

This is a (final) review of the manuscript entitled “Dynamically-optimal models of atmospheric motion” by A. G. Voronovich.

The author has addressed most of my concerns in his response, which I appreciate. I believe that, with now minor changes, the manuscript can be accepted for publication.

First, and not in order of the author’s responses, the reason I emphasized both “standard” test cases, and to paying homage to potential performance benefits, is because the very first line of the abstract says that the paper is a “derivation of a dynamical core...”. This leads many readers down a very well-worn path of expectations. If this alone were changed to “A derivation of the dynamical equations for the dry atmosphere...”, I think the concerns regarding the test problems and performance would be alleviated. Even in this case, however, it would still be helpful to point out in the conclusion potential additional development, *including* optimization benefits that can be examined, and standard tests for a dynamical core that could be run. I would try to withhold stating anything about dynamical cores until the conclusion, in fact (although the comment in the Introduction about the method being of interest to developers of dycores is ok, IMO).

I want to thank the author for the detailed responses to my other comments. Looking at my previous comment 2 on the missing boundary term, I fully see my misconception, and no further explanation is required in the manuscript. The author’s explanation for the affine transformation is also appreciated; it is, in fact, the same as one would use in the barycentric approximation.

Regarding the author’s response #4 to my question(s), however, I still have some questions, but this response engenders the only modification that I recommend. First, on line 345, you indicated that $\Delta t = 0.25$, but you do not state how you arrived at this. But I believe this is very much related to your response # 4 where you state that the time step in essence ensures uniqueness and well-posedness. You need to state how you chose this time step. But you might effectively include this in a modified discussion of uniqueness and nondegeneracy that you started on line 228: It seems to me that by limiting the time step, you are limiting the distance over which the fluid can flow to each \vec{a}_i , which is essentially what up-winding attempts to capture in the Eulerian context as it constrains the amount of fluid that flows over one time step. The time step would be governed by the characteristic width of a tetrahedron and the maximum wave speed in the problem (here, the sound speed). That said, I would recommend replacing the sentence on line 228 beginning “The existence and uniqueness of such a solution ...” with a new paragraph that contains essentially your response #4, something like the following (but please read carefully as there are some changes):

The existence and uniqueness (non-degeneracy) of solutions to Eq. (29) may be explained as follows. Let us consider the trajectories of fluid particles. The fluid particle that at $t = 0$ was located at the vertex with coordinates \vec{a}_i at a later moment of time t will be located at a point $\vec{r}_i(t) = \vec{a}_i + \vec{\xi}_i$. Points \vec{r}_i form vertices of a shifted tetrahedron onto which the initial tetrahedron is mapped. Note, that shifts $\vec{\xi}$ of the fluid particles inside a tetrahedron are assumed to be linear

functions of the shifts of the fluid particles located at the vertices of the tetrahedron: this is our basic (linear) approximation of the forward operator (see the first paragraph of Sec. 3). Thus, the initial tetrahedron with the vertices at the points \vec{a}_i linearly (more precisely, affinely) mapped onto a shifted tetrahedron with vertices at \vec{r}_i ; the transformation is linear regardless of the trajectories of the fluid particles at the vertices \vec{a}_i being linear or curved. In particular, faces and edges of the initial tetrahedron are mapped onto corresponding faces and edges of the shifted tetrahedron. Since shifts of the internal points of the tetrahedron are linear functions of \vec{a}_i the Jacobian of the linear transformation of the initial tetrahedron within it is constant (the constants for different tetrahedrons are, of course, also different, and they depend on time t). Thus, piecewise linearity of the forward operator ensures that the mapping of the whole initial volume onto the shifted volume is also piecewise linear, and the mapping is one-to-one provided neither tetrahedron in the course of evolution degenerates (i.e. tetrahedra volumes never become zero). The latter is achieved by adopting a Courant-limited timestep based on the fastest wave-mode that the equations admit (here, the sound speed (correct??)), which also ensures that time integration errors remain small. Non-degeneracy of the initial tetrahedrons can be checked easily, since trajectories of the fluid particles at the vertices are calculated in the course of numerical integration.

On line 345, you can then just point to this discussion when selecting a time step.

Finally, on line 342: 'periodic in horizontