Ensemble design for seasonal climate predictions: Studying extreme Arctic sea ice lows with a rare event algorithm

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Abstract. Initialized ensemble simulations can help identify the physical drivers and assess the probabilities of weather and climate extremes based on a given initial state. However, the significant computational burden of complex climate models makes it challenging to quantitatively investigate extreme events with probabilities below a few percent. A possible solution to overcome this problem is to use rare event algorithms, i.e., computational techniques originally developed in statistical physics that increase the sampling efficiency of rare events in numerical simulations. Here, we apply a rare event algorithm to ensemble simulations with the intermediate complexity coupled climate model PlaSim-LSG to study extremes of pan-Arctic sea ice area reduction under pre-industrial greenhouse gas conditions. We construct seven pairs of control and rare event algorithm ensemble simulations each starting from seven different initial winter sea ice states. The rare event simulations produce sea ice lows with probabilities of at least two orders of magnitude smaller than feasible with the control ensembles, and drastically increase the number of extremes compared to direct sampling. We find that for a given probability level, the amplitude of negative late summer sea ice area anomalies strongly depends on the baseline winter sea ice thickness, but hardly on the baseline winter sea ice area. The experiments furthermore indicate a quasi-zero probability to internally generate a seasonally sea ice-free Arctic in this set-up. Finally, we investigate the physical processes in two trajectories leading to sea ice lows with conditional probabilities of less than 0.001%. In both cases, negative late summer pan-Arctic sea ice area anomalies are preceded by negative spring sea ice thickness anomalies. These are related to enhanced surface downward longwave radiative and sensible heat fluxes in an anomalously moist, cloudy and warm atmosphere. During summer, extreme sea ice area reduction is favoured by enhanced open-water-formation efficiency, anomalously strong downward solar radiation and the sea ice-albedo feedback. This work highlights that the most extreme summer sea ice conditions result from the combined effects of preconditioning and weather variability, emphasizing the need for thoughtful ensemble design when turning to real applications.

1 Introduction

Anthropogenic emissions of greenhouse gases have contributed to the loss of Arctic sea ice during the last 45 years (Notz and Marotzke, 2012; Stroeve and Notz, 2018; Gregory et al., 2002; Ding et al., 2017). Despite the steadily increasing rate of greenhouse gas concentrations (Meinshausen et al., 2017), the decline of the pan-Arctic sea ice area is non-linear, and internal climate variability and feedback mechanisms have been shown to modulate the downward trend of the sea ice (Ding et al.,

25 2017; Baxter et al., 2019; Ono et al., 2019; Francis and Wu, 2020; Tietsche et al., 2011). Accelerated decline of the late summer sea ice area occurred from the mid-2000s to 2012, resulting in record reductions of sea ice area compared to the trend line in 2007 and 2012.

30 The 2007 and 2012 drastic sea ice loss events have been attributed both to climate change via preconditioning through the ongoing winter sea ice thinning and to internal climate variability via particular weather and climate events during the melting season (Lindsay et al., 2009; Kauker et al., 2009; Zhang et al., 2008, 2013; Parkinson and Comiso, 2013; Kirchmeier-Young et al., 2017). The precise relative contributions of anthropogenically forced *vs.* internal climate variability to these events is, however, uncertain (Kirchmeier-Young et al., 2017; Ono et al., 2019). By applying an extreme event attribution analysis to en-

35 semble simulations with the Second Generation Canadian Earth System Model (CanESM2), Kirchmeier-Young et al. (2017) concluded that a sea ice low with an amplitude as observed in 2012 (i.e. a 2012-like sea ice area anomaly relative to the observed 1981-2010 mean) would have been extremely unlikely to occur without global warming. Ono et al. (2019), instead, used a 200-year control simulation with a fixed year-2000 greenhouse gas forcing in the Model for Interdisciplinary Research on Climate (MIROC) version 5.2 and simulated a sea ice low larger in magnitude than the 2012 sea ice area anomaly relative

40 to a linear trend line fitted to observed September sea ice area values between 1979 and 2006. In accordance with Kirchmeier-Young et al. (2017), however, the authors do not managed to simulate a sea ice area anomaly with a larger amplitude than the observed 2012 anomaly relative to the observed climatological mean between 1981 and 2010.

45 Various studies have analyzed the physical drivers of the 2007 and 2012 sea ice lows. In 2007, sea ice reduction was favoured by enhanced inflow of warm Pacific water through Bering Strait (Woodgate et al., 2010) and by anomalously persistent southerly winds in the Pacific sector associated with the Arctic Dipole Anomaly (ADA) pattern (Wang et al., 2009; Lindsay et al., 2009; Overland et al., 2012; Kauker et al., 2009). In 2012, a summer storm contributed to enhanced sea ice reduction by leading to increased bottom melt via anomalously strong vertical mixing in the oceanic boundary layer (Guemas

50 et al., 2013; Zhang et al., 2013). Further possible triggers of anomalously low summer Arctic sea ice area include enhanced North Atlantic oceanic heat transport (Årthun et al., 2012), the positive and negative phases of the winter and summer Arctic Oscillation (AO) (e.g. Rigor et al. (2002); Ogi et al. (2016)), reduced cloudiness during summer (Schweiger et al., 2008), and increased surface downward longwave radiation related to enhanced poleward atmospheric moisture transport during spring (Kapsch et al., 2013, 2019).

55 Even though the physical drivers of individual extremes of Arctic sea ice reduction have been suggested, their quantitative statistical analysis is hampered by the small number of extreme events that can be sampled from observations and numerical simulations. Likewise, it is the small number of events in the lower tail of the distribution of sea ice area values that makes it

60 challenging to quantify the probability of a 2012-like sea ice low event for a given background climate. The record of satellite-based sea ice observations includes only a few annual sea ice minima with orders of magnitude comparable to the ones in September 2007 and 2012 (Fetterer et al., 2017). Moreover, the large computational cost of complex climate models makes it unrealistic to run ensembles with a few thousands of trajectories and to quantitatively study extreme events with probabilities of less than 1%. One workaround is to estimate these very small probabilities with extreme value theory models (Coles, 2001).

65 However, even when applicable, these methods provide only a statistical extrapolation of the probabilities and do not provide information on the dynamics. A better understanding of the precursors of extremes of summer Arctic sea ice reduction and a more precise estimate of their probabilities are in turn crucial to improve seasonal predictions of these events and to assess
70 their risk of occurrence under different climate change scenarios.

In this work, we study extreme negative summer pan-Arctic sea ice area anomalies using initialized ensemble simulations with the intermediate complexity coupled climate model PlaSim-LSG under pre-industrial greenhouse gas conditions. We investigate the statistical properties of extreme summer sea ice lows as a function of different initial winter states and examine the physical drivers favouring extremes of sea ice area reduction within a single melting season. In order to improve the
75 sampling of extreme sea ice loss events, we use a rare event algorithm. Rare event algorithms are computational techniques developed in statistical physics to improve the sampling efficiency of rare events in numerical simulations (e.g Ragone et al., 2018; Ragone and Bouchet, 2020, 2021; Sauer et al., 2024). Compared to conventional numerical simulations with the same computational cost, rare event algorithms enable to increase the number of simulated extreme events by several orders of magnitude while preserving the dynamical consistency of the model. In this way, these techniques allow to reduce the uncertainty
80 of probability and return time estimates and of conditional statistics on extreme events (e.g. composites) compared to conventional simulation strategies, and to generate ultra-rare events that are very unlikely to be observed using direct sampling. Rare event algorithms have been introduced in the 1950s (Kahn and Harris, 1951) and have been used since for a wide range of applications (for an overview and the mathematical analysis see e.g. Del Moral (2004); Giardina et al. (2011); Grafke and Vanden-Eijnden (2019)). Recently, some of these techniques have been applied in climate science and in fluid dynamics to
85 study heat waves (Ragone et al., 2018; Ragone and Bouchet, 2020, 2021), midlatitude precipitation (Wouters et al., 2023), tropical storms (Plotkin et al., 2019; Webber et al., 2019), weakening and collapse of the Atlantic Meridional Overturning Circulation (AMOC) (Cini et al., 2023), extreme Arctic sea ice lows related to unconditional probability distributions (Sauer et al., 2024) and turbulence (Bouchet et al., 2018; Grafke et al., 2015; Lestang et al., 2020).
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Here we use a genealogical selection algorithm (Ragone et al., 2018; Ragone and Bouchet, 2020, 2021; Sauer et al., 2024) adapted from Del Moral and Garnier (2005); Giardina et al. (2011), that is efficient to study persistent, long lasting events. A first application of this algorithm to study extreme Arctic sea ice lows in the intermediate complexity coupled climate model PlaSim under pre-industrial greenhouse gas conditions is given in Sauer et al. (2024). In that study, ensemble simulations with
95 the rare event algorithm were initialized with independent initial conditions sampled from a stationary 3000-year control run such that the statistics of extreme events is related to unconditional probability distributions. In such an approach, the rare event algorithm seeks to oversample trajectories leading to low sea ice states in an absolute sense, i.e., independent from the extent to which the sea ice lows are related to multi-annual fluctuations in the sea ice-ocean system (referred to as "preconditioning" in Sauer et al. (2024)) or driven by the dynamics occurring on intra-seasonal time scales. Here, instead, we use a seasonal climate
100 prediction set-up where an ensemble simulation is initialized from a single initial condition to which a small perturbation is added. We perform different ensemble simulations starting from different initial conditions sampled from the same control run as in Sauer et al. (2024). The goal of these experiments is to disentangle the roles of the initial condition vs. seasonal scale

fluctuations in favouring the occurrence of extremely negative summer sea ice area anomalies. Since each individual ensemble simulation is initialized from a same, slightly perturbed initial condition, statistical and physical properties of low sea ice states

105   inferred from that ensemble need to be interpreted in terms of conditional probabilities.

