends to underpredict the reduction rate of surface complexity during the early stages of landscape recovery following a catastrophic landslide. The timescale for this reduction appears to be scale-dependent: smaller-scale features (e.g., ~5–15 m) experience underprediction within  $10^1$  to  $10^2$  years, while larger features (e.g., >25 m) are affected over a longer period of  $10^2$  to  $10^3$  years. After  $10^3$  to  $10^4$  years, the model tends to over-smooth small-scale features. These observations suggest the need for a mechanism that accelerates complexity reduction during the initial recovery phase, a roughening process to counteract over-smoothing by diffusion over longer timescales, and an explanation for the observed scale dependency.

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Since the position of the NFSR river bed at the toes of the studied landslides remained relatively stationary during the Holocene (Lahusen et al., 2016), the rate of landform smoothing on these landslide deposits is unlikely to have been affected by changes in the local base level within the timeframe of our investigation (the last ~15,000 years). Thus the observed initial acceleration of smoothing can be attributed to higher in-situ diffusivity  $D$ , reflecting increased hillslope sediment transport efficiency at the beginning of landscape recovery when loose and unconsolidated materials covered the barren landslide deposits (Booth et al., 2017). The reactivation of landslides, which commonly occurs in the early stages of recovery before the deposits settle, could also offer an additional roughening process over shorter timeframes. An example is the Hazel landslide in the NFSR valley, where hillslope movement persisted for decades before the Oso landslide occurred in the same area (Miller and Sias, 1998). However, since the natural reestablishment of vegetation and soil development typically spans  $10^1$  to  $10^2$  years (e.g., Fu et al., 2017; Kennedy et al., 2012; Seidl et al., 2014; White et al., 2022; Russell and Michels, 2011), other roughening mechanisms are necessary to sustain surface complexity over longer periods. One possible factor that maintains topographic complexity after revegetation is tree throw, which can continuously create new small-scale (~7.5 m) roughening features (Roering et al., 2010). Long-term climate change could be another important factor. In the Pacific Northwest, research suggests a drier climate from approximately 10,000 to 6,000 years ago, followed by a transition to wetter conditions similar to those of the present day in the last ~6,000 years (Leopold et al., 1982; Brubaker, 1991). This climate shift implies that hillslope diffusivity may have increased over the Holocene as precipitation levels rose (Richardson et al., 2019), potentially explaining the complexity-age patterns observed in smaller-scale features.

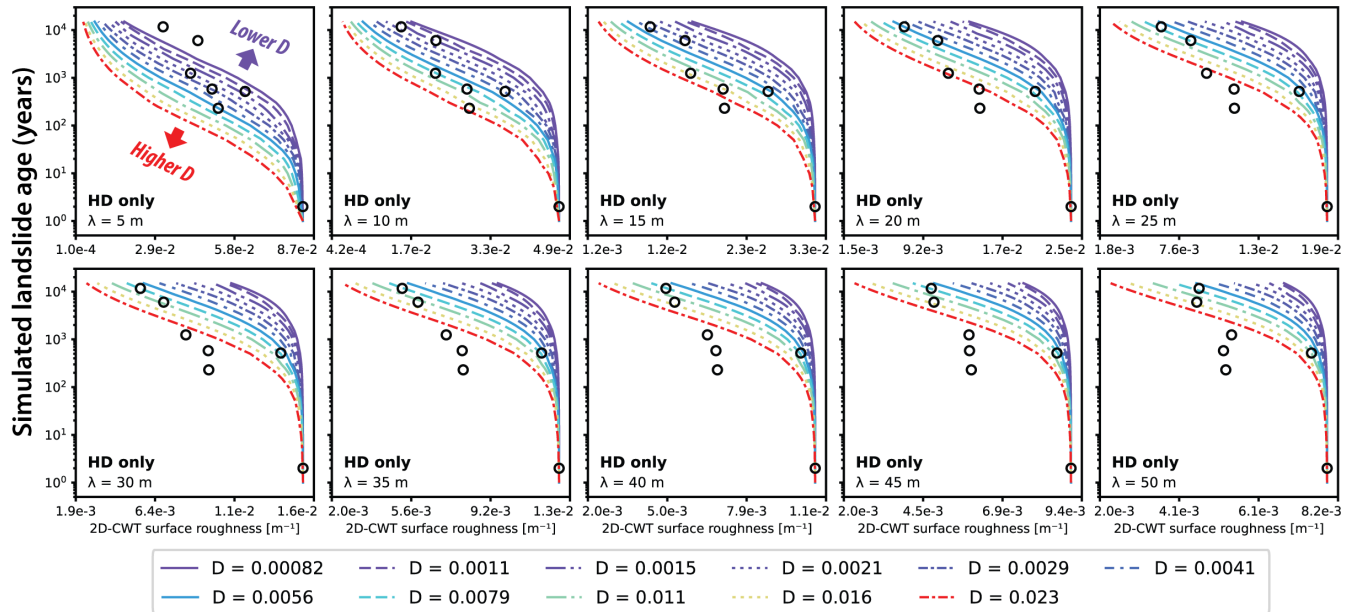
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**(a) Nonlinear Hillslope Diffusion only**



