

1 Supplementary Materials

2 3 Supplementary Text 1. Pre-calculated ocean function $O(t)$ for first outer 4 iteration

5 As discussed in section 2.3, a pre-calculated ocean function accounting for the
6 load/ocean transition is used for the first outer iteration in our SLE solver to make the
7 solutions from the first outer iteration as good as possible. The goal is to make it possible to
8 obtain accurate RSL results by running just one forward GIA calculation instead of 3-4 outer
9 iterations, at least under some circumstances. It should be mentioned that the outer iterations
10 are necessary if the initial topography used in the first outer iteration is not ideal, although in
11 our case the present-day topography is a good approximation to the initial topography at the
12 last interglacial period.

13 Here we describe our approach to derive the pre-calculated ocean function $O(\theta, \phi, t)$
14 for the first outer iteration. The idea is calculating the change of ocean area (i.e., ocean-
15 continent transitions) based on ice volume change (i.e., $\Delta I(t_i)$) and the present-day
16 topography $T_0(\theta, \phi)$, assuming barostatic sea level change on a rigid Earth (i.e., no radial
17 surface displacement). Note by assuming barostatic sea level change, we mean the total ocean
18 water volume change causes a uniform sea level change globally, which caused the change of
19 ocean area. First, it is easy to find ocean function if the barostatic sea level is known as H
20 (meters higher than the present-day sea level), that is, $O(\theta, \phi, H) = 1$ if $T_0(\theta, \phi) - H > 0$,
21 and $O(\theta, \phi, H) = 0$ otherwise. Then, the barostatic sea level at time t_i can be determined
22 from ice volume change $\Delta I(t_i)$. Define the ocean area $A(H) = \int_{\Omega} O(\theta, \phi, H) ds$, where Ω is
23 the surface. By mass conservation, $\int_0^H \rho_w A(h) dh = -\rho_{ice} \Delta I(t_i)$, then, the barostatic sea
24 level H can be determined at stage t_i . Since the $O(\theta, \phi, H)$ is known, the $O(\theta, \phi, t_i)$ is thus
25 determined for each stage t_i .

27 **Supplementary Text 2. A one-iteration solution method for the sea level**
28 **equation.**

29 For our GIA benchmark with ICE-6G_D, we implemented the multiple outer iteration
30 algorithm by Kendall et al., (2005) for the sea level equation in our semi-analytical code (A et
31 al., 2013). For ICE-6G and VM5a, calculation K3 represents the reference case with
32 convergent solutions after three outer iterations, based on Kendall's original approach. The
33 normalized ocean area which is a measure of the ocean function $O(t)$ for K3 varies between
34 ~ 0.66 at the last glacial maximum (LGM) and ~ 0.71 at 122 kybp and the present-day (Fig.
35 S1). Figure S1 also shows the ocean area after the first outer iteration for calculation K3,
36 which, denoted as K1, differs significantly from that of K3. Calculation AS1 represents a
37 single outer iteration model run using our pre-calculated ocean function $O(t)$ as discussed in
38 section 2.3, and AS2 represents the results from the second outer iteration after AS1 using the
39 updated ocean functions $O(t)$ and initial topography T_0 , Figure S1 clearly demonstrates that
40 AS1, different from K1, is very similar to K3 and AS2, while the latter two are identical,
41 indicating that the ocean function $O(t)$ for our first outer iteration (AS1) is a fairly accurate
42 representation of the convergent solutions of the Kendall's original approach (K3). Note that
43 the present-day topography is used as initial topography T_0 for calculations AS1 and K1. As
44 discussed below, the reliability of solution from AS1 depends on the differences between the
45 true initial topography and the assumed one (i.e., present-day topography as used here). If
46 there is significant difference between the assumed and true initial topography, updating
47 initial topography is necessary. In our case, the initial stage is the last interglacial period
48 which has similar ice volume to the present day, so the present-day topography is a good
49 approximation to the initial topography.

50 Using RSL from K3 as standard results, Fig. S2 shows that the maps of RSL
51 difference (i.e., the accuracy) to K3 from calculations AS1, K1 and AS2 at 5 kybp, 10 kybp
52 and 15 kybp. The absolute error in RSL from AS1 is negligibly small for most regions (Fig.
53 S2a, S2d and S2g), whereas the absolute error from K1 is much worse, especially at 20 kybp

54 (Fig. S2h). AS2 is identical to K3, the standard results (Fig. S2c, S2f and S2i). Admittedly,
55 there are relatively large errors in some localized regions for AS1, such as Hudson Bay and
56 the Arctic Ocean near Fennoscandia for some periods (Fig. S2a and S2d), because we ignore
57 the change in surface radial displacement when deriving the pre-calculated ocean function
58 used in AS1. However, the largest errors in those areas mostly occur in the ocean, while along
59 the coastlines where paleo-relative sea level records are available, the absolute errors are all
60 less than 10 meters (Fig. S2a and S2d). Figure S3 shows the modeled RSL curves at four
61 representative sites including Hudson Bay and Fennoscandia from K3, K1, AS1 and AS2
62 calculations. The results are consistent with that from Figure S2 in that the errors in modeled
63 RSL from AS1 (i.e., the single outer iteration model run using our revised method for ocean
64 functions) are negligible, whereas the errors from K1 are evident, especially for far-field sites.
65 Note that even at Churchill, which is on the coastline of Hudson Bay, AS1 has negligible
66 errors in RSL calculations.

