Supplementary Material: Modeling memory in gravel bed rivers: A flow history-dependent relation for evolving thresholds of motion

Claire C. Masteller[1], Joel P.L. Johnson[2], Dieter Rickenmann[3], and Jens M. Turowski[4]

[1]Department of Earth, Environmental, and Planetary Sciences, Washington University in St. Louis.

[2]Jackson School of Geosciences, University of Texas at Austin.

[3]WSL Swiss Federal Institute for Forest, Snow, and Landscape Research.

[4]Helmholtz Centre Potsdam, GFZ German Research Centre for Geosciences

Corresponding author: Claire Masteller (cmasteller@wustl.edu)

Introduction

We provide three supplementary figures to support the model calibration (Fig. S1, S2) and to illustrate model performance (Fig. S3). We provide a description of the calibration of the γ parameter in the state function model (Text S1).

Text S1. Description of the γ parameter calibration

Paphitis and Collins (2005) conducted experiments using fine, medium and coarse sand, in which they systematically varied both the conditioning time (E_D, the duration of flow between threshold condition, between 5 and 120 minutes), and also the ratio of shear velocity (u_τ) to initial critical shear velocity ($u_{\tau ci}$). The ratio of shear velocities they explored was between 70% and 95% of critical, which corresponds to conditioning flow shear stresses between 50% and 90% of the initial critical shear stress. Paphitis and Collins (2005) present a rather complex equation that they fit to their experimental data:

$$\frac{u_{\tau c(t)}}{u_{\tau ci}} = 1.05\left[1 - 0.01e^{(-0.005E_D)}\right] + \left[0.005 + 0.1\left(\frac{u_\tau}{u_{\tau ci}} - 0.7\right)\right] ln(E_D) + 0.06\left[10^{-7\left(0.97-\frac{u_\tau}{u_{\tau ci}}\right)}\right]$$

(Equation S1) for $0.7 \leq \frac{u_\tau}{u_{\tau ci}} \leq 0.95$ and $E_D \leq 120$ minutes.

The function they present fits their data with a correlation coefficient of 0.83 (i.e., R2=0.69). To calibrate γ, we calculate $u_{\tau c(t)}/u_{\tau ci}$ for a range of E_D and u_τ, using values of $u_{\tau ci}$ they report. We then calculate τ_C^* from $u_{\tau c(t)}$. We then numerically calculate the partial derivative of equation (S1) with respect to time, $\partial\tau_C^*/\partial t$. Figure S2 shows a nonlinear regression (using Matlab's cftool) of the strengthening term in our model (equation 1) to $\partial\tau_C^*/\partial t$ calculated from equation S1. This regression provides a best-fit estimate of γ, including empirical regression uncertainties. Remember that the weakening term in equation (1) is zero for the below-threshold flow conditions explored in their experiments, and so the fitting of gamma is not influenced by other model parameters, in particular weakening exponent ε.

Our reported 95% confidence interval on gamma only represents the empirical regression uncertainty when fitting our function (the strengthening term) to their function (equation Sx). Therefore, it is likely that a somewhat wider range of gamma may be able to fit the range of their data, and the true range of possible values may be somewhat larger than 2.5+-0.32. We hypothesize that a range of γ =2 to γ =3 may be possible.

We use the data and fitting function of Paphitis and Collins (2005) to calibrate γ because it is the most complete and internally consistent data set we are aware of with sufficient constraints on how thresholds evolve with both tau*/tau*c and time. Nonetheless, a possible limitation of applying these experimental data to calibrate our model and then applying it to gravel-bed channels is that the Paphitis and Collins (2005) experiments were conducted with unimodal sand. In particular, boundary Reynolds numbers in their experiments are transitional between hydraulically smooth and hydraulically rough flow. For the coarsest grains they use (D_{50}=0.0774 mm), boundary Reynolds number $Re_w = u_\tau k_s/\upsilon \approx 15$, where k_s is a roughness length scale assumed to be D_{50}, and υ is the kinematic viscosity of water. If ks was instead assumed to be a multiple of D50 (such as ks=3.5D84) then Re_w would be closer to the hydraulically rough flow criteria of $Re_w \geq 100$. It is also worth noting that grain size did not explicitly factor into equation (S1), beyond its implicit control on $u_{\tau ci}$. The insensitivity of their results to grain size suggests that the results may not depend significantly on grain size or on hydraulically rough flow being fully developed.