**(b) Stream Power Incision only**

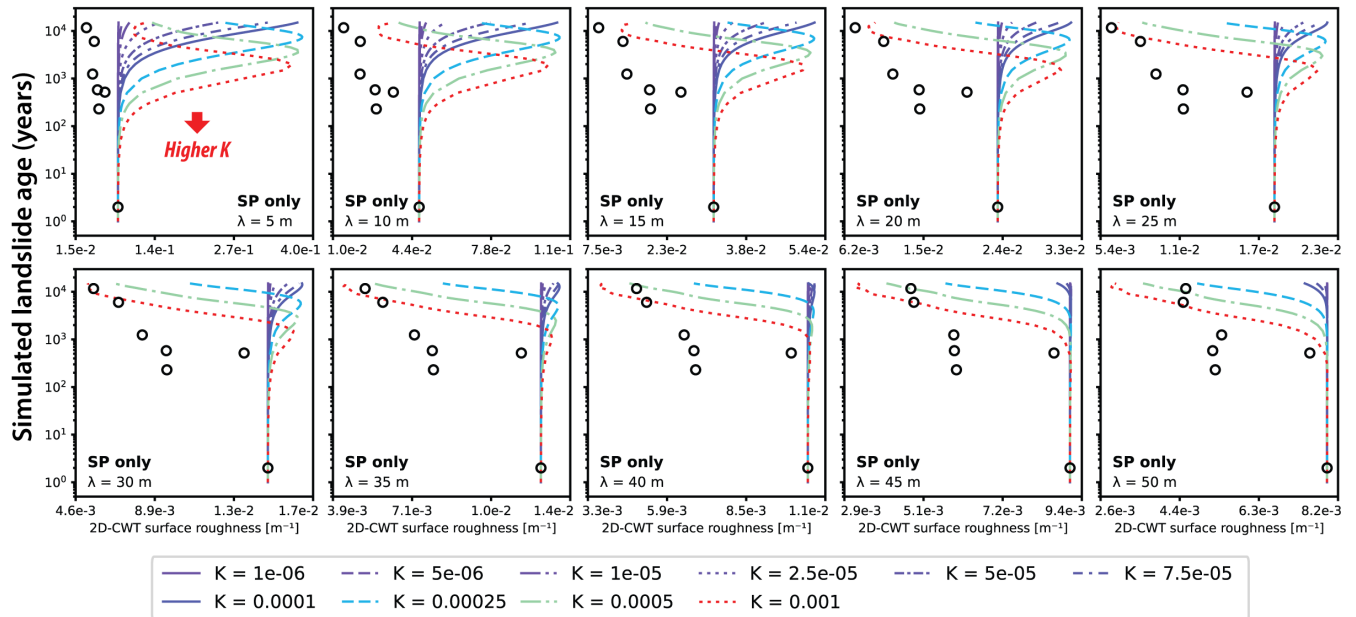


Figure 4: Changes in mean 2D-CWT surface complexity measures for the simulated landscape of the 2014 Oso landslide over a 15,000-year period, predicted by models of nonlinear hillslope diffusion (a) and stream power incision (b) in *Landlab\_simulation.ipynb*, with varying hillslope diffusivity ( $D$  in  $\text{m}^2 \text{yr}^{-1}$ ) and channel erodibility ( $K$  in  $\text{m}^{0.5} \text{yr}^{-1}$ ). Black circles represent real-world data from the seven dated landslide measurements in the NFSR valley (Fig. 3a).