67 To further assess the errors in RSL from our AS1 model, we tested two additional
68 GIA calculations with extremely strong or weak mantle viscosity models. For both cases, the
69 lithospheric thickness is 100 km. For the strong mantle case, the entire mantle below the
70 lithosphere has a viscosity of 5×10^{22} Pas. For the weak mantle case, the 200 km thick
71 asthenosphere below the lithosphere and the rest of the mantle have viscosities of 5×10^{18} Pas
72 and 10^{20} Pas, respectively. Figure S4 shows similarly small errors for both cases to that of
73 VM5a (Fig. S2), indicating the reliability of our AS1 model.

74 Other pre-calculated ocean functions $O(t)$ for any given ice model may be constructed
75 to obtain more accurate RSL results in our AS1 method by replacing the “rigid Earth”
76 approximation with others, for example, the isostasy approximation in which surface
77 elevation changes to compensate the surface loads. Another possible way is to perform a full
78 GIA modeling with three outer iterations (i.e., for outer iterations to converge) for a reference
79 viscosity model and use the ocean functions from the last outer iteration as the pre-calculated
80 ocean functions for any other GIA calculations with reasonable viscosity models in our AS1

81 method. We test such a strategy by using a reference viscosity model which has a 100-km
 82 thick elastic lithosphere and its underlying mantle with a uniform viscosity of 10^{21} Pas and
 83 then applying the resulting pre-calculated ocean functions for those same two GIA cases with
 84 extremely strong or weak viscosity models as in Figure S4. The resulting errors in RSL for
 85 those two cases (Fig. S5) are similar to that in Figure S4 for which the “rigid Earth”
 86 approximation was used in building the pre-calculated ocean functions.

87 To quantify the upper bound of errors in RSL by using one outer iteration (e.g., our
 88 AS1 method), we compute 806 GIA models covering a wide range of mantle viscosities and
 89 determine RSL histories for a large number of sites in three regions including North America,
 90 Fennoscandia, and far fields using both AS1 and K3 methods. The numbers of sites are 18,
 91 12, and 36 for North America, Fennoscandia, and far fields, respectively. The North
 92 American and Fennoscandian sites are from Peltier et al., (2015), and the far-field sites are
 93 from Lambeck et al., (2014). These models, same as those in Kang et al., (2024), have three
 94 viscosity layers: a lithosphere of 100 km thick, the upper and lower mantles, and use ICE-
 95 6G_D as the ice history (Peltier et al., 2015, 2018). The viscosity varies from 10^{19} Pas to
 96 $10^{21.5}$ Pas in the upper mantle and from $10^{20.5}$ Pas to $10^{23.5}$ Pas in the lower mantle. The
 97 relative error (i.e., the relative difference from the reference case K3) in modeled RSL for
 98 each site is defined as $\epsilon_i = \frac{\int_0^T |RSL_{x,i}(t) - RSL_{K3,i}(t)| dt}{\int_0^T |RSL_{K3,i}(t)| dt}$, where $RSL_{x,i}$ is the modeled RSL at site i
 99 for case K1, AS1, or AS2, $RSL_{K3,i}$ is for the reference case K3, and the integral is for the total
 100 model time duration. The regionally averaged relative error ϵ is defined as the average error
 101 among all sites within each region, i.e., $\epsilon = \frac{\sum \epsilon_i}{N}$, where N is the total number of sites within
 102 each region. The maximum regionally averaged relative error among those 806 GIA models
 103 is less than 5% (Supplement Table 2) for our AS1 method.

104 We also quantify the maximum absolute error in RSL, defined as the maximum of
 105 $|RSL_x(t) - RSL_{K3}(t)|$ among all time periods t and all sites in each region from those 806
 106 calculations (Supplement Table 2). For far-field sites where RSL is mainly controlled by

107 ocean functions and ice volume changes, the maximum absolute error in RSL is less than 3
108 meters for the AS1 method but more than 10 meters for the K1 method, consistent with Fig.
109 S1 in that AS1 provides more accurate ocean functions than K1. However, the maximum
110 absolute error in near-field RSL is more significant and up to ~23 meters for both AS1 and
111 K1 methods, reflecting the fact that near-field ocean functions and paleo-topography are more
112 affected by visco-elastic deformation. Fig. S6 shows the RSL curves for the site and viscosity
113 model corresponding to the maximum absolute error of ~23 meters in RSL for AS1. Note that
114 at the site for this case with the maximum absolute error, the total RSL change exceeds 600
115 meters and the RSL from AS1 is not significantly different from that from K3 (Fig. S6).
116 Depending on factors including the user's goal, RSL data quality, and requirements for
117 accuracy and efficiency of GIA calculations, AS1 could be a viable method to obtain reliable
118 RSL in both far fields and near fields with minimal computational cost.