Owing to the rare event algorithm, we generate extremely low sea ice summers with conditional probabilities of less than 0.001%. Likewise, we investigate the impact of different initial conditions on the probabilities and amplitudes of extreme sea

110   ice lows. Finally, we elaborate physical drivers in two trajectories leading to late summer sea ice lows with less than 0.001%. The paper is structured as follows. In section 2, we present the set-up of the model, the methodology of the rare event algorithm and the design of control and rare event algorithm ensemble simulations. In section 3, we show that the rare event algorithm improves the sampling efficiency of extreme sea ice lows conditional on starting from a certain initial condition compared to standard control ensemble simulations. We analyze the impact of the initial condition on the probabilities and amplitudes of

115   the extremes. In section 4, we discuss physical processes and conditions in individual trajectories prior to extreme sea ice lows with probabilities with less than 0.001%. In section 5, we present our conclusions.

2   Materials and methods

2.1   Model and data

All simulations are performed with a coupled set-up of the intermediate complexity climate model Planet Simulator (PlaSim)

120   version 17 (Fraedrich et al., 2005). This set-up includes a dynamic Large-Scale Geostrophic (LSG) ocean (Maier-Reimer et al., 1993; Drijfhout et al., 1996), a mixed-layer ocean and a thermodynamic sea ice model (PlaSim coupled to LSG is referred to as "PlaSim-LSG" in the following).

The atmosphere is run with a horizontal spectral resolution of T21 (triangular truncation at wavenumber 21 $\sim 5.625°$ x

125    $5.625°$ on a Gaussian grid), ten levels up to 40 hPa in the vertical and a computational time step of 45 minutes. The LSG is configured on a $2.5°$ x $5°$ staggered E-type grid (Arakawa and Lamb, 1977) in the horizontal, with 22 levels and a maximum ocean depth of 6000 m in the vertical and a computational time step of five days. The sea ice model is based on the zero-layer model of Semtner (1976). It computes the sea ice thickness and sea surface temperature evolution from the energy balances at

130   the top and bottom of a sea ice-snow layer. The sea ice-snow layer is assumed to have a linear temperature gradient and to have no capacity to store heat. No sea ice drift is taken into account. The sea ice concentration is binary, i.e., a grid cell is fully sea ice covered or open water. The computational time step in the sea ice and mixed-layer ocean models is one day.

135   PlaSim-LSG is run with a fixed pre-industrial effective CO_2 volume mixing ratio of 280 ppmv. We choose a constant greenhouse gas forcing since we are interested in the statistics and dynamics of Arctic sea ice lows related to internal climate variability under a stationary climate. Solar radiation includes the seasonal cycle but not the diurnal cycle. Each model month

140   is 30 days long.

We use data from a stationary 3000-year control run (Sauer et al., 2024) and from a set of control and rare event algorithm ensemble simulations (see section 2.2 for more details). We consider the statistics of the pan-Arctic sea ice area

$$A(t) = \sum_{\phi \geq \phi_{min}} \sum_{\lambda} SIC_{\phi,\lambda}(t) \cdot G_{\phi,\lambda}, \tag{1}$$

where $SIC_{\phi,\lambda}(t)$ is the sea ice concentration at time t in a grid cell centered at latitude ϕ and longitude λ, $G_{\phi,\lambda} = \int_{\phi-\frac{\Delta\phi}{2}}^{\phi+\frac{\Delta\phi}{2}} \int_{\lambda-\frac{\Delta\lambda}{2}}^{\lambda+\frac{\Delta\lambda}{2}} R^2 cos(\phi')d\lambda'd\phi'$ is the grid cell area, R is the earth radius, $\Delta\phi$ and $\Delta\lambda$ are the angular distances between two grid points in the meridional and zonal direction. The summation in (1) includes all ocean grid boxes north of 40°N (i.e. $\phi_{min} \cdot \frac{180°}{\pi} = 40°$) with a binary land-sea mask (i.e. a grid cell is either completely ocean or land). The annual average, amplitude of the seasonal cycle and the timing of the annual minimum and maximum of the pan-Arctic sea ice area produced by the model are representative of the observed Arctic sea ice climatology between 1979 and 2015 (cf. Figure 1(a,c) of Sauer et al. (2024) and Figure S1 in the Supplementary Information of Sauer et al. (2024)). Differences in the representation of sea ice in PlaSim-LSG compared to observations include a delayed melting period, a positive sea ice concentration (SIC) bias from the Greenland to the Kara Sea and a negative SIC bias western of Greenland (cf. Figure 1(a,c) of Sauer et al. (2024) and Figure S1 in the Supplementary Information of Sauer et al. (2024)). All in all, however, the differences between the pan-Arctic sea ice area in PlaSim-LSG and in observational data are small compared to the range of sea ice area values available from the Coupled Model Intercomparison Project Phase 6 (CMIP6) models (Notz and SIMIP Community, 2020). The representation of sea ice in PlaSim-LSG is therefore sufficient for the purpose of this study.

We study extreme anomalies of February-September and August-September mean pan-Arctic sea ice area. We classify a sea ice area anomaly $I'(t)$ as extremely negative if $I'(t) \leq n\sigma_{ctrl}$, where σ_{ctrl} is the standard deviation of sea ice area in the control run or in a control ensemble (see section 2.2) and n is a real-valued number. In the following, we will vary n continuously to study an extensive range of extreme event amplitudes.

2.2 Rare event algorithm: Methodology and set-up

One key difficulty in the study of climate extremes is the lack of robust statistics as the large computational burden of complex climate models makes it unfeasible to run them long enough to sample a large amount of trajectories corresponding to the tail of the distribution of a target observable.

The rare event algorithm presented in recent studies (Giardina et al., 2011; Ragone et al., 2018; Ragone and Bouchet, 2020, 2021; Sauer et al., 2024) is a genealogical selection algorithm applied on top of an ensemble simulation. It is designed to improve the sampling efficiency of trajectories populating the tail of the distribution of the time-average of a target observable $A(t)$. At constant intervals of a resampling time τ_r, we assign to each trajectory a weight. The weight is a function of the time-average of the observable during the past interval of duration τ_r for that trajectory. Before simulating the next time window of length τ_r, trajectories with small weights are removed from the ensemble and are substituted by slightly perturbed copies of trajectories with large weights. The algorithm includes a parameter k which controls the relative amplitude of the weights for given values of the target observable, where the case $k = 0$ would correspond to a regular ensemble simulation. If τ_r is not larger than the decorrelation time of the observable, then the selection will favour the survival of trajectories leading to extreme

anomalies of the time-average of a target observable such as the pan-Arctic sea ice area (positive for k positive and viceversa). We refer to Ragone et al. (2018); Ragone and Bouchet (2020, 2021); Sauer et al. (2024) for more details about the method and present here the main outcome.

We denote $X(t)$ the vector of all model variables at time t and T_a the total simulation time. We consider an ensemble of N trajectories $\{X_n(t)\}$ $(n = 1, 2, ..., N)$. Let $\mathbb{P}_0(\{X_n(t)\}_{0 \leq t \leq T_a})$ be the probability density of observing a given trajectory $X_n(t)$ from time 0 to time T_a in a direct numerical simulation with the model, and $\mathbb{P}_k(\{X_n(t)\}_{0 \leq t \leq T_a})$ be the probability density of the same trajectory in an ensemble simulation with the rare event algorithm with a given value of the parameter k. For a large ensemble size N the relation between the two is given by the importance sampling formula

$$\mathbb{P}_k(\{X_n(t)\}_{0 \leq t \leq T_a}) \underset{N \to \infty}{\sim} \frac{e^{k \int_0^{T_a} A(\{X_n(t)\}) \, dt}}{\mathbb{E}_0[e^{k \int_0^{T_a} A(\{X_n(t)\}) \, dt}]} \mathbb{P}_0(\{X_n(t)\}_{0 \leq t \leq T_a}), \tag{2}$$

where $\mathbb{E}_0$ is the expectation value with respect to $\mathbb{P}_0$, $A\{X_n(t)\}$ the target observable and k a biasing parameter controlling the strength of the selection. Thanks to Equation (2), the expectation value according to $\mathbb{P}_0$ (the real statistics of the system) of a generic quantity $O(\{X_n(t)\}_{0 \leq t \leq T_a})$ can be estimated from data generated with the algorithm (which are instead distributed according to $\mathbb{P}_k$) as

$$\mathbb{E}_0[O(\{X_n(t)\}_{0 \leq t \leq T_a})] \underset{N \to \infty}{\sim} \frac{1}{N} \sum_{n=1}^{N} Z_k e^{-k \int_0^{T_a} A(\{X_n(t)\}) \, dt} \cdot O(\{X_n(t)\}_{0 \leq t \leq T_a}). \tag{3}$$

where Z_k is a constant factor computed by the algorithm (see Ragone et al. (2018); Ragone and Bouchet (2020)). Equation (3) shows that statistical quantities like composites, probabilities or return times with respect to the real model statistics can be estimated with data generated by the algorithm by weighting the contribution of each trajectory to sample averages by the inverse of the exponential factor which appears in the importance sampling formula. In Ragone et al. (2018), Ragone and Bouchet (2020, 2021) and Sauer et al. (2024), the trajectories appearing in (2) and (3) were generated from ensembles initialized with different initial conditions randomly selected from a control run to uniformly sample the attractor of the dynamics. In such an application, Equation (2) would refer to unconditional probabilities. In this work, however, the ensembles are initialized from single initial conditions and the probabilities and expectation values in (2) and (3) have to be interpreted as conditional on the selected initial state.

We use as target observable $A\{X_n(t)\}$ the pan-Arctic sea ice area and perform $M = 7$ ensemble simulations with the rare event algorithm initialised from seven different initial conditions. In order to have a baseline of the statistics, each experiment is accompanied by a control ensemble simulation (i.e. a simulation without applying the algorithm) with the same ensemble size, simulation length and initial condition as the one with the rare event algorithm. Each ensemble contains $N = 600$ trajectories and is run for a total simulation time $T_a = 240$ days from 01 February to 30 September.

The biasing parameter in the simulations with the rare event algorithm is $k = -0.075 \cdot 10^{-6}$ km^{-2} day^{-1}. The precise value of k is chosen empirically. However, a reasonable order of magnitude of the parameter can be derived from the standard deviation

220 and decorrelation time of the sea ice area anomalies in the control ensembles using a scaling argument presented in Ragone and Bouchet (2020) and in the Supporting Information of Ragone and Bouchet (2021). According to this scaling argument, the selected k-value corresponds to a shift of the mean of the distribution of the February-September mean sea ice area to values between approximately $0.15 \cdot 10^6$ km^2 and $0.35 \cdot 10^6$ km^2 below the means of the control ensembles. This corresponds to a range of estimated probabilities between about 10% and 1% (see section 3.1).