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Another possibility, without changing  $D$  over a longer timescale, is to consider the combined effects of channel incision (which roughens the landscape) and hillslope diffusion (which smooths it). In our modeling scenario with these coupled processes, we produce a complexity-age trend that more closely matches the observed NFSR valley data at smaller spatial scales while keeping  $D$  and  $K$  constant (**Fig. 5**). For the 5 m scale, the NFSR valley data are roughly bounded by modeled curves with  $K$  values between  $0.0001 \text{ m}^{0.5} \text{ yr}^{-1}$  and  $0.00037 \text{ m}^{0.5} \text{ yr}^{-1}$ , and  $D$  values between  $0.0011 \text{ m}^2 \text{ yr}^{-1}$  and  $0.0041 \text{ m}^2 \text{ yr}^{-1}$ . Comparing the simulation results between fixed  $D$  and fixed  $K$  indicates that the pattern of complexity reduction is more sensitive to variations in  $K$ . Only a narrow range of  $K$  values for a given  $D$  results in the observed exponential decay relationship between surface complexity and landslide age, suggesting a natural scaling relationship between  $D$  and  $K$  for a given environmental and geological setting. This finding is consistent with observations from steady-state landforms, where the length scale (i.e., the scale of landform complexity) between evenly spaced channels and ridgelines is governed by a specific constant  $D/K$  ratio (Perron et al., 2008a; Perron et al., 2009), reflecting the balance and self-regulation between diffusive smoothing and channel dissection. If this model-coupling hypothesis holds, it suggests that the time required for a landscape to recover from a rugged landslide surface to its background complexity level may be longer than previously estimated using diffusion-only simulations (c.f. Woodard et al., 2024) due to the prolonged resistance to smoothing by continuous channel incision.

While our estimated  $D$  range at 5 m generally aligns with the estimation from Booth et al. (2017) at 15 m scale, we note that we are unable to reproduce the same bracketed  $D$  value range using 15 m scale complexity measurements. In fact, the complexity reduction trend varies across different spatial scales in all of our simulation scenarios, which is different from the consistent exponential decay pattern in the real-world observation across the scales (at least for 5 m to 50 m in 2D-CWT analysis) (**Fig. S2**). Specifically, for the diffusion-only models, larger values of  $D$  better match the observed landslide age-complexity data as the spatial scale increases. The  $D$  values most consistent with those that have been independently inferred from hilltop curvature in the Pacific Northwest only fit our landslide data reasonably well when surface complexity is quantified at relatively short length scales (e.g., <15 m). We suggest that this is because  $D$  values are typically inferred from hilltop curvature measured from smoothed lidar data at similarly short length scales.

When stream power incision is introduced into the model, it roughens the landscape at smaller scales through channel dissection (**Fig. 2b-c** and **Fig. 3b**). This indicates that a higher initial diffusivity ( $D$ ), as previously discussed, may still be necessary to counterbalance the complex landforms created by channel incision within the first  $10^1$  to  $10^2$  years following a landslide event. On the other hand, the modeling bias toward higher complexity at larger spatial scales suggests that the intricacies of natural behavior in landslide-prone landscapes cannot be fully captured using simple models of nonlinear diffusion (**Eqs. 10–11**) and stream power incision (**Eq. 12**) with constant erosivity parameters. The deviation between the simulation results between coupled modelling scenario and real-world data at larger spatial scale, particularly at the timeframe of  $10^2$  to  $10^3$  years, may reflect that  $D$  and  $K$  values are scale-dependent. To approach the observed real-world data trend using



these simple models, it requires a much higher  $D$  (and/or a smaller  $K$ ) to explain the reduction rate of the landform features with larger length scale (**Fig. 5**). It seems plausible because the landslides produce a vast range of materials at different length scale, and the mechanical properties and mobility of these materials are likely size dependent (Collins and Reid, 2019; Wartman et al., 2016). Large, sharp, and fractured landslide blocks may be more susceptible to rapid hillslope erosion and  
425 smoothing, requiring a higher  $D$  to account for the age-complexity relationship at larger scales. In contrast, the hillslope erosion rate for smaller, looser particles are more likely limited by local variability in topography, hydrology, lithology, and bioturbation, reflecting a smaller  $D$ .