119 We summarize our attempts to get accurate RSL results from a single complete GIA
120 model run as follows. Since the purpose of multiple outer iterations is to update ocean
121 function history and initial topography successively to be consistent with the present-day
122 topography and a given ice model (Kendall et al., 2005), our strategy is to construct pre-
123 calculated ocean functions and initial topography that would lead to RSL solutions with an
124 adequate level of accuracy with a single complete GIA model run (i.e., the AS1 method). The
125 present-day topography would be a good approximation for initial topography if a model
126 starts with an ice-sheet distribution similar to that of the present day (i.e., the interglacial
127 period), as in the benchmark study here. We found that three outer iterations of complete
128 model runs with successively updated ocean functions and initial topography could be
129 replaced with our AS1 method, depending on users' goals and requirements for the error
130 levels. For example, studies on global properties of RSL could achieve adequately accurate
131 results from one single complete run (i.e., AS1) with properly constructed pre-calculated
132 ocean functions, as we discussed. If the goal is to model the RSL for one particular near-field
133 site as accurately as possible, it would be more prudent to run two or three outer iterations of

134 complete GIA runs with successively updated ocean functions and initial topography
135 following Kendall et al. (2005). It is worthwhile to mention that, when modeling RSL
136 changes, one should also consider other factors including the errors in RSL records (often
137 exceeding 10 m in near field during the rapid deglaciation (Peltier et al., 2015; Lambeck et
138 al., 2017)), the relatively low resolution of global ice models, inherent numerical errors, and
139 unaccounted processes in the current sea level equation (e.g., erosion and sedimentation).

140

141 **Supplement Table 1: Comparison of Load Love Numbers h_l , k_l , and l_l Between**
 142 **CitcomSVE and Semi-Analytical Solutions and the corresponding errors.**

case	$h(0)$	$k(0)$	$ l(0) $	$h(40)$	$k(40)$	$ l(40) $	ϵ_{ha}	ϵ_{ka}	ϵ_{la}	ϵ_{hd}	ϵ_{kd}
Analytic_l1	-1.2543	-1.0000	0.8866	-1.4964	-1.0000	1.9090					
l1m0_R1	-1.2527	-1.0000	0.8869	-1.4943	-1.0000	1.9171	1.37E-03	2.00E-06	2.67E-03	3.99E-04	5.56E-05
l1m0_R2	-1.2548	-1.0000	0.8859	-1.4967	-1.0000	1.9115	2.29E-04	1.00E-06	8.68E-04	1.01E-04	1.50E-05
l1m0_R3	-1.2547	-1.0000	0.8846	-1.4968	-1.0000	1.9105	2.66E-04	0.00E+00	5.10E-04	4.39E-05	6.58E-06
l1m0_R4	-1.2546	-1.0000	0.8864	-1.4968	-1.0000	1.9101	2.91E-04	0.00E+00	4.00E-04	2.50E-05	3.94E-06
Analytic_l2	-0.9577	-0.3041	0.0200	-2.4066	-0.9396	0.8216					
l2m0_R1	-0.9549	-0.3037	0.0201	-2.4000	-0.9373	0.8305	2.43E-03	1.91E-03	7.19E-03	3.64E-04	4.13E-04
l2m0_R2	-0.9578	-0.3039	0.0202	-2.4060	-0.9388	0.8242	1.24E-04	4.98E-04	2.19E-03	9.54E-05	1.04E-04
l2m0_R3	-0.9585	-0.3042	0.0200	-2.4064	-0.9391	0.8232	1.79E-04	2.51E-04	1.38E-03	4.25E-05	4.61E-05
l2m0_R4	-0.9574	-0.3038	0.0203	-2.4066	-0.9392	0.8229	2.30E-04	1.85E-04	1.10E-03	2.30E-05	2.65E-05
Analytic_l2m1	-0.3058	1.0944	0.1118	0.6151	2.1973	0.1884					
l2m1_R1	-0.3094	1.0836	0.1103	0.5583	2.1294	0.1653	7.10E-02	2.08E-02	8.97E-02	8.10E-03	5.82E-04
l2m1_R2	-0.3063	1.0925	0.1116	0.6077	2.1885	0.1847	1.01E-02	2.89E-03	1.46E-02	1.07E-03	6.46E-05
l2m1_R3	-0.3100	1.0900	0.1111	0.6144	2.1964	0.1878	1.78E-03	4.93E-04	3.08E-03	8.14E-04	6.37E-05
l2m1_R4	-0.3056	1.0948	0.1118	0.6178	2.2003	0.1891	2.99E-03	7.52E-04	2.34E-03	4.99E-04	3.60E-05
Analytic_l4	-1.0251	-0.1342	0.0568	-4.4402	-0.9416	0.3411					
l4m0_R1	-1.0194	-0.1341	0.0565	-4.4105	-0.9339	0.3480	6.18E-03	7.58E-03	1.25E-02	3.70E-04	1.42E-03
l4m0_R2	-1.0254	-0.1343	0.0569	-4.4354	-0.9397	0.3432	9.21E-04	1.94E-03	3.60E-03	9.21E-05	3.58E-04
l4m0_R3	-1.0253	-0.1342	0.0569	-4.4384	-0.9407	0.3425	3.29E-04	9.63E-04	2.29E-03	4.22E-05	1.58E-04
l4m0_R4	-1.0247	-0.1341	0.0569	-4.4395	-0.9410	0.3423	1.25E-04	6.08E-04	1.89E-03	2.39E-05	9.24E-05
Analytic_l8	-1.2376	-0.0772	0.0302	-8.8405	-0.9605	0.0958					
l8m4_R1	-1.2172	-0.0767	0.0301	-8.5145	-0.9171	0.1048	3.09E-02	3.94E-02	3.54E-02	6.22E-04	4.94E-03
l8m4_R2	-1.2354	-0.0772	0.0302	-8.7607	-0.9492	0.0980	7.26E-03	1.01E-02	8.95E-03	1.60E-04	1.21E-03
l8m4_R3	-1.2359	-0.0771	0.0304	-8.7960	-0.9544	0.0977	3.85E-03	5.29E-03	8.55E-03	7.12E-05	5.91E-04
l8m4_R4	-1.2372	-0.0772	0.0303	-8.8084	-0.9563	0.0977	2.63E-03	3.56E-03	8.50E-03	4.21E-05	3.02E-04
Analytic_l16	-1.6868	-0.0574	0.0229	-17.8470	-0.9726	0.0479					
l16m8_R1	-1.5913	-0.0544	0.0225	-15.0636	-0.7883	0.0329	1.39E-01	1.74E-01	2.55E-01	1.00E-03	2.05E-02
l16m8_R2	-1.6660	-0.0568	0.0228	-17.0264	-0.9179	0.0418	3.88E-02	4.92E-02	9.26E-02	2.52E-04	4.86E-03
l16m8_R3	-1.6781	-0.0572	0.0228	-17.3994	-0.9437	0.0430	2.06E-02	2.54E-02	7.45E-02	1.12E-04	2.14E-03
l16m8_R4	-1.6825	-0.0573	0.0228	-17.5347	-0.9530	0.0435	1.40E-02	1.69E-02	6.71E-02	6.60E-05	1.20E-03
l16m8_R5	-1.6805	-0.0572	0.0228	-17.6230	-0.9579	0.0464	1.04E-02	1.31E-02	2.26E-02	6.50E-05	1.33E-03