225 The resampling time in the simulations with the rare event algorithm is $\tau_r = 5$ days. It is chosen to be not larger than the persistence time scale of the large-scale atmospheric circulation (Baldwin et al., 2003) and to be only slightly larger than the typical persistence of synoptic-scale atmospheric fluctuations (Hoven, 1957). The rare event simulations are therefore suitable 230 to improve the sampling efficiency of extremely low sea ice states both driven by oceanic processes and due to anomalies in the atmospheric circulation that are in the order of or larger than the upper range of the synoptic time scale.

Within an individual ensemble, each trajectory starts from the same initial condition as done for ensemble weather and 235 seasonal climate predictions. In order to allow the trajectories to diverge from each other, we slightly perturb the surface pressure field of each trajectory as described in the Supporting Information of Ragone et al. (2018). The seven different initial conditions for the seven ensembles are sampled from the 3000-year control run according to different anomaly values of the January-February mean pan-Arctic sea ice area, i.e. the observable itself, and of the January-February mean cumulative area with sea ice thickness equal or larger than 1.93 m (SIT1.93, Figure 1 and table 1). The choice of SIT1.93 is motivated 240 by the studies of Chevallier and Salas-Mélia (2012) and Lindsay et al. (2008). According to those studies, the late winter-early spring cumulative area with sea ice thickness larger than a critical threshold has a larger impact on the late summer sea ice area than the winter-spring sea sea ice volume and area itself. Here we apply a composite analysis to the PlaSim-LSG control run to identify a sea ice-related variable whose anomaly value during the timing of the ensemble initialization has a potentially large connection with the probability of an extreme late summer sea ice low (Figure 1(a); note that the critical 245 thickness values of 0.2 m, 0.5 m, 0.66 m, 0.9 m, 1.5 m, 1.93 m, 2.5 m, 4 m, 4.2 m, 6 m are selected from Chevallier and Salas-Mélia (2012) and Lindsay et al. (2008)). Compared to the various variables shown in Figure 1(a), extremely negative February-September and August-September mean sea ice area anomalies are most strongly related to the January-February sea ice volume and cumulative area with sea ice thickness equal or larger than a critical thickness between 1.8 m and 2.1 m respectively. We choose SIT1.93 following the critical threshold used in Lindsay et al. (2008). We emphasize, however, that 250 the labeling ("SIT1.93−", "SIT1.93=", "SIT1.93+") of the selected initial conditions would be the same as in this study if the labeling were chosen according to the anomaly values of the cumulative area with sea ice thickness equal or larger than the remaining thresholds between 1.80 m and 2.10 m. In PlaSim-LSG, the correlations between SIT1.93 and the cumulative area with sea ice thickness equal or larger than the remaining thresholds between 1.8 m and 2.1 m are between 0.96 and 0.99, and the precise choice of a threshold between 1.8 m and 2.1 m does not affect the interpretation of the results.
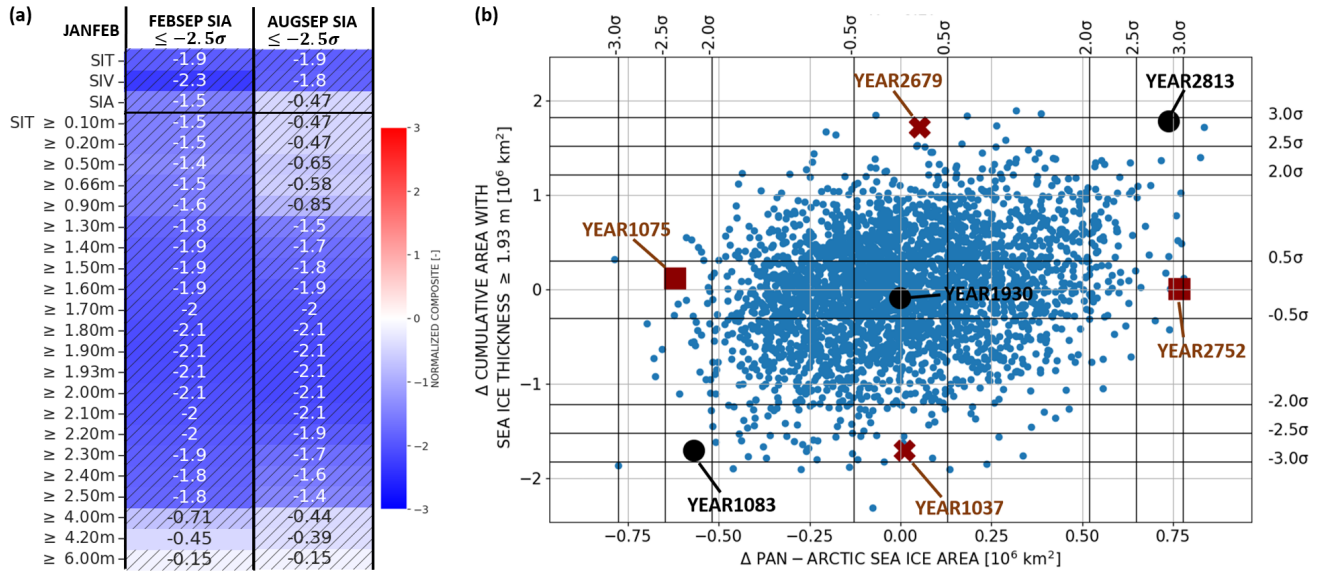
Figure 1. PlaSim-LSG 3000-year control run (Sauer et al. (2024)): (a) Normalized mean anomalies of January-February mean quantities conditional on extreme negative (left) February-September (FEBSEP) and (right) August-September (AUGSEP) mean pan-Arctic sea ice area anomalies equal or smaller than -2.5 standard deviations. "SIV", "SIA" and "SIT" are the pan-Arctic sea ice volume, sea ice area and mean sea ice thickness. "SIT ≤ threshold" are anomalies in the cumulative area with sea ice thickness equal or larger than a critical threshold. Hatching denotes statistical significance at the 5% level assessed from a two-sided t-test applied to five composite estimates after subdividing the 3000-year control run into five 600-member ensembles. (b) Scatter plot of January-February mean anomalies of SIT1.93 vs. pan-Arctic sea ice area including the years from the selected initial conditions.

Table 1. 600-member control and rare event algorithm ensemble simulations running between 01 February and 30 September. Labeling of the experiments according to the different initial conditions with (middle column) the full label and (right column) short labels used in the text.

Control run year	Full label	Short label
1083	SIA-LOW SIT1.93-LOW	SIA− SIT1.93−
1037	SIA-NEUTRAL SIT1.93-LOW	SIA= SIT1.93−
1075	SIA-LOW SIT1.93-NEUTRAL	SIA− SIT1.93=
1930	SIA-NEUTRAL SIT1.93-NEUTRAL	SIA= SIT1.93=
2752	SIA-LARGE SIT1.93-NEUTRAL	SIA+ SIT1.93=
2679	SIA-NEUTRAL SIT1.93-LARGE	SIA= SIT1.93+
2813	SIA-LARGE SIT1.93-LARGE	SIA+ SIT1.93+

3 Results

3.1 Importance sampling of extreme sea ice lows and estimation of their probabilities

We exploit initialized ensemble simulations to investigate the statistical properties of extreme negative February-September and August-September mean pan-Arctic sea ice area anomalies as a function of seven different initial winter sea ice states (cf. section 2.2). The goal of using the rare event algorithm is to obtain a better statistics of extremely low sea ice area values than available from standard control ensemble simulations. Compared to the latter, the algorithm likewise allows to better infer the order of magnitude of a lower bound of sea ice area values that can be generated internally via atmosphere-ocean dynamics.

We show the time evolution of pan-Arctic sea ice area anomalies relative to the control ensemble mean and the distributions of summer sea ice area for the ensemble simulations starting from a neutral initial winter sea ice state (Figure 2(a,c,e); control run model year 1930 and labeled as "SIA= SIT1.93="; see section 2.2 for more details) and from an initial winter state characterized by neutral sea ice area and extremely low cumulative area with sea ice thickness equal or larger than 1.93 m (SIT1.93; Figure 2(b,d,f); control run model year 1037 and labeled as "SIA= SIT1.93−"). Among all seven experiments, the "SIA= SIT1.93=" rare event simulation delivers the trajectory with the most extreme negative August-September mean sea ice area anomaly relative to the corresponding control ensemble mean (i.e. relative to the climatological mean value defined for the neutral sea ice initial condition), and the "SIA= SIT1.93−" experiment produces the trajectory leading to the lowest August-September mean sea ice area value in an absolute sense (cf. Figure 2 and Supplement Figures S2-S4).

Both in the "SIA= SIT1.93=" and "SIA= SIT1.93−" experiments, the differences between the sea ice area in the rare event algorithm and the control ensembles are small compared to the control intra-ensemble standard deviation until June (Figure 2(a,b)). From the end of June onwards, trajectories generated with the rare event algorithm show a systematic shift towards lower sea ice area values compared to the control ensembles, reaching an ensemble mean shift of about $-1.2 \cdot 10^6$ km^2 in each case.

The biasing towards negative sea ice area anomalies with the rare event algorithm compared to the control ensemble reflects the importance sampling of extreme negative sea ice area anomaly values on average over the entire simulation period between February and September (Figure 2(c-d)) [cf. Sauer et al. (2024); Ragone and Bouchet (2021)]. The distributions of February-September mean sea ice area obtained with the rare event algorithm fluctuate around a mean value in the lower tail of the control distribution, and their minimum sea ice area values are smaller than the minima obtained with the corresponding control ensembles (minimum February-September mean sea ice area values of $9.69 \cdot 10^6$ km^2 and $9.26 \cdot 10^6$ km^2 in the "SIA= SIT1.93=" and "SIA= SIT1.93−" rare event simulations vs. $9.88 \cdot 10^6$ km^2 and $9.34 \cdot 10^6$ in the corresponding control ensemble simulations). As the February-September mean sea ice area is strongly related to the August-September one (the minimum-maximum range of the correlation between both quantities across the members in all seven control ensembles is [0.89 0.94]), the algorithm likewise improves the sampling efficiency of extremely low August-September mean sea ice area exceeding the lower range of sea ice area values obtained with the control ensembles (Figure 2(e-f)); minimum August-September mean sea ice area values of $2.82 \cdot 10^6$ km^2 and $2.07 \cdot 10^6$ km^2 in the "SIA= SIT1.93=" and "SIA= SIT1.93−" rare event simulations vs.

