Report on
On the hydrostatic approximation in rotating stratified flow
by Achim Wirth
Review 2

The new version of the text presented by the author has been much improved. Nevertheless, there are still grey areas that need to be clarified. I suggest that the result of **this studies would be appropriate for publication** if the author revised the manuscript taking into account the comments below. I am confident that author will carefully consider them.

A) According to author: "The equation for the evolution step is the same in both formalisms, it is Eq. 1 (the third component is never used in the hydrostatic formalism as the vertical eulerian-acceleration is calculated based on the horizontal divergence). I now added in the beginning of section 3:...".
I cannot translate this sentence into a clear mathematical equation and there are still some elements that are mathematically unclear. For example, I think that my equations (R2) to (R6) written in my previous review correspond to the mathematical translation of the author's sentence. If so, then the author should specify equations (R2) - (R6), if not, then the author should write the equation that gives (19) (new version). This is an important point that I maintain, because it creates some confusion about the mathematical formulation. However, the core of this article is the quantification of the differences between two formalisms.

B) According to author: "I disagree with the reviewer $\mathbb{P}^N$ is perpendicular to $\mathbf{k}$, but $\mathbb{P}^H$ is not"
In fact, to be precise, according to your Figure 2 (new version), $\mathbb{P}^N(\hat{\mathbf{a}})$ is the **orthogonal** projection of $\hat{\mathbf{a}}$ onto the plane perpendicular to $\mathbf{k}$ while $\mathbb{P}^H(\hat{\mathbf{a}})$ is the **vertical** (non-orthogonal) projection (non-orthogonal) projection of $\hat{\mathbf{a}}$ in the plane perpendicular to $\mathbf{k}$. These projections for the two formalisms are always in a plane perpendicular to $\mathbf{k}$ to respect the free divergence. This geometric difference between the orthogonal projection and the vertical projection reflects the central difference between the two formalisms. This distinction must be clearly indicated in the legend of Figure 2.

C) According to Author "The reviewer is right, there is a difference in the definition of $\mathbb{P}^N$ and its representation in the Figure explaining the projection, because $\mathbb{P}^H$ is not only the projection in the vertical "down" on the subspace of zero divergence (this only the last part), there are two more steps involved: the forgetting of the vertical dynamic acceleration and the addition of the buoyancy acceleration to the horizontal acceleration. The

Fig. and its legend are now corrected. In the text it was ok. When I give a talk on the subject I spent 10 min on this fig. as it explains all the projection formalism. I added in the text:..."

For a reader like me, we need to know where we are going when we start a new section. Indeed, we do not know why to extend the fourth dimension to the operator, and it takes time (several hours in my case) to finally understand the interest of the mathematical tricks. This means that the author must start explaining the issues at the beginning of the section, for example:

"The purpose of this section is to express the operator $\mathbb{A}$ that measures the difference between the two formulations $\mathbb{P}^N$ and $\mathbb{P}^H$ that are defined in (20) and (22) (new version). However, the problem comes from the difference in the input vectors a^N and a^H in the operators $\mathbb{P}^N$ and $\mathbb{P}^H$. In order to express a single input of the operator $\mathbb{A}$ we use a vector $\tilde{a}$ which is defined in (24) (new version) by finding a common basis between the operator $\mathbb{P}^N$ and $\mathbb{P}^H$."

D) According to author "The square-root of difference in the square of the horizontal wave velocity between the two formalisms... phase speed."

What is the definition of c^N and c^H? It should be specified in the text (I assume they are the phase velocities)..