143 ϵ_{ha} , ϵ_{ka} , and ϵ_{la} are amplitude errors for Love numbers h, k, and l, respectively. ϵ_{hd} and ϵ_{kd} are

144 dispersion errors for Love numbers h and k, respectively.

145

146 **Supplement Table 2: Maximum errors in modeled RSL from different cases**
 147 **compared to the reference case (K3) among an ensemble of semi-analytical**
 148 **calculations with a wide range of viscosity models**

	North America ^a	Fennoscandia	Far Field
K1 ^c	6.54% (22.8 m) ^b	4.81% (10.0 m)	4.69% (10.6 m)
AS1	4.53% (23.2 m)	4.89% (11.0 m)	2.97% (2.7 m)
AS2	0.49% (4.5 m)	0.32% (0.8 m)	0.19% (0.4 m)

149

150 a. “North America”, “Fennoscandia”, and “Far Field” are regions with groups of sites used in
 151 calculating RSL. The numbers of sites are 18, 12, and 36 for North America, Fennoscandia,
 152 and Far Field, respectively. The North American and Fennoscandian sites are from (Peltier et
 153 al., 2015). The far-field sites are from (Lambeck et al., 2014).

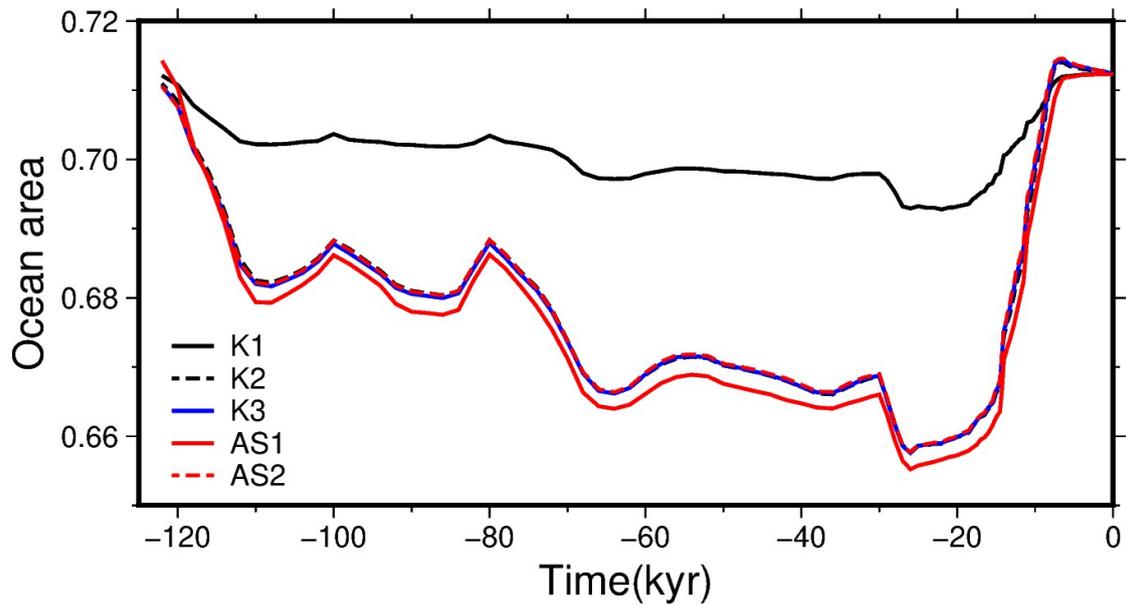
154 b. The relative error in modeled RSL for each site is defined as $\epsilon_i = \frac{\int_0^T |RSL_{x,i}(t) - RSL_{K3,i}(t)| dt}{\int_0^T |RSL_{K3,i}(t)| dt}$,
 155 where $RSL_{x,i}$ is modeled RSL at site i for case K1, AS1, or AS2, and $RSL_{K3,i}$ is for the
 156 reference case K3. The regionally averaged relative error is defined as the average error
 157 among all sites within each region, i.e., $\frac{\sum \epsilon_i}{N}$, where ϵ_i is the error for each site i and N is the