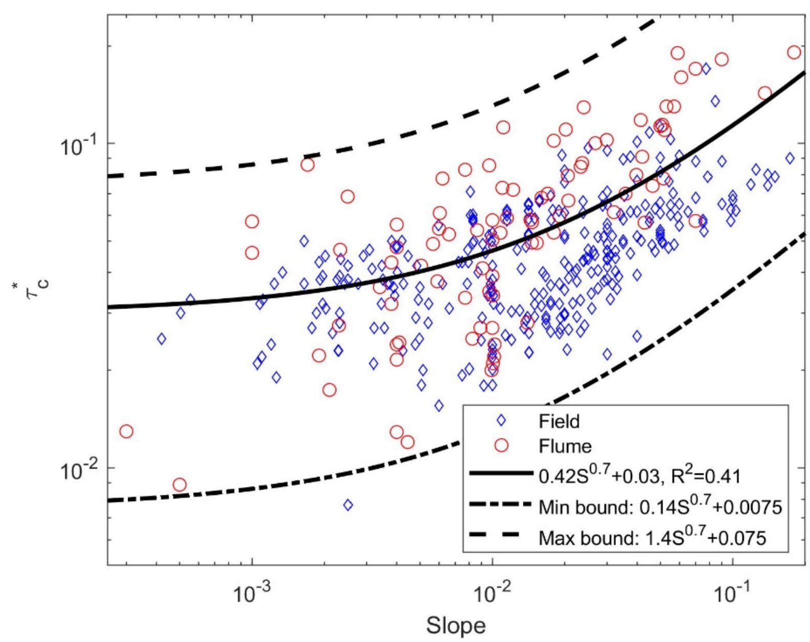

Figure S1. Field and flume data are weighted equally in the best-fit regression, removing possible bias from there being ~3.5 times more flume data points. Minimum and maximum bounds were determined visually to accommodate almost all data points, assuming the best-fit exponent (0.36). Compiled data are limited to slopes < 0.2 and D50 ≥ 2 mm. Most data were compiled by Prancevic and Lamb (2015), building on Buffington and Montgomery (1997), with additional data from Olinde (2015) and Lenzi et al. (2006).

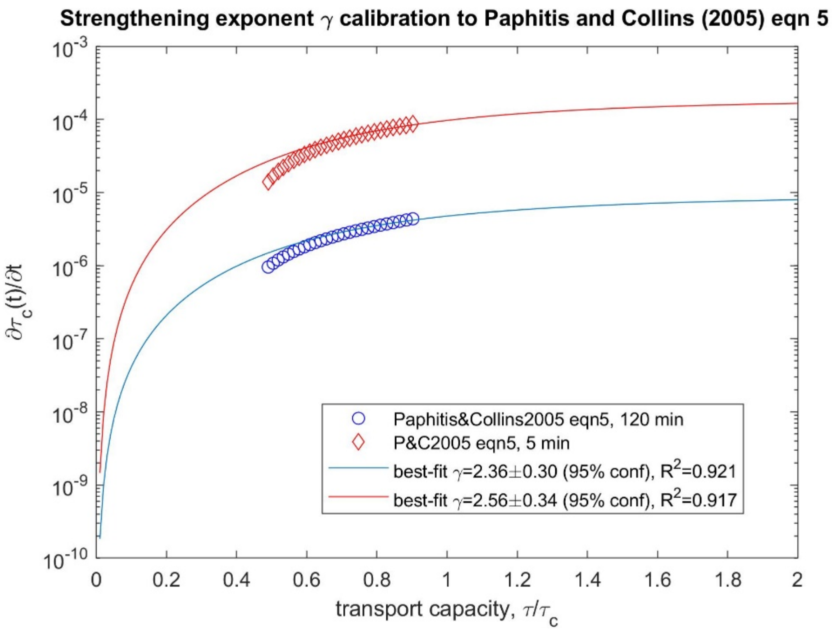

Figure S2. Calibration of strengthening term exponent gamma based on Paphitis and Collins (2005). For both the shortest conditioning time (5 minutes) and longest conditioning time (120 minutes) spanned by the Paphitis and Collins (2005) data, regressions to their best-fit empirical equation give gamma exponents within uncertainty of each other.

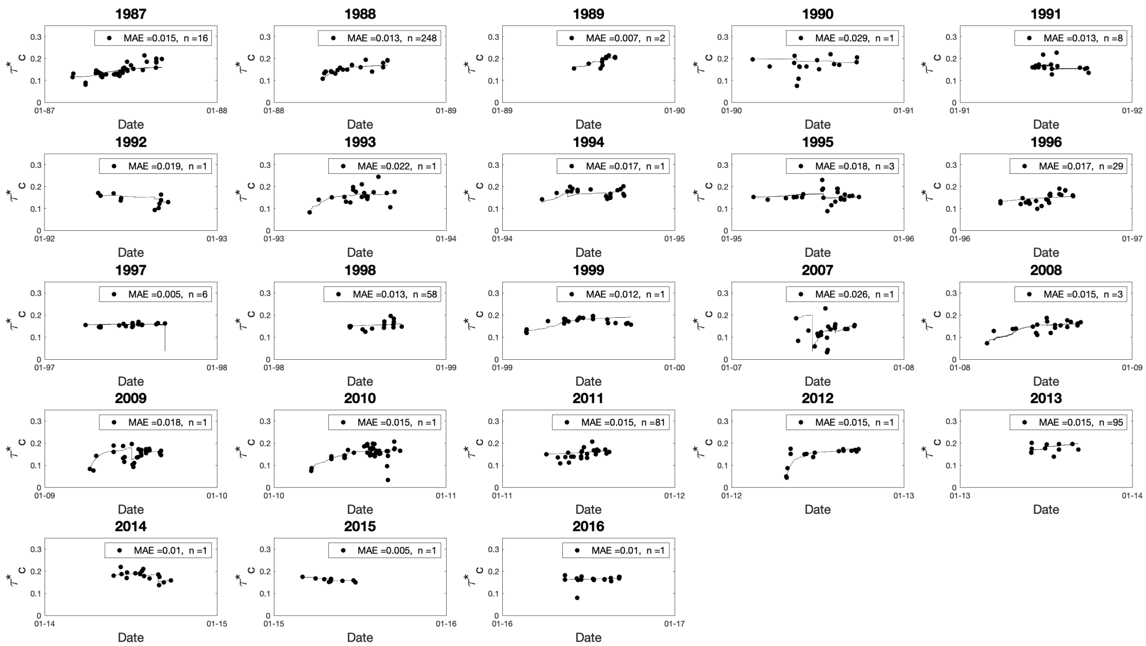

Figure S3. Best-fit model runs for every year on the record. "n" gives the number of models (i.e., parameter combinations) that minimize MAE and provide an equivalent best fit to the data. Annual best-fit parameters are also specified.