Although these simple modeling tests presented in this short communication do not fully resolve all observed geomorphic  
430 questions in the NFSR valley, we emphasize the potential of studying topographic complexity in landslide-prone terrains to gain insights into the fundamental erosivity parameters that drive landscape evolution at various spatial scales. The integrated approach of Landlab and pyTopoComplexity offers a quantitative method for evaluating the efficiency of landform adjustments to erosion and understanding dominant surface processes. Additionally, Landlab provides several other components with advanced erosion models that account for complex processes, such as variability in soil production rates on  
435 hillslopes, mass wasting, stream power incision thresholds (e.g., in **Eq. 12**), and sediment transport and cover effects in channels (e.g., Barnhart et al., 2020; Hobbey et al., 2017; Shobe et al., 2017; Campforts et al., 2022), which were not included in our simulations. In our notebook *Landlab\_simulation.ipynb*, researchers can directly import those components from Landlab and explore landscape evolution through the lens of topographic complexity measures.

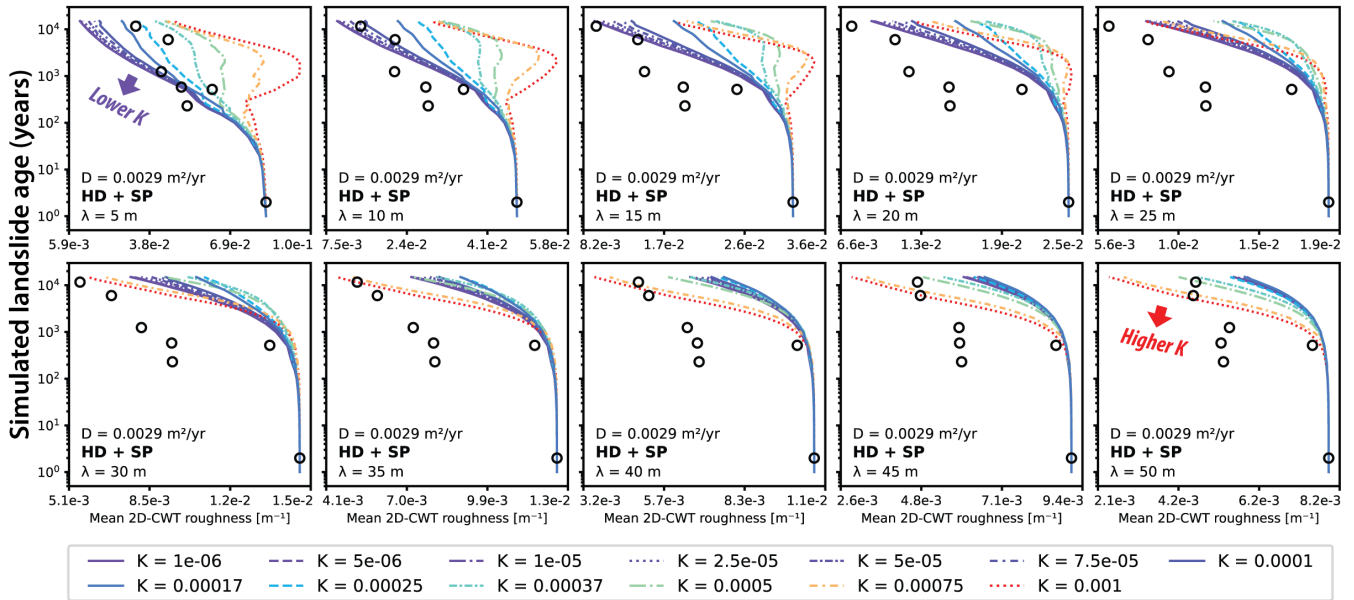
## 5 Conclusions

440 pyTopoComplexity is an open-source software package designed for the efficiently quantifying topographic complexity using advanced methods such as two-dimensional continuous wavelet transform analysis, fractal dimension estimation, Rugosity Index, and Terrain Position Index calculations. By bridging the gap between traditional terrain analysis tools and modern quantitative geomorphology, it offers researchers with robust and reproducible measures of surface complexity across multiple spatial scales, generating insights across the fields, including geology, geomorphology, geography, ecology, and oceanography.