$3.54 \cdot 10^6$ km^2 and $2.37 \cdot 10^6$ in the control ensemble simulations).

It is noticeable that the sea ice area obtained with the control and rare event ensemble simulations depends on the chosen initial condition (i.e. its location relative to the distribution of the 3000-year control run varies among the different experiments; cf. Figure 2(c-f) and Supplement Figure S2-S4). In order to quantify the impact of the initial condition on extreme summer sea ice lows, we compare the probability of extreme negative February-September and August-September mean sea ice area anomalies between the seven experiments (Figure 3). We compute these probabilities as

$$P(a) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{1}_a(I_n) \text{ with } \mathbb{1}_a(I_n) = \begin{cases} 1, & I_n \leq a \\ 0, & I_n > a \end{cases}, \tag{4}$$

where I_n is the February-September or August-September time-averaged pan-Arctic sea ice area in trajectory n, a is the amplitude of the sea ice area anomaly, N the number of trajectories and $\mathbb{1}_a(I_n)$ is the indicator function. As Equation (4) represent the probability as an expectation value of the indicator function, probabilities from the simulations with the rare event algorithm can be computed by applying the estimator in Equation (3) to this quantity (see Lestang et al. (2018), Ragone et al. (2018); Ragone and Bouchet (2020) and Sauer et al. (2024)).

In Figure 3, the solid and dashed lines show the probabilities estimated with the control and rare event algorithm ensemble simulations respectively. The control and rare event algorithm estimates are consistent with each other where they overlap. Compared to the control ensembles, the major advantage of the algorithm is the access to much rarer events with the former than the latter, reaching probability values of 10^{-5} (0.001%) for a computational cost of order 10^2-10^3. The plateaus at probability values smaller than 10^{-4} are likely due to undersampling as discussed in Lestang et al. (2018) and in the Supporting Information of Ragone et al. (2018).

For a fixed probability level, the amplitudes of February-September and August-September mean sea ice area span a range of about five and four and a half standard deviations among the different initial conditions. This confirms a strong impact of the initial condition on the amplitudes and probabilities of extreme negative summer sea ice area anomalies evaluated relative to the 3000-year control run baseline climatology (Figure 3). Regarding the February-September seasonal average, both a low state in the winter SIT1.93 and in the winter sea ice area contribute to the preconditioning of an extreme (Figure 3(a)). Consequently, the most extremely negative seasonally averaged sea ice area anomalies occur following years with both a low winter state in the sea ice area and in SIT1.93 (Figure 3(a)). The contribution of low winter sea ice area and low SIT1.93 to these anomalies, however, is not fully additive (Figure 3(a)). For a fixed probability level, the sum of the February-September mean sea ice area anomalies relative to the control run mean over the "SIA= SIT1.93−" and "SIA− SIT1.93=" experiments exceeds in magnitude the sea ice area anomalies obtained with the "SIA− SIT1.93−" experiment. We hypothesize that the preconditioning of low states of the seasonally mean sea ice area through low winter sea ice area and SIT1.93 is efficient only for a restricted geographic region where the climatological mean sea ice thickness is small enough to be able to form open water within one season. In parts of that region, the contribution of low winter sea ice area and SIT1.93 to an anomalously large area of open water during summer is likely to overlap spatially (i.e. open water conditions prevail in certain grid boxes independent from

whether both winter sea ice area and SIT1.93 are in a low state or only one of them). Regarding the August-September mean, the contribution of a low winter sea ice area to an extreme is small compared to the contribution of a low winter state in SIT1.93 (Figure 3(b)). Thus, similar amplitudes of extremely low August-September sea ice area occur for all three experiments with a
335   neutral SIT1.93, independent from the state of the winter sea ice area. Likewise, the most extremely low August-September sea ice area occurs in the experiment starting from an initial condition with a low SIT1.93 and a neutral winter sea ice area. A low winter sea ice area is therefore not required to produce the most extremely negative August-September sea ice area anomalies.

340   Despite the important role of preconditioning in favouring extremely low sea ice states, preconditioning represents only a necessary and not a sufficient condition for extremes with the largest amplitudes available in this study (Figure 3 and table 2). The most extreme sea ice lows occur as a consequence both of winter sea ice-ocean preconditioning and anomalous intra-seasonal processes (e.g. weather patterns; see section 4) that enhance sea ice reduction within one single melting season. The relative contribution of preconditioning to extreme sea ice lows *vs.* anomalous dynamics on an intra-seasonal time scale is
345   larger for February-September than August-September mean sea ice area (Figure 3 and table 2). Thus, the February-September time-averaged control ensemble mean sea ice area anomaly for the "SIA− "SIT1.93−" initial condition explains about 50% of the most extremely negative February-September mean sea ice area anomaly occurring in the "SIA− "SIT1.93−" rare event algorithm simulation. In contrast, the August-September time-averaged control ensemble mean sea ice area anomaly for the experiments starting from "SIT1.93−" are in the order of 30%-35% of the most extremely negative August-September mean
350   sea ice area anomalies appearing in the ensemble simulations with the rare event algorithm (Figure 3 and table 2).

Finally, it is striking that despite the fact that the algorithm favours the sampling of extremely low sea ice area values, no seasonally sea ice-free Arctic occurs in the simulations in the present study. Even in the experiment with a "SIT1.93-" initial
355   condition, i.e. an initial state being particularly favourable for low late summer sea ice states according to Figure 1, the most extremely low August-September mean sea ice area does not show values below the order of $2 \cdot 10^6$ km^2. Consequently, a seasonally sea ice-free Arctic generated by internal climate variability under the pre-industrial greenhouse gas forcing used in this study has a quasi-zero probability.
360

