

Dear Hirohiko Nagano,

Thank you for your interest in our work.

We would like to answer your questions as follows:

1. *"I have a question regarding the relationship between the diffusion coefficients obtained from the ratio of measured CO₂ flux to CO₂ conc gradient and from the model. I am unsure, but such evaluation may contribute to considering the limitations and further improving the novel measurement system. What is the relationship between those two diffusion coefficients?"*

We agree that in general, estimating diffusion coefficients obtained from measured CO₂ flux and CO₂ concentration gradient is a good approach as recommended by, for example, Sánchez-Cañete et al., (2017, 2018).

D_s (in situ CO₂ diffusion coefficient [$\text{m}^2 \text{s}^{-1}$]) can be calculated as (e.g., Sánchez-Cañete et al., 2018):

$$D_s = -\frac{F_{chamber} \cdot \Delta z}{\Delta C} \quad (1)$$

where:

- $F_{chamber}$ is surface flux measured by chamber method
- $\Delta z = 0.05$ m in our case
- ΔC is gradient of CO₂ molar density

The calculated D_s will be used to optimize a diffusion model specific to the soil of the research site (e.g., Sánchez-Cañete et al., 2018).

$$\frac{D_s}{D_a} = a\theta_a^b \quad (2)$$

where:

- D_a is the diffusion coefficient of CO₂ in free air [$\text{m}^2 \text{s}^{-1}$]
- θ_a is the soil air porosity ($\theta_a = \text{soil porosity} - \text{SWC}$, where SWC is soil water content)
- a and b are fitting coefficients.

While this approach is theoretically sound, its practical application is sometimes challenging due to limitations in accurately measuring θ_a . To obtain reliable θ_a , reliable SWC measurements are required. However, in our hyper-arid soil (aridity index = 0.07), SWC remains consistently below 5%, which is outside the accuracy range of available moisture sensors. Consequently, θ_a , also suffers from low accuracy. This inaccuracy prevents us from capturing the real-time fluctuations of θ_a , leading to unreliable estimates of a and b when optimizing the diffusion model.

Since improving the accuracy of gradient flux calculations is not the focus of this work, we have not discussed this aspect in detail. However, as requested, Figure 1 below compares D_s values obtained using the chamber-based method ($F_{chamber}$ and ΔC , labeled as $D_{chamber}$) with those derived from the Buckingham model (Buckingham, 1904) (labeled as $D_{Buckingham}$).