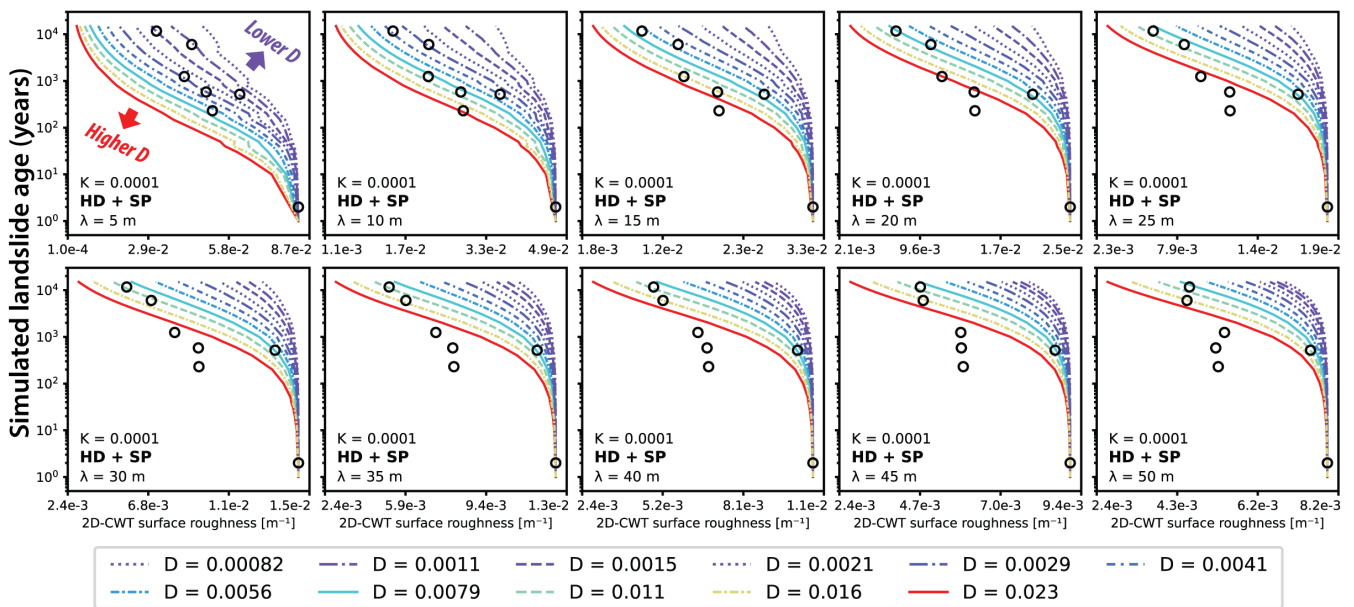
445 A case study in the North Fork Stillaguamish River valley showcases software capability to accurately assess the morphometric properties of landslide deposits, revealing characteristic landform scales and enhancing our understanding of geomorphic processes. The integrated approach using the notebook *Landlab\_simulation.ipynb* combines dated landslide inventories, Landlab's landscape evolution modeling components, and multiscale topographic complexity analysis, demonstrating  
450 pyTopoComplexity's effectiveness in linking real-world data with simulation-based insights. This framework allows researchers to explore landscape recovery rates and mechanisms by estimating in-situ hillslope diffusivity ( $D$ ) and channel erodibility ( $K$ ), critical factors in understanding landscape evolution and response to disturbances. Leveraging the modular flexibility of Landlab, researchers can also integrate other erosion models to investigate how different processes shape the dynamic evolution of topographic complexity in response to environmental forces and geomorphic events like landslides.