The results obtained in Figure 3 are consistent with the memory properties of the sea ice in the Arctic. Typically, late summer sea ice area anomalies are poorly connected to the late winter sea ice area (Blanchard-Wrigglesworth et al., 2011; Chevallier and Salas-Mélia, 2012), explaining the non-existing contribution of winter sea ice area anomalies to extreme late summer sea ice area lows. Quantities related to the sea ice thickness, as shown in Chevallier and Salas-Mélia (2012), instead, act as a
365   preconditioning for extreme reduction of sea ice area via its impact on the open-water formation efficiency during summer. The dependency of extreme negative seasonally-averaged sea ice area anomalies on the winter sea ice area is explainable by the persistence of negative sea ice area anomalies from late winter to spring (Blanchard-Wrigglesworth et al., 2011).
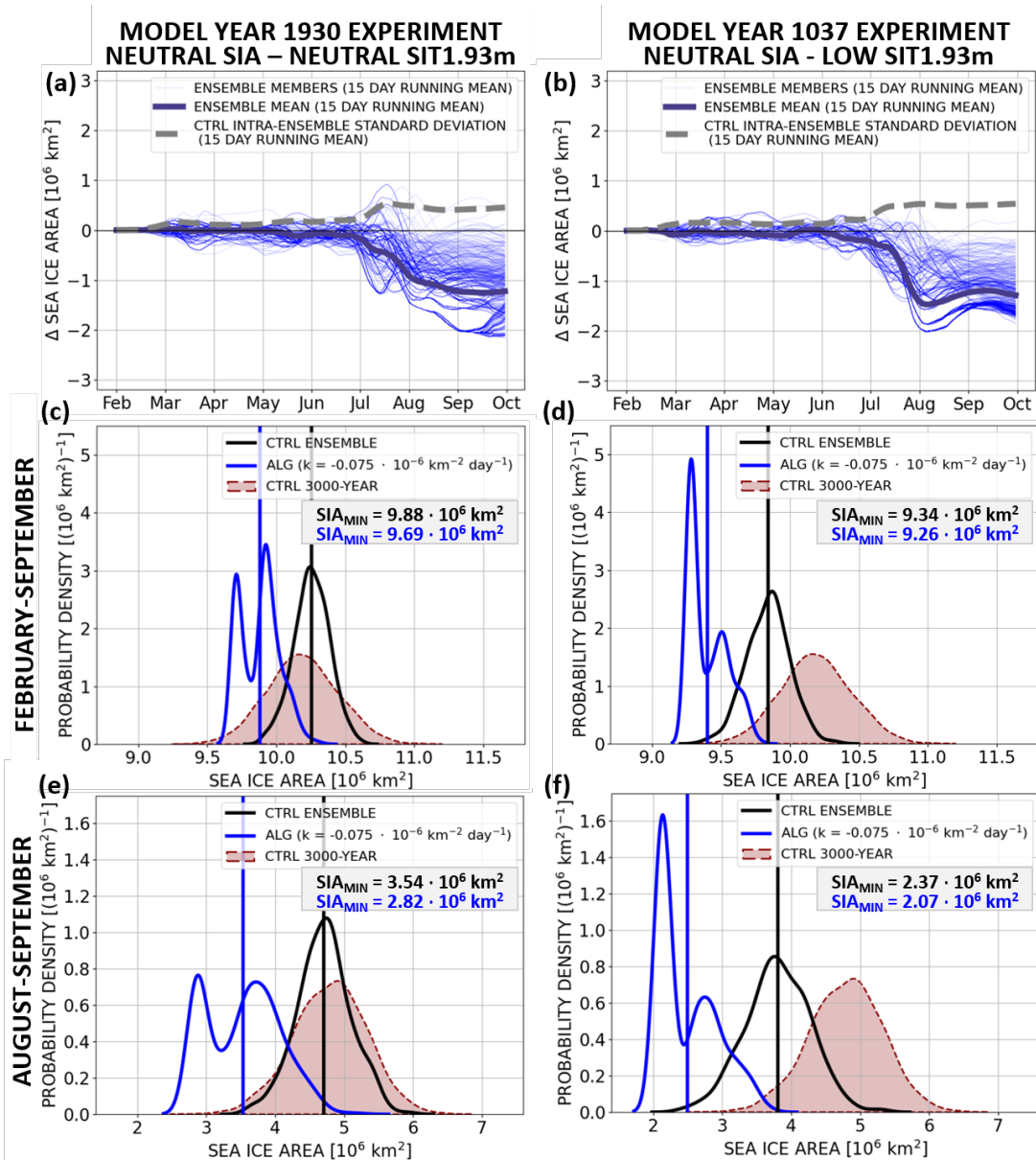
Figure 2. Ensemble simulations initialized on 01 February (a,c,e) 1930 and (b,d,f) 1037 of the control run. (a-b) Rare event simulations: Trajectories (thin blue lines) and ensemble mean (thick blue line) of daily pan-Arctic sea ice area anomalies relative to the daily climatology of the corresponding control ensembles. The gray dashed lines show the intra-ensemble standard deviations in the control ensembles. All lines are presented as 15-day running means. (c-f) Probability distribution functions of (c-d) February-September and (e-f) August-September mean pan-Arctic sea ice area for (blue) the rare event simulation, (black) the control ensembles and (red) the full 3000-year control run. The vertical lines show the mean of the distributions. The black and blue values indicate the smallest February-September and August-September mean sea ice area value sampled with the control and rare event ensemble simulations respectively.
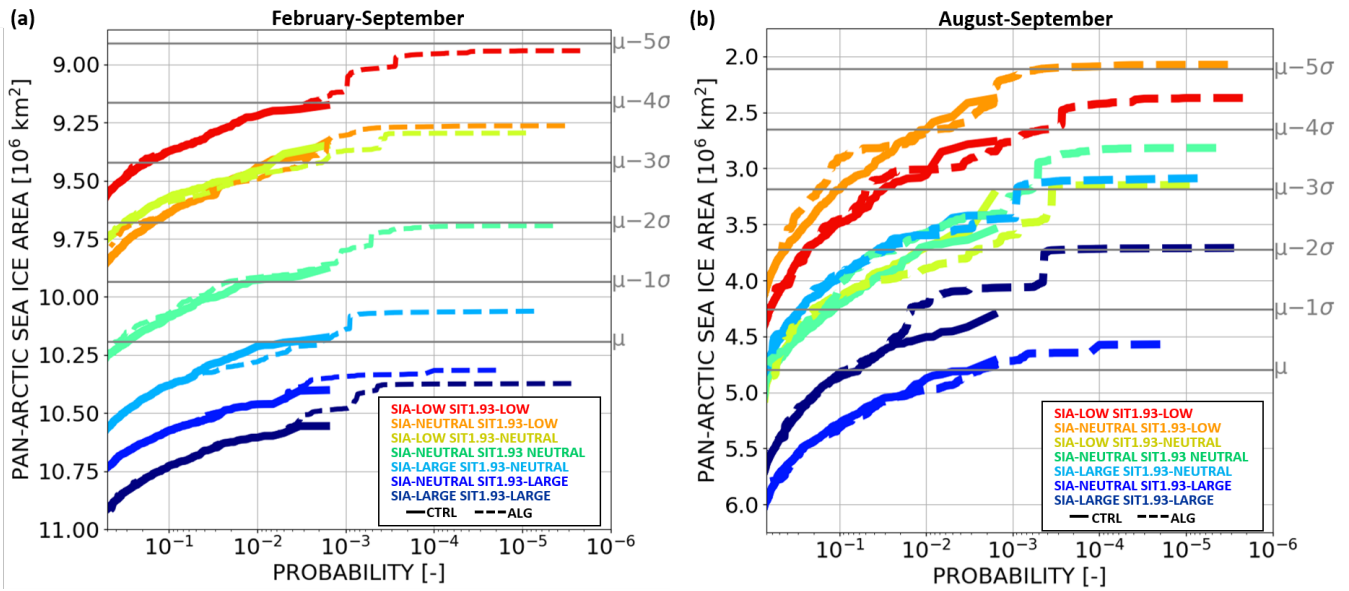
Figure 3. Probabilities (x-axes) of (a) February-September and (b) August-September mean sea ice area equal or smaller than a given threshold (y-axes) as a function of seven different initial conditions for (solid lines) the control ensembles and (dashed lines) for the algorithm. Note that the y-axes are displayed in reverse order. In the legend, "SIA" and "SIT1.93" indicate the state of the January-February mean anomaly of the pan-Arctic sea ice area and the cumulative area with sea ice thickness equal or larger than 1.93 m respectively. The grey labels on the right of each panel show how many standard deviations a sea ice area value is below the mean of the 3000-year control run.

Table 2. February-September (FEBSEP) and August-September (AUGSEP) mean sea ice area anomalies relative to the mean μ of the 3000-year control run for the trajectory with the most extremely negative sea ice area anomaly obtained with the rare event algorithm ($SIA_{MIN}-μ$; i.e. preconditioning + anomalous weather) and for the control ensemble means ($μ_{ENS}-μ$; i.e. the impact of preconditioning on the mean of the control ensemble). "nσ" denotes the unit of multiple control run standard deviations with $n \in \mathbb{R}$.

	FEBSEP		AUGSEP	
Initial condition	$(SIA_{MIN}-μ)$ [nσ]	$(μ_{ENS}-μ)$ [nσ]	$(SIA_{MIN}-μ)$ [nσ]	$(μ_{ENS}-μ)$ [nσ]
SIA− SIT1.93−	−4.87	−2.46	−4.52	−1.32
SIA= SIT1.93−	−3.61	−1.36	−5.07	−1.84
SIA− SIT1.93=	−3.49	−1.72	−3.07	−0.19
SIA= SIT1.93=	−1.94	+0.25	−3.68	−0.16
SIA+ SIT1.93=	−0.51	+1.48	−3.18	−0.48
SIA= SIT1.93+	+0.48	+2.14	−0.42	+1.81
SIA+ SIT1.93+	+0.71	+2.81	−2.02	+1.19

4 Physical processes and conditions prior to sea ice lows with probabilities of less than 0.001%

In this section we study the physical processes and conditions favouring extremely low late summer sea ice area. In contrast to the work in Sauer et al. (2024), the development of low sea ice states relative to the initial condition-specific control ensemble mean are entirely attributable to intra-seasonal drivers as all trajectories within an ensemble start from the same initial condition. Using the "SIA= SIT1.93=" and "SIA= SIT1.93−" experiments, we discuss the trajectory leading to the lowest August-September mean pan-Arctic sea ice area value available from the two experiments (Figures 4-7). The lowest August-September mean sea ice area values in the "SIA= SIT1.93=" and "SIA= SIT1.93−" ensembles are $2.82 \cdot 10^6$ km^2 and $2.07 \cdot 10^6$ km^2 and thus about 40% and 45% smaller than the control ensemble mean values of $4.71 \cdot 10^6$ km^2 and $3.81 \cdot 10^6$ km^2. These sea ice area anomalies are therefore larger in magnitude than the deviation of the observed 2012 mean August-September mean sea ice area from a trend line fitted to the period 1979-2006 (about -32%; $3.09 \cdot 10^6$ km^2 compared to $4.53 \cdot 10^6$ km^2; see Figure S1 in the Supplementary Information). The estimated probabilities to generate a sea ice area equal or smaller in absolute value than the one of the trajectories of the two ensemble simulations are P=$0.45 \cdot 10^{-5}$ ("SIA= SIT1.93=") and P=$0.33 \cdot 10^{-5}$ ("SIA= SIT1.93−") (see Figure 3).

4.1 Seasonal evolution of the state of the sea ice

The most dominant negative sea ice area anomalies in both trajectories start to develop in June-July and reach their peak amplitudes in August-September (Figure 4(a,d)). Negative August-September mean sea ice area anomalies are characterized by enhanced open water area in the Barents and Kara Seas, the eastern central Arctic Ocean and northern of the Canadian Archipelago (Figure 4(b,e)). From late winter to late spring, sea ice area anomalies are close to zero in the "SIA= SIT1.93=" experiment and slightly negative with a magnitude in the order of the intra-ensemble standard deviation in the "SIA= SIT1.93−" experiment. The fact that winter-spring sea ice area anomalies are small compared to the late summer supports the conclusion from Figure 3 that winter-spring sea ice area anomalies do not play an important role for the formation of late summer sea ice lows.

It is noticeable that late summer negative sea ice area anomalies are preceded by negative pan-Arctic sea ice volume anomalies starting to develop in spring (Figure 4(a,d)). These are related to the development of negative sea ice thickness anomalies before an anomalous reduction of the pan-Arctic sea ice area. These negative sea ice thickness anomalies develop predominantly on the Eurasian side of the Arctic and northern of the Canadian Archipelago (Figure 4(c,f)). They act as a preconditioning for enhanced open water formation in late summer as described in Chevallier and Salas-Mélia (2012).