 158 total number of sites within each region. The numbers out of parenthesis represent the
 159 maximum region-averaged relative error among the 806 calculations of varying mantle
 160 viscosity. The numbers inside parenthesis represent the maximum absolute error (i.e.,
 161 $\max(|RSL_x(t) - RSL_{K3}(t)|)$) among all time periods t and all sites in each region from those
 162 806 calculations of varying mantle viscosity. Those numbers measure the maximum possible
 163 error for each case among reasonable mantle viscosity structures.

164 c. The meaning of case K1, AS1, and AS2 can be found from the main text.

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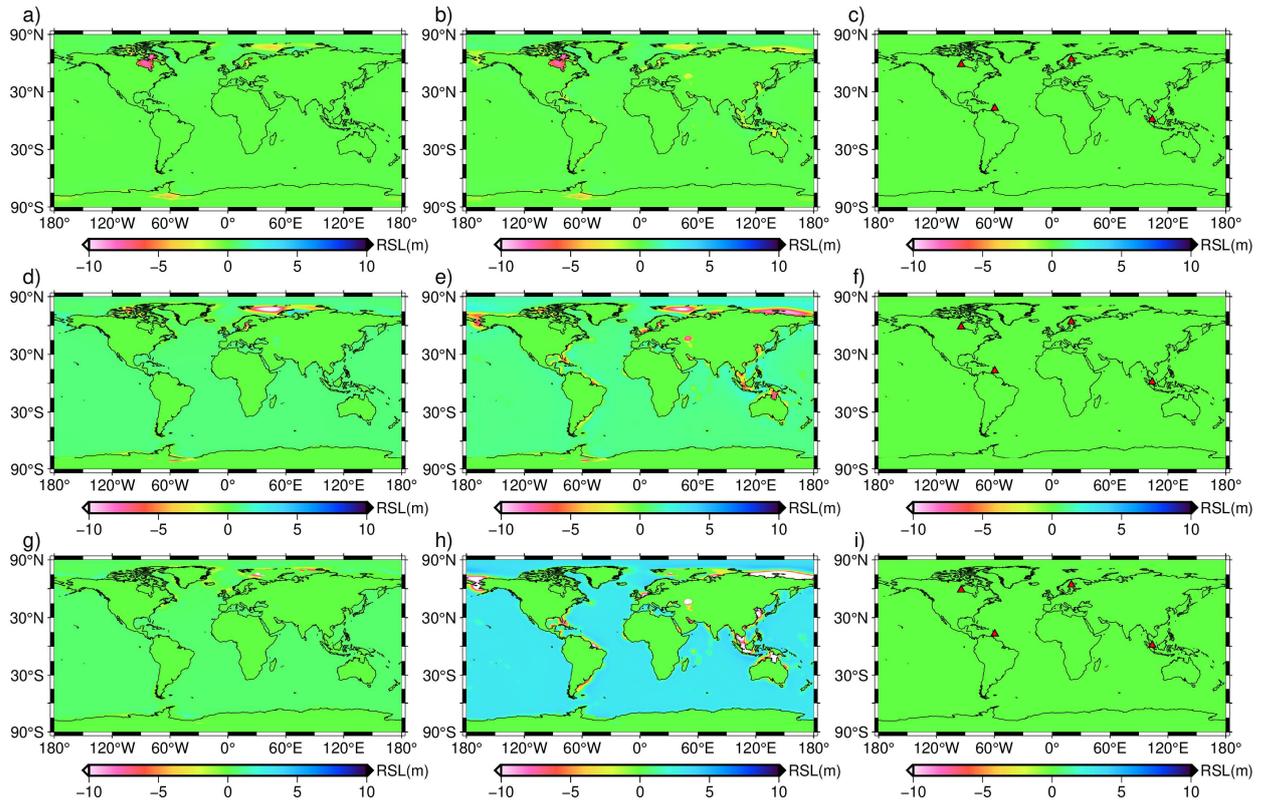
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169 Figure S1. Normalized ocean areas determined from the ocean functions from five different
170 semi-analytical calculations: K1, K2, K3, AS1, and AS2.). Calculation K3, the reference case,
171 represents the convergent solutions after the third outer iteration based on the algorithm from
172 Kendall et al., (2005), while K1 is that after the first outer iteration and K2 is the after the
173 second outer iteration. AS1 represents the first outer iteration based on the pre-calculated
174 ocean functions determined assuming rigid Earth, and AS2 is the second outer iteration with
175 updated ocean functions and initial topography following AS1. Note that K2, K3 and AS2 are
176 almost overlapping with each other.