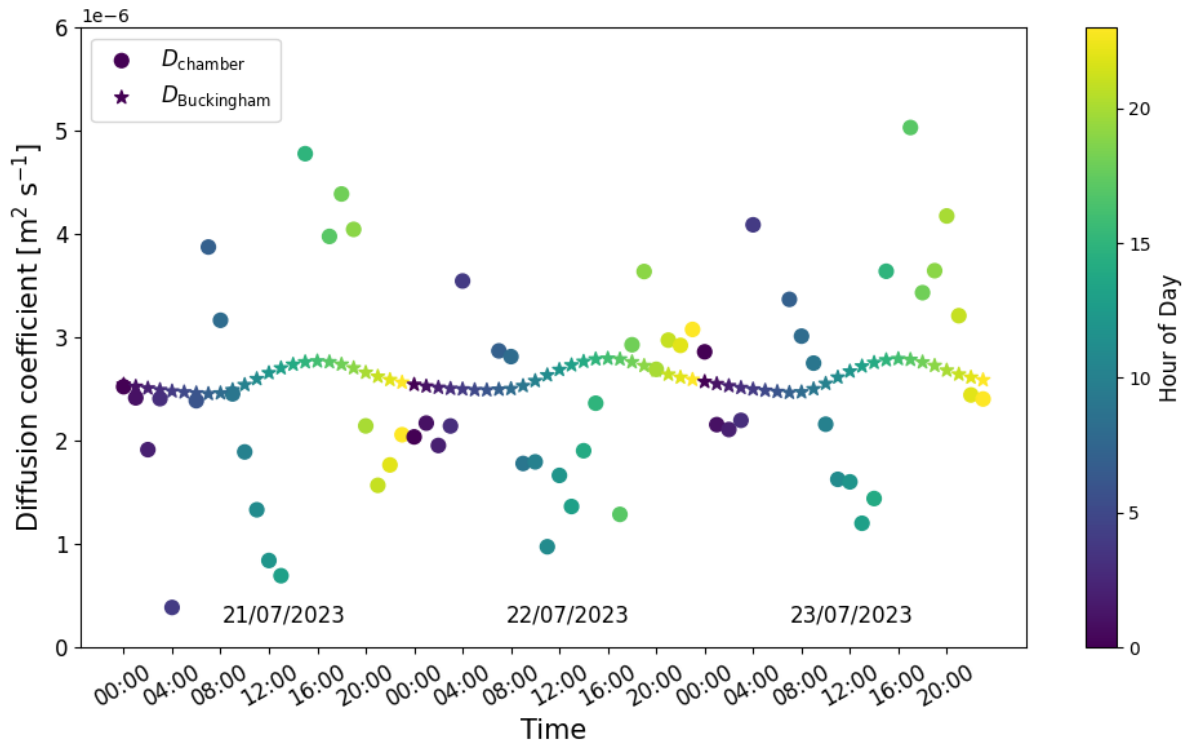


Figure 1. Comparison between $D_{chamber}$ and $D_{Buckingham}$

2. "Also, I want to see the relations among measured CO_2 flux, CO_2 conc gradient, and modeled diffusion coefficient."

The relations between measured CO_2 flux vs. modelled diffusion coefficient and measured CO_2 flux vs. CO_2 concentration gradient are presented in Fig. 2. From the two correlations, it is shown that flux is mainly driven by CO_2 concentration gradient (as expected).

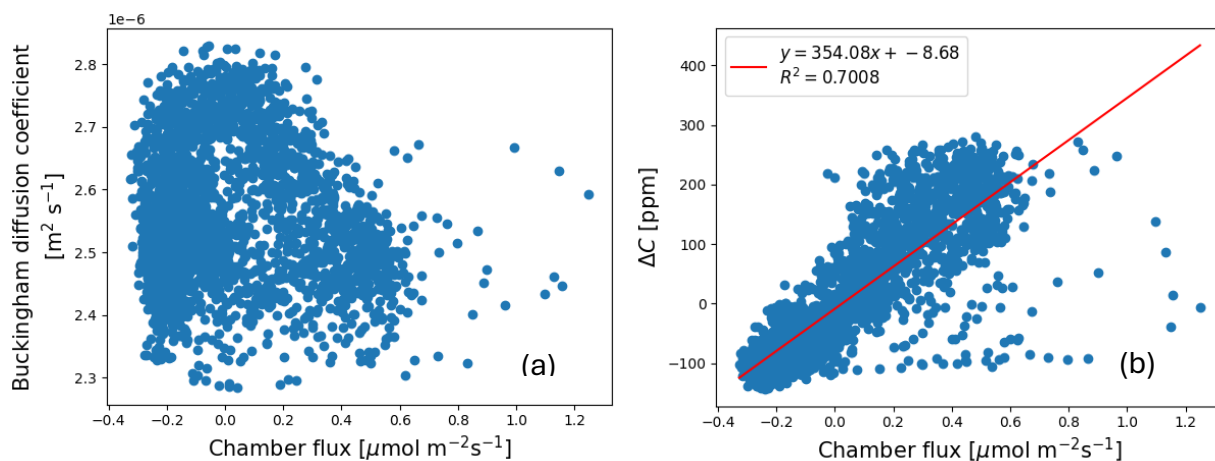


Figure 2. Relation between measured CO_2 flux vs. Buckingham diffusion coefficient (a) and vs. CO_2 concentration gradient (b).

References

Buckingham, E. (1904), Contributions to Our Knowledge of the Aeration of Soils, U.S. Dept. of Agriculture, Bureau of Soils, Washington, D. C.

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