4.2 Seasonal evolution of the state of the atmosphere

We are interested in the processes favouring the development of negative sea ice thickness and ensuing negative sea ice area anomalies. Firstly, we analyze the seasonal evolution of two metre temperature (T2M) and 500 hPa geopotential height (Z500) anomalies to establish a link between sea ice lows and the thermodynamic and dynamic states of the atmosphere (Figure 5). The evolution of the atmospheric state differs between the "SIA= SIT1.93=" and "SIA= SIT1.93−" trajectories. In the "SIA=
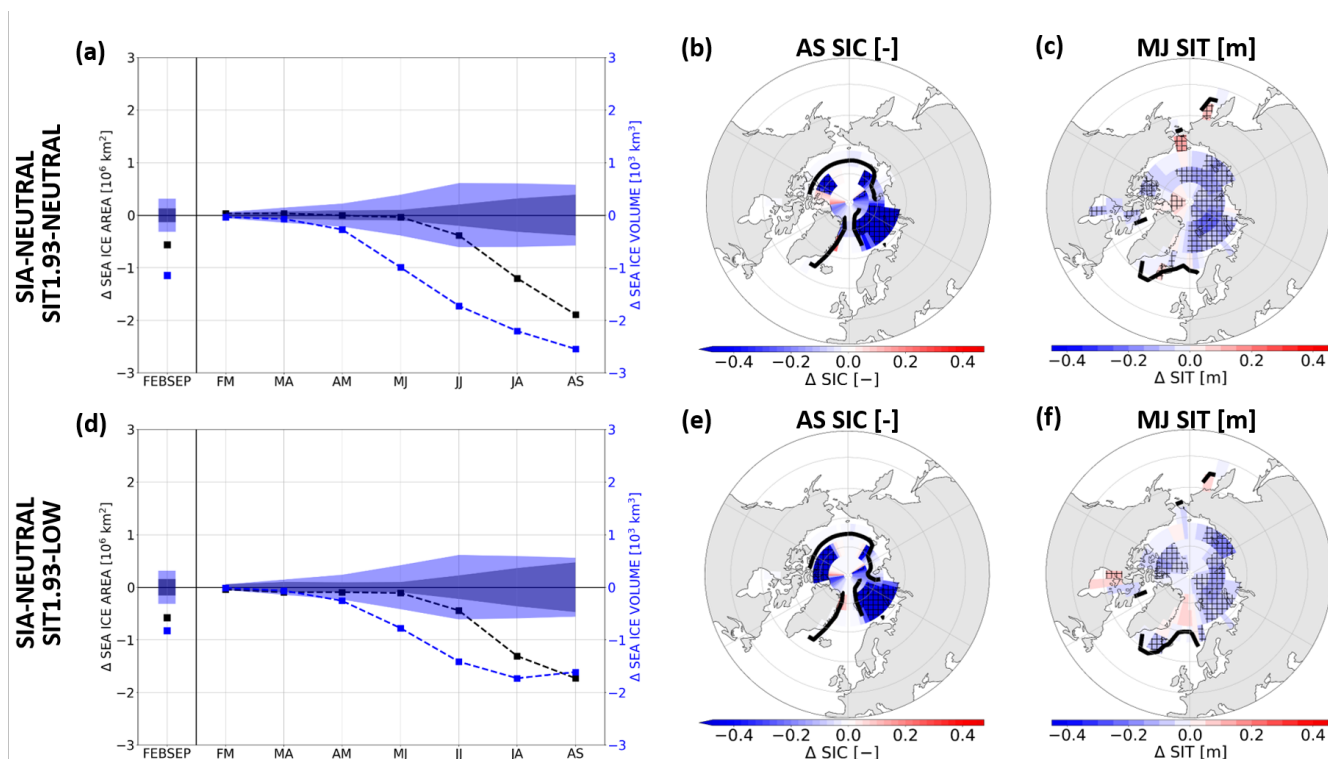
Figure 4. Trajectory with the lowest February-September and August-September mean pan-Arctic sea ice area in the rare event algorithm ensemble simulation starting from a winter initial condition characterized by (a-c) neutral sea ice area and neutral SIT1.93 (control run year 1930) and (d-f) neutral sea ice area and low SIT1.93 (control run year 1037). (a,d) Pan-Arctic sea ice (black) area [10^6 km^2] and (blue) volume [10^3 km^3] anomalies. Shading shows the intra-ensemble standard deviation of the corresponding control ensembles. (b,e) August-September mean sea ice concentration and (c,f) May-June mean sea ice thickness anomalies. Hatching indicates anomalies larger in magnitude than the intra-ensemble standard deviation. (a-f) All anomalies are computed with respect to the climatology of the corresponding control ensembles.

SIT1.93=" trajectory, the Arctic region is in an anomalously warm state from late winter to late spring. Positive T2M anomalies occur over the central Arctic Ocean in February-March and show a wavenumber 3-like positive T2M pattern in spring associated with a baroclinic signal in the 500 hPa geopotential height anomaly field (Figure 5(b,c,g,h)). In May-June, anomalously warm conditions prevail over the eastern Arctic and north of the Canadian Archipelago, and thus spatially coincide with the region of negative sea ice thickness anomalies (cf. Figures 5(c) and Figure 4(c)). In June-July, the magnitude of T2M anomalies declines as two metre temperatures are constrained by the climatological sea ice melting to be close to the freezing point. The reemergence of anomalously warm conditions in July-August may both be a driver of sea ice reduction and a consequence of reduced sea ice conditions. In contrast to the "SIA= SIT1.93=" experiment, the "SIA= SIT1.93−" trajectory shows an anomalously cold near-surface atmosphere in February-March followed by a strong warming with a barotropic positive geopotential height anomaly response during spring (Figure 5(k,l,m,p,q,r)). Positive 500 hPa geopotential height anomalies persist until summer, while T2M are constrained to be close to the freezing point in June-July before reemerging in August-September as

for the "SIA= SIT1.93=" trajectory (cf. Figures 5(n,o,s,t) and 5(d)).
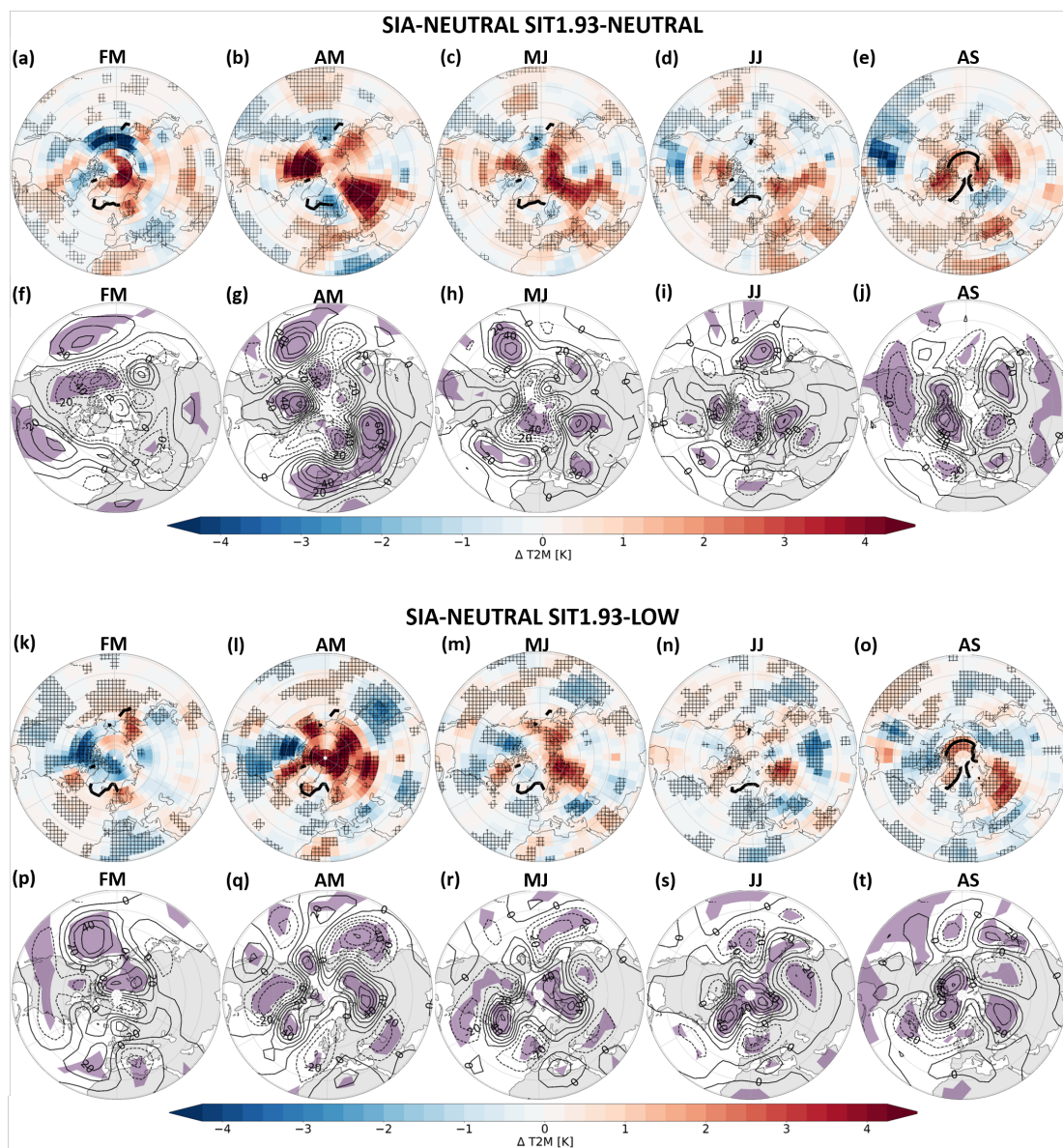
Figure 5. Trajectory with the lowest February-September and August-September mean pan-Arctic sea ice area in the rare event algorithm ensemble simulation starting from a winter initial condition characterized by (a-j) neutral sea ice area and neutral SIT1.93 (control run year 1930) and (k-t) neutral sea ice area and low SIT1.93 (control run year 1037). (a-e) and (k-o) February-March (FM), April-May (AM), May-June (MJ), June-July (JJ) and August-September (AS) mean T2M anomalies [K] and (f,j) and (p-t) FM, AM, MJ, JJ, AS mean Z500 anomalies [gpm]. Contour interval for Z500 is 10 gpm. Hatching indicates anomalies larger in magnitude than the intra-ensemble standard deviation. All anomalies are computed with respect to the climatology of the corresponding control ensembles. The black contour line in (a-e) and (k-o) shows the climatological sea ice edge defined as the 15% sea ice concentration contour line.

4.3 Surface energy budget analysis

We perform a surface energy budget analysis to understand how the anomalous reduction of sea ice is physically related to anomalous atmospheric conditions (Figures 6-7). We define the surface energy budget for an infinitesimally thin interface without heat storage located between the atmosphere and the snow-sea ice-ocean system (cf. Serreze and Barry (2014)). The different terms of the budget are then given by

$$R_{SW}(1-\alpha) + R_{LW} + S + L = -\epsilon\sigma T_s^4 + Q + M, \tag{5}$$

where R_{SW} and R_{LW} are the downward shortwave and downward longwave radiative fluxes, respectively; α is the surface albedo, S and L are the sensible and latent heat fluxes, ϵ is the surface emissivity, $\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4} is the Stefan–Boltzmann constant and T_s is the surface temperature. Q summarizes conductive and turbulent energy fluxes between the surface and the snow-sea ice-ocean system. M is the energy flux associated with melting and freezing at the surface. Energy fluxes related to bottom sea ice growth, to temperature changes in the snow-sea ice-ocean system and the vertical energy flux from the ocean into the sea ice are implicitly included in Q. In Figure 6, we complement the analysis of the atmosphere-surface energy fluxes by the analysis of the vertical oceanic heat flux. If not stated otherwise, we define upward (downward) energy fluxes and associated flux anomalies to be positive (negative).