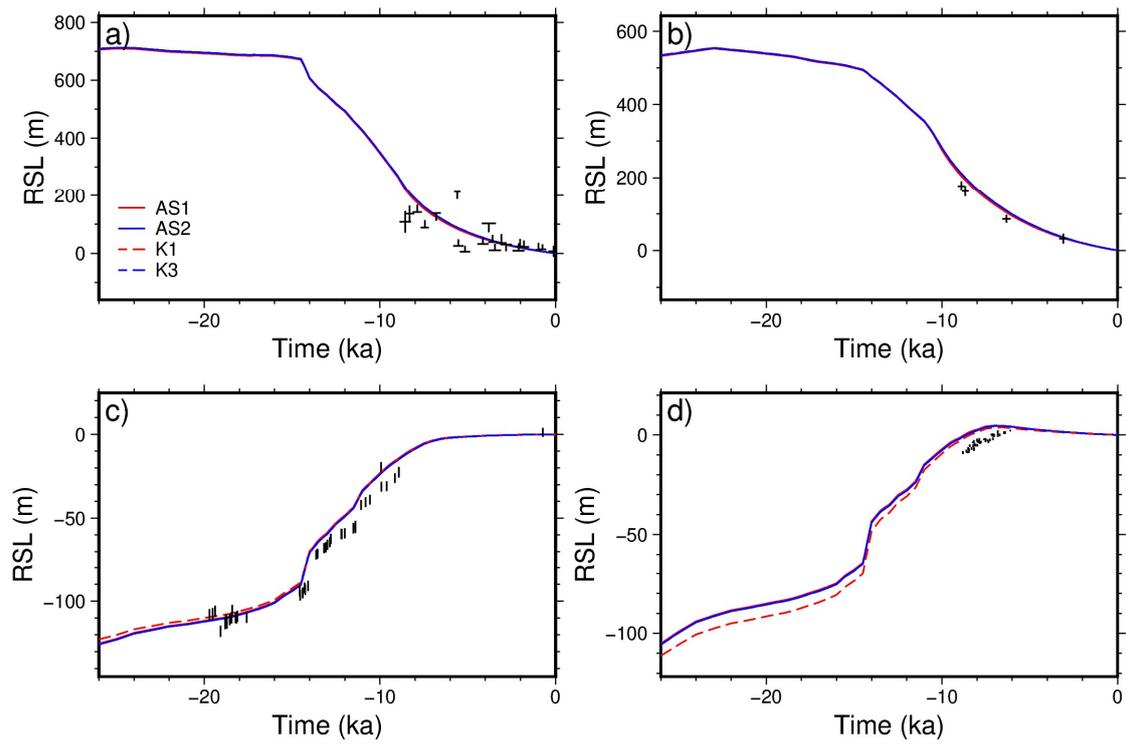
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178

179 Figure S2. Comparison of modeled relative sea level (RSL) at 5 kybp (the top row), 10 kybp (the
 180 middle row), and 15 kybp (the bottom row) among four different semi-analytic calculations: K1,
 181 K3, AS1 and AS2. Shown here are the differences (or the errors) in RSL to reference case K3 from
 182 AS1 (the left column, a, d, and g), K1 (the middle column, b, e and h), and AS2 (the right column,
 183 c, f, and i), respectively. The difference (or error) at a given time is defined as $(RSL_x - RSL_{K3}) \cdot$
 184 O_x , where x is AS1, K1, or AS2, O_x is the ocean function. Note that RSL is only meaningful for
 185 ocean regions (including coastlines), since RSL records at one site can exist only if this site is
 186 ocean for that time period. The red triangles in the last column represent sites in Figure S3. Note
 187 the ice model and viscosity model used are ICE6G and VM5a.

