Response to Reviewer 1: Our comments are provided in blue. Text modifications are provided in green.

The manuscript provides a derivation of the well-know and well-studied Gassmann's equations. It is not clear why another derivation is needed and if so, how different is it from their previous work that they reference.

General Response and Main Contribution

Thank you very much for the detailed and constructive review. Before addressing each comment in detail, we would like to first clarify the main contribution of this manuscript.

Previous thermodynamic derivations of poroelasticity (such as Yarushina and Podladchikov, 2015) **were based on the assumption that the second derivative matrix (the Hessian) of the internal energy is diagonal.** This means that certain couplings between solid and fluid variables were neglected from the start. In particular, this approach leads directly to a situation where two solid bulk moduli — often denoted K' and K'' — are identical.

In this work, we remove the diagonal assumption and keep the full form of the Hessian matrix, allowing for couplings between the solid and fluid. As a result, K' and K'' appear naturally as two independent solid-phase moduli within the general thermodynamic formulation. We believe this generalization significantly strengthens the theoretical foundation of Gassmann's and Biot's equations, and provides a fully transparent and extendable thermodynamic framework.

We appreciate this important point and have revised the manuscript to better clarify the novelty and motivation. The primary purpose of our work is to provide a structured, thermodynamically admissible derivation of the extended Biot poroelastic framework, in which Gassmann's equations emerge as a special limiting case. This is particularly relevant in light of recent debates questioning the validity of Gassmann's equations (e.g., Thomsen 2023–2025), where it has been argued that an additional solid bulk modulus is necessary even for monomineralic rocks.

In our revised manuscript, we emphasize that the self-similarity condition (which underpins Gassmann's derivation) is not simply an arbitrary assumption but follows rigorously from the requirement of zero entropy production in reversible poroelastic deformation. By explicitly incorporating the full matrix of second derivatives of internal energy (i.e., off-diagonal coupling terms), we provide a general formulation that includes the standard Gassmann, Brown & Korringa, Detournay–Cheng, and Biot models as limiting cases.

Furthermore, the present manuscript offers complete symbolic derivations with reproducible Maple code, allowing full transparency and reproducibility. This level of detail and reproducibility distinguishes our approach from previous works.

The manuscript has been significantly revised and reorganized based on your valuable feedback. Given the extent of the changes (approximately 90% of the manuscript has been restructured, expanded, or rewritten), we respectfully suggest that the reviewer refer directly to the revised manuscript for a complete view of the updated content. Including all modifications inline here would not be efficient or practical.

They first make the assumption about linear, elastic, homogeneous material, but then they give derivations with viscoelastoplasticity and elastic versus non-elastic pore deformation. It would be better to re-structure the more general parts first and then make the assumptions about linear elastic materials. Why the reference to chemical potentials when Gassmann's theory is not about chemical reactions?

We fully agree that a clearer structure was needed. In the revised manuscript, we carefully reorganized the presentation:

First, we introduce the general thermodynamically consistent formulation based on classical irreversible thermodynamics (CIT), in which all terms are derived systematically from conservation laws and internal energy potentials.

Then, we derive specific forms for linear elastic poroelasticity, including the standard Biot and Gassmann equations as special cases of the general formulation.

Regarding chemical potentials: we acknowledge that Gassmann’s theory does not involve chemical reactions. The reference to chemical potentials was unnecessary and potentially confusing. We have removed this terminology throughout the revised manuscript.

Check equation 11. Should be 1/K_phi, otherwise not consistent.

We fully agree with the reviewer. The term should represent compliance rather than incompressibility. We have corrected Equation 11 (with a new number) accordingly in the revised manuscript.

References to their own previous numerical work is to cursory and not clear.

We appreciate this suggestion. In the revised manuscript, we have significantly expanded the discussion and added detailed numerical results (Figure 1) to better illustrate and validate the theoretical framework. These numerical simulations explicitly demonstrate the convergence of the full 3D finite-element model towards the Gassmann solution. We also provide the material properties, pore geometry, and convergence analysis in detail, fully linking the present derivation to our previous numerical studies.

If the basic assumption is about a homogeneous mineralogy then why at the end the discussion about heterogeneous minerals?

This is an important clarification. While Gassmann’s original formulation applies to homogeneous monomineralic media, recent critiques (e.g., Thomsen 2023–2025) argue that even in these cases additional moduli are needed. In response, we include a brief discussion on heterogeneous mineralogy to address the relevance of the Brown & Korringa (1975) framework and clarify under what conditions distinctions between the solid grain moduli become practically significant.

Our numerical results demonstrate that for monomineralic, deviations between these extended formulations and Gassmann’s original equations are negligibly small. This justifies the continued validity of Gassmann’s equations in many practical cases, while providing a more general framework for future extensions.

We thank the reviewer once again for their constructive feedback, which helped us substantially improve the clarity and completeness of our manuscript.

Sincerely,
Yury Alkhimenkov and Yury Podladchikov