Both in the "SIA= SIT1.93=" and "SIA= SIT1.93−" trajectories, strongly negative net surface-atmosphere energy flux anomalies occur over the Arctic Ocean from April-May to August-September, indicating an anomalous energy accumulation within the snow-sea ice-ocean system (Figure 6(a,b)). Anomalies in the vertical oceanic heat flux are much smaller than the atmospheric ones, but still systematically positive in summer in both trajectories (Figure 6(a,b)). Slightly positive vertical oceanic heat flux anomalies in late winter in "SIA= SIT1.93−" together with enhanced heat loss from the surface to the atmosphere indicate a small oceanic forcing on the reduction of sea ice, contributing to the small negative late winter-spring sea ice area anomalies shown in Figure 4(d).

The contribution of different flux components to the negative net atmosphere-surface energy flux anomalies varies among the season and between the two trajectories. In the "SIA= SIT1.93=" trajectory, two distinct phases exist. In spring, enhanced net energy accumulation in the snow-sea ice-ocean system is predominantly explained by negative net longwave radiative flux anomalies and enhanced downward sensible heat fluxes (Figure 6(c,e)). During summer, the anomalies in the atmosphere-surface energy transfer are dominated by the shortwave fluxes. Qualitatively, the two phases are apparent also in the "SIA= SIT1.93−" trajectory (Figure 6(d,f)). Compared to the "SIA= SIT1.93=" trajectory, however, the enhanced longwave radiative forcing on the sea ice has a smaller duration exceeding the intra-ensemble standard deviation exclusively in April-May, and spring sensible heat flux anomalies are close to zero. During summer, the "SIA= SIT1.93−" trajectory exhibits a strong shortwave radiative forcing available to melt sea ice.

We further subdivide the net shortwave and longwave radiative surface flux anomalies into their downward and upward components to trace back the net radiative surface flux anomalies to anomalous atmospheric conditions (Figure 7). In the

"SIA= SIT1.93=" trajectory, reduced downward solar radiation and enhanced downward longwave radiation occurs in April-May and May-June (Figure 7(a,c)). The signal in the longwave radiative fluxes are associated with enhanced atmospheric
460 water vapour and cloudiness (Figure 7(e)). In the "SIA= SIT1.93−" trajectory, instead, enhanced downward radiative flux anomalies associated with clouds and water vapour primarily occur in April-May (Figure 7(b,d,f)). In summer, a reduction in clouds occurs and the shortwave radiative fluxes in combination with the sea ice-albedo feedback become the dominant driver of sea ice reduction in both experiments.

5 Discussion and conclusions

465 We exploit seven initialized ensemble simulations to study the statistical properties of extreme negative summer and late summer pan-Arctic sea ice area anomalies as a function of seven different initial winter states. Likewise, we investigate physical processes in two trajectories leading to extreme late summer sea ice lows with larger amplitudes than the one observed in 2012. In order to circumvent the poor sampling of rare events in numerical simulations, we apply a rare event algorithm to improve the sampling efficiency of trajectories leading to extremely low pan-Arctic sea ice area on average over the melting season and
470 during the annual sea ice minimum. The simulations with the rare event algorithm produce several hundreds of times more extremes than conventional control ensemble simulations for the same computational cost, and allow to generate extreme sea ice lows with probabilities two to three orders of magnitude smaller compared to direct sampling.

475 The approach used in this study is complementary to the rare event algorithm study of Sauer et al. (2024). In the latter, ensemble simulations are initialized with independent initial conditions sampled from an entire 3000-year control run and the statistics computed with the rare event algorithm corresponds to unconditional probability distributions. Such an approach allows to improve the sampling efficiency of low sea ice states both related to multi-annual variability in the sea ice-ocean system (referred to as "preconditioning" in Sauer et al. (2024)) and driven by the dynamics occurring on intra-seasonal time scales.
480 The single-initial condition approach used in this study, instead, corresponds to a seasonal climate prediction set-up giving access to the probability to observe an extreme late summer sea ice low as a function of a given initial state. This approach also enables a targeted study of intra-seasonal drivers of anomalously strong sea ice reduction during the melting season. Within each ensemble simulation, a contribution of multi-annual sea ice-ocean preconditioning to the generation of low sea ice states compared to the control ensemble mean is by definition excluded as each trajectory starts from the same sea ice-ocean state.

485 The statistics obtained from the seven different experiments indicates a strong impact of the late winter sea ice initial condition on the probability and amplitudes of extremely negative summer pan-Arctic sea ice area anomalies computed relative to a 3000-year control run baseline climatology. For a fixed probability level, amplitudes of the most extreme February-September and August-September mean sea ice area anomalies span a range of about five and four and a half control run standard deviations over the seven experiments (Figure 3 and table 2). While winter preconditioning is a necessary condition to generate a sea ice low with an amplitude larger than a certain threshold, the most extreme sea ice lows require both preconditioning and anomalous intra-seasonal dynamics (e.g. anomalies in the atmospheric circulation) favouring enhanced reduction of sea ice area during the melting season (Figure 3 and table 2). The relative importance of preconditioning compared to intra-seasonal
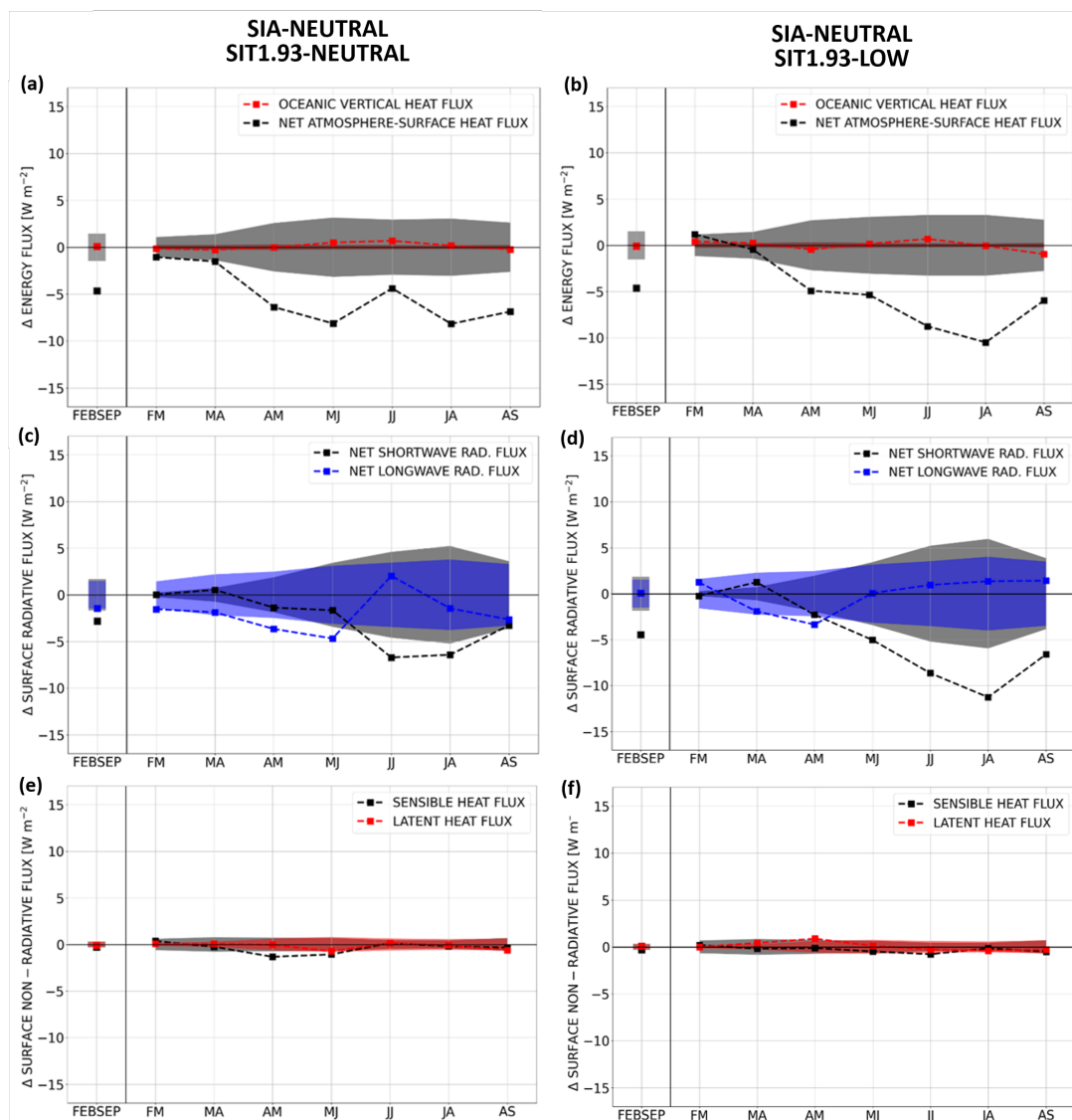
Figure 6. Trajectory with the lowest February-September and August-September mean pan-Arctic sea ice area in the rare event algorithm ensemble simulation starting from a winter initial condition characterized by (a,c,e) neutral sea ice area and neutral SIT1.93 (control run year 1930) and (b,d,f) neutral sea ice area and low SIT1.93 (control run year 1037). Mean energy flux anomalies [W m^{-2}] on average over all ocean grid boxes northern of $70°$N. (a-b) (black) Net atmosphere-surface energy flux (sensible + latent + net longwave + net shortwave) and (red) vertical ocean heat flux. (c-d) Net surface (blue) longwave radiative and (black) shortwave radiative flux. (e-f) Surface (red) latent and (black) sensible heat flux anomalies. The sign convention is chosen such that positive anomalies correspond to enhanced upward fluxes. (a-f) Shading indicates anomalies larger in magnitude than the intra-ensemble standard deviation. All anomalies are computed with respect to the climatology of the corresponding control ensembles.
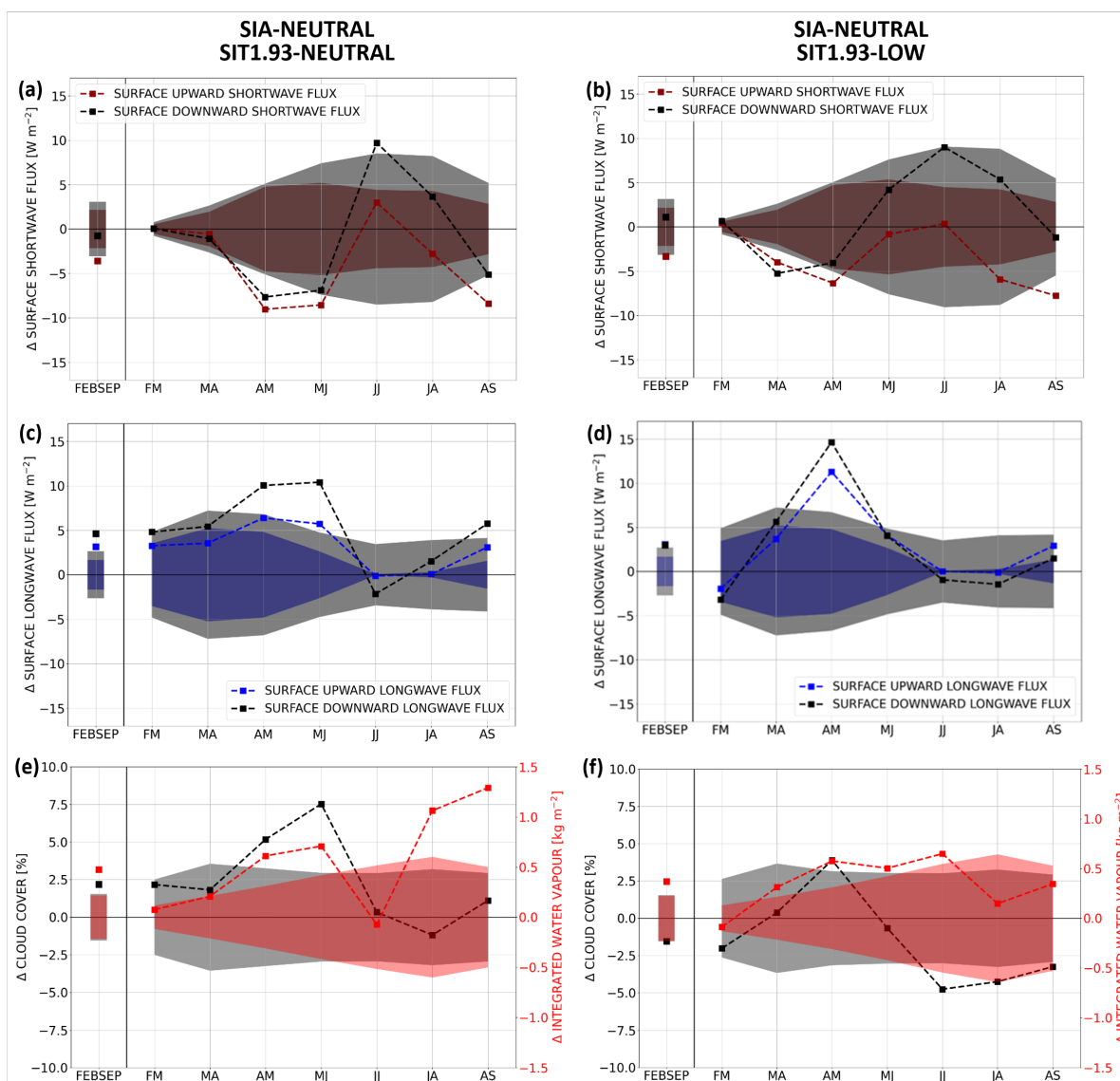
Figure 7. Trajectory with the lowest February-September and August-September mean pan-Arctic sea ice area in the rare event algorithm ensemble simulation starting from a winter initial condition characterized by (a,c,e) neutral sea ice area and neutral SIT1.93 (control run year 1930) and (b,d,f) neutral sea ice area and low SIT1.93 (control run year 1037). Mean anomalies of different quantities spatially averaged over all ocean grid boxes northern of 70°N. (a-b) Surface (darkred) upward and (black) downward shortwave radiative flux anomalies [W m^{-2}]. (c-d) Surface (blue) upward and (black) downward longwave radiative flux anomalies [W m^{-2}]. (a-d) Direction-independent absolute values of the downward and upward fluxes are considered, i.e., a positive (negative) anomaly indicates a radiative flux that is stronger (weaker) in magnitude than the climatology. (e-f) (Black) Total cloud cover [%] and (red) integrated water vapour [kg m^{-2}] anomalies. (a-f) Shading indicates anomalies larger in magnitude than the intra-ensemble standard deviation. All anomalies are computed with respect to the climatology of the corresponding control ensembles.

dynamics in driving extremely low sea ice area is larger for the February-September than August-September mean sea ice area (table 2). Thus, at least 50% of the most extremely negative February-September mean sea ice area anomaly in the "SIA− SIT1.93−" experiment is explainable by the impact of the preconditioning on the control ensemble mean sea ice area (table 2). For August-September, instead, about 36% of the most extremely negative sea ice area anomalies are explainable by the impact of the preconditioning on the control ensemble mean (table 2).

An extremely negative February-September mean pan-Arctic sea ice area anomaly is preconditioned by low January-February mean states of both the pan-Arctic sea ice area and the cumulative area with sea ice thickness larger than a given threshold (the precise value used in this study is 1.93 m). In contrast, a source of predictability of an extreme late summer sea ice low is given by the winter sea ice thickness information, but not from the winter sea ice area. Both results are in line with the memory properties of the sea ice in the Arctic (Blanchard-Wrigglesworth et al., 2011; Chevallier and Salas-Mélia, 2012). Using a preindustrial simulation with the Centre National de Recherches Météorologiques Coupled Global Climate Model version 3.3 (CNRM-CM3.3), Chevallier and Salas-Mélia (2012) finds that the cumulative area with sea ice thickness larger than a threshold between 0.9 and 1.5 m provides a potential source of predictability of the September pan-Arctic sea ice area six months in advance. The authors argue that winter-spring sea ice thickness influences summer sea ice area by modulating the open-water-formation efficiency during the melting season. The dependency of extremely negative February-September mean sea ice area anomalies both on the winter SIT1.93 and winter pan-Arctic sea ice area itself can be explained by the fact that late winter sea ice area anomalies have a non-zero persistence until spring (Blanchard-Wrigglesworth et al., 2011; Chevallier and Salas-Mélia, 2012). All in all, we emphasize that the precise critical sea ice thickness threshold used to define the cumulative area covered by sea ice thickness larger than a given value and the precise time lag may vary between different models.

We furthermore utilize the rare event algorithm to detect an approximate lower bound of sea ice area values that can be generated under the constant pre-industrial greenhouse gas forcing used in this study. The lowest August-September mean sea ice area values occurring in this study are in the order of $2 \cdot 10^6$ km^2. Conditional on starting from a "SIT1.93−" initial winter state, the estimated probabilities for a sea ice area value equal or below this order of magnitude is 0.001%. Even though this study may not have detected the initial condition with the most optimal sea ice-ocean properties in favouring low late summer sea ice area as possible, we conclude that the probability of observing a seasonally sea ice-free Arctic under this set-up is either close to zero or even zero. We hypothesize that a lower bound of sea ice area values larger than zero is given by sea ice being too thick to be melt away within one season.

Owing to the rare event algorithm, we analyze the physical properties in two individual trajectories leading to extremely negative August-September mean sea ice area anomalies with conditional probabilities of less than 0.001%. While the precise thermodynamic and dynamical evolution of the atmosphere differs between both trajectories, common mechanisms can be detected between both trajectories. In both cases, an anomalously strong reduction of the pan-Arctic sea ice area in mid to late summer is anticipated by the development of negative sea ice thickness anomalies during spring. Apart from a small contribution of enhanced vertical ocean heat flux anomalies to the reduction of sea ice, the largest amount of net energy accumulation in the sea ice-ocean system is related to enhanced downward net atmosphere-surface energy fluxes. Within the season, we

535 observe two phases with two different physical properties. During spring, enhanced cloudiness and atmospheric water vapour results in negative (i.e. downward) net surface longwave radiative flux anomalies, providing enhanced energy to warm or melt sea ice. This is consistent with the finding of Kapsch et al. (2013, 2019) showing that observed extreme September sea ice lows have been preceded by enhanced downward longwave radiative fluxes in an anomalously moist and cloudy atmosphere. During summer, anomalous reduction of sea ice area is favoured by enhanced downward solar radiative flux, the sea ice-albedo

540 feedback and enhanced open water formation efficiency in an anomalously thin sea ice environment.

This study shows how rare event algorithms can be applied to initialized ensembles to study the probability of extreme states conditional on given initial conditions. This approach could be extremely useful in the context of seasonal to decadal

545 predictions, both in terms of risk quantification, and to inform ensemble initialization strategies by the identification of important physical drivers for the development of extremes. This would be particularly relevant for applications with complex numerical models. The present study is based on a relatively low resolution climate model with a purely thermodynamic sea ice model. A direct dynamic forcing of the atmospheric circulation on the sea ice, e.g., related to synoptic-scale storms or to the Arctic Oscillation and the Arctic Dipole Anomaly pattern, are therefore not captured by the model. Nevertheless, based on

550 the analysis of two individual trajectories, it illustrates possible thermodynamics states of the atmosphere to generate sea ice lows larger in magnitude than the one observed in 2012. A similar rare event algorithm study with the EC-Earth model version 3.3.1, including a more complex physics than PlaSim and a dynamic sea ice model is in preparation.

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Appendix A

670 *Code and data availability.* The code and data availability will be indicated after acceptance of the article.

Author contributions. All authors contributed to the study conception and design. Model simulations, data collection and analyses were performed by the first author Jerome Sauer. The first draft of the manuscript was written by the first author Jerome Sauer and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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