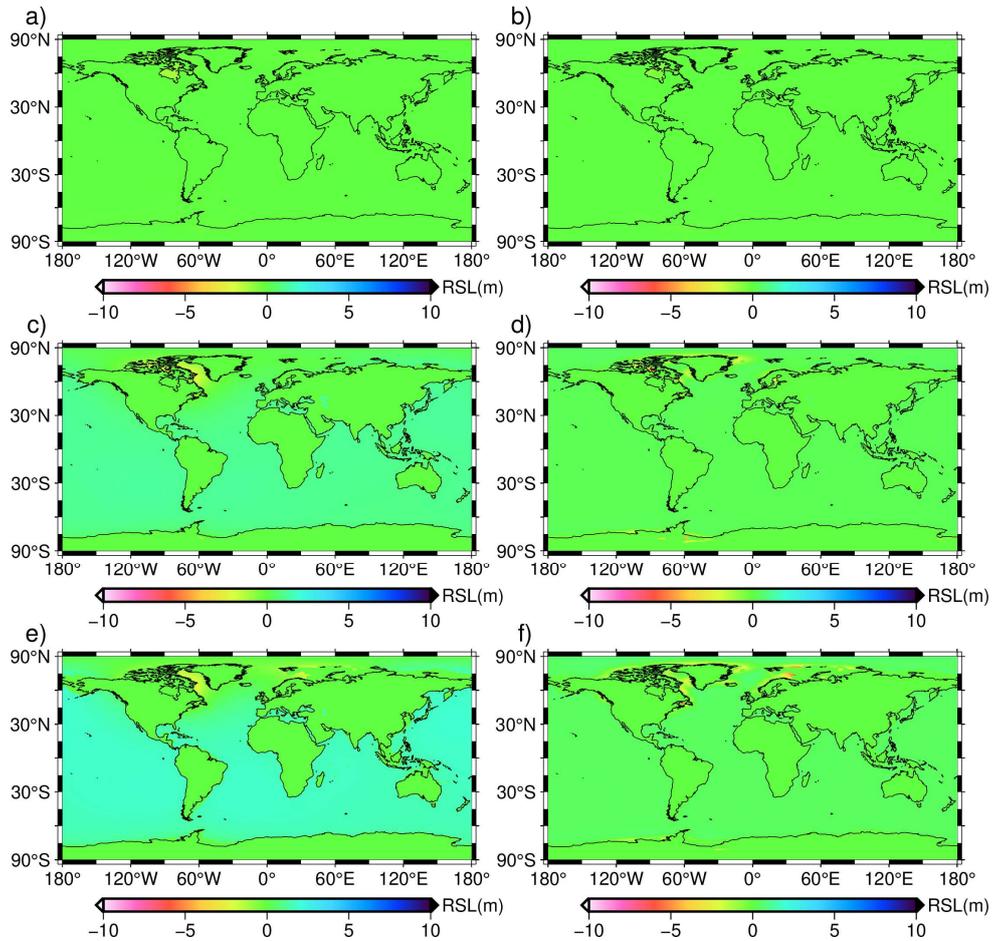
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190

191 Figure S3. Comparison of modeled RSL curves at four sites from four semi-analytic calculations
 192 (AS1, K1, AS2, and K3 as in Figure S1): Churchill (Hudson Bay) (a), Vasterbotten (b), Barbados
 193 (c), and Geylang (d) (i.e., same sites as in Figure 6 where their longitudes and latitudes are given).
 194 The locations of those four sites are also shown in Figure S2.

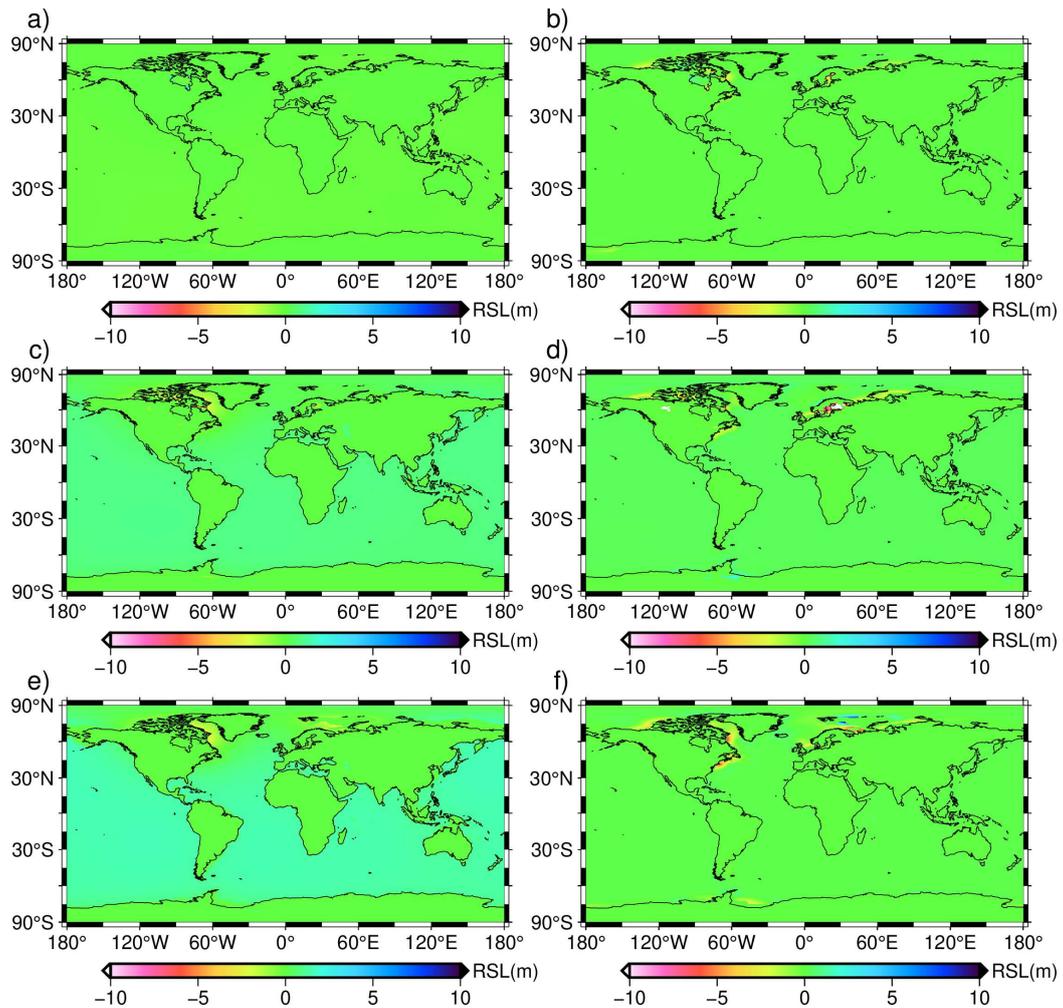
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196

197 Figure S4. The RSL differences to calculation K3 from calculation AS1, or $(RSL_{AS1} - RSL_{K3}) \cdot$
 198 O_{AS1} , for the case with an extremely strong mantle (the left column, a, c, and e for 5 kybp, 10 kybp
 199 and 15 kybp, respectively), and the case with an extremely weak mantle (the right column, b, d,
 200 and f for 5 kybp, 10 kybp and 15 kybp, respectively). Note that the pre-calculated ocean functions
 201 for both cases are constructed assuming a “rigid Earth”.

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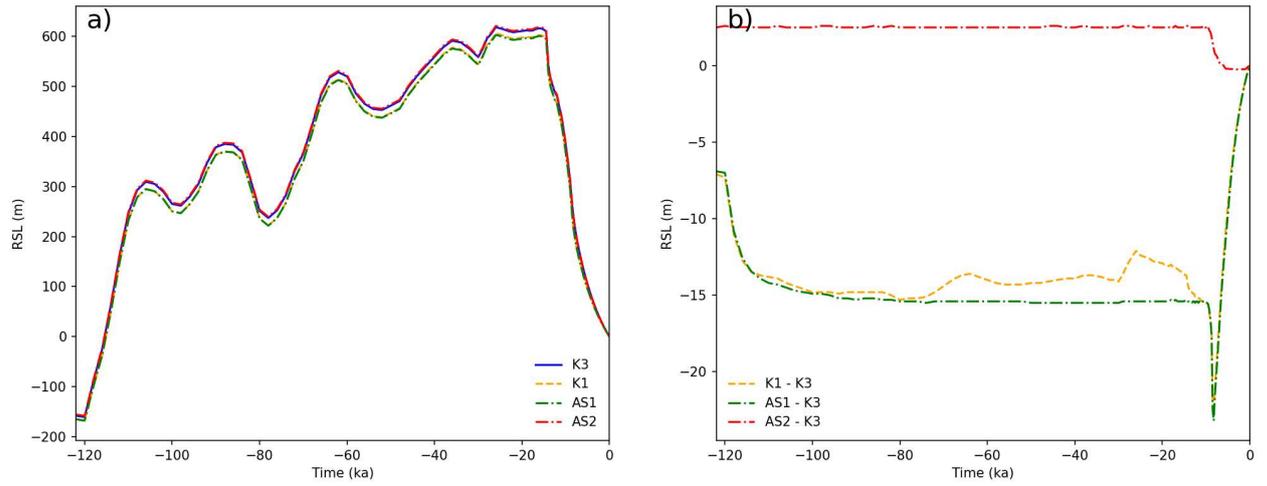


203

204 Figure S5. The same as in Figure S4, except for using different pre-calculated ocean functions.

205 The RSL differences to calculation K3 from calculation AS1, or $(RSL_{AS1} - RSL_{K3}) \cdot O_{AS1}$, for the
 206 case with an extremely strong mantle (the left column, a, c, and e for 5 kybp, 10 kybp and 15 kybp,
 207 respectively), and the case with an extremely weak mantle (the right column, b, d, and f for 5 kybp,
 208 10 kybp and 15 kybp, respectively). The pre-calculated ocean functions for both cases are
 209 constructed from the convergent solutions (i.e., with three outer iterations) using Kendall et al.,
 210 (2005) for a reference viscosity model (i.e., 100-km thick lithosphere overlying the mantle with
 211 uniform viscosity of 10^{21} Pas). Note that the reference viscosity model is only used for
 212 constructing the pre-calculated ocean functions.

213



214

215 Figure S6. The modeled RSL (a) for calculations for the site and the viscosity model that yields
 216 the maximum absolute error presented in Supplement Table 2 (i.e. 23.20 m, the maximum
 217 absolute error for AS1 in North America). (b) shows the differences in RSL between K1, AS1,
 218 and AS2 to the reference case K3 for that site and viscosity model. The site is Churchill, and the
 219 mantle viscosities are 1.26×10^{20} Pas and 1.26×10^{23} Pas for upper and lower mantle, respectively.

220