**(a) Nonlinear Hillslope Diffusion (fixed D) + Stream Power Incision (varying K)**



**(b) Nonlinear Hillslope Diffusion (varying D) + Stream Power Incision (fixed K)**



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**Figure 5: Changes in mean 2D-CWT surface complexity measures for the simulated landscape of the 2014 Oso landslide over a 15,000-year period, predicted by coupled models of nonlinear hillslope diffusion and stream power incision in *Landlab\_simulation.ipynb*. (a) Simulations with a fixed  $D = 0.0029 \text{ m}^2 \text{ yr}^{-1}$  and varying channel erodibility ( $K$  in  $\text{m}^{0.5} \text{ yr}^{-1}$ ). (b) Simulations with a fixed  $K = 0.0001 \text{ m}^{0.5} \text{ yr}^{-1}$  and varying  $D$  in  $\text{m}^2 \text{ yr}^{-1}$ . Black circles represent real-world data from the seven dated landslide measurements in the NFSR valley (Fig. 3a).**

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## 6 Code and data availability

The pyTopoComplexity software, related Jupyter Notebooks, and example data are available in <https://doi.org/10.5281/zenodo.11239338> and <https://github.com/GeoLarryLai/pyTopoComplexity>. An early version of the codes in MATLAB for the 2D-CWT analysis, following methods in Booth et al. (2009), is available on Adam M. Booth's personal website (<https://web.pdx.edu/~boothad/tools.html>). Data of mapped landslide polygons and radiocarbon dating results in the NFSR valley are from Booth et al. (2017).

## 7 Author contributions

LShL spearheaded the design of the pyTopoComplexity package and manuscript preparation. AMB and ARD conceptualized the research idea and secured funding for its development. AMB contributed to the core fundamental functionality of the *pycwtmexhat.py* module, while EH assisted with refactoring. All authors edited the manuscript.

## 8 Competing interests

The author declares that they have no conflict of interest.

## 9 Acknowledgements

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## Appendix A: Notation

$A$	Upstream drainage area [ $\text{m}^2$ ]
$A_c$	Contoured area (i.e., true geometric surface area) within a square window of $\Delta^2$ [ $\text{m}^2$ ]
$A_p$	Planimetric area within a square window of $\Delta^2$ [ $\text{m}^2$ ]
$C$	Resultant wavelet coefficient of 2D-CWT analysis [ $\text{m}^{-1}$ ]





$D$	Hillslope diffusivity [ $\text{m}^2 \text{yr}^{-1}$ ]
$E$	Erosion rate [ $\text{m yr}^{-1}$ ]
$K$	Channel erodibility coefficient (units vary with $m$ and $n$ ) [ $\text{m}^{0.5} \text{yr}^{-1}$ when $m = 0.5$ and $n = 1$ ]
$m$	Exponent of drainage area ( $A$ ) in stream power equation
$N$	Number of terms in the Taylor expansion of hillslope sediment flux equation
$n$	Exponent of channel slope ( $S_{sp}$ ) in stream power equation
$p, P$	Lag distances ( $p = 1, \dots, P$ ) along the one-dimensional topological profile for variogram analysis [m]
$\vec{q}_s$	Two-dimensional vector of the sediment flux per unit slope width at the surface
$r, R$	Length steps ( $r = 1, \dots, R$ ) along the one-dimensional topological profile for variogram analysis [m]
$S_c$	Asymptotic maximum hillslope gradient
$\vec{S}_{hd}$	Two-dimensional vector of topographic downslope gradient at each grid point
$S_{hd}$	Hillslope gradient (i.e., the magnitude of $\vec{S}_{hd}$ )
$S_{sp}$	Channel slope
$s$	A wavelet scale parameter for 2D-CWT analysis
$t$	Time [yr]
$x$	Distance in easting direction of each grid cell in the DEM raster [m]
$y$	Distance in northing direction of each grid cell in the DEM raster [m]
$z$	Distance in up direction (i.e., elevation) of each grid cell in the DEM raster [m]
$\bar{z}_{nb}$	Mean elevation of grid cells surrounding each grid point within a square window of $\Delta^2$ [m]
$\alpha$	Coefficient of the ideal power-law one-dimensional variogram function
$\beta$	Exponent of the ideal power-law one-dimensional variogram function
$\gamma_1$	One-dimensional variogram function
$\Delta$	Designated window size for spatial analysis (number of grids times $\delta$ ) [m]
$\delta$	Grid spacing of the DEM raster [m]
$\lambda$	Fourier wavelength of the wavelet function $\psi$ [m]
$\psi$	2D Ricker or Marr wavelet (i.e., Mexican hat wavelet) function
$\Omega$	Threshold of channel erosion [ $\text{m yr}^{-1}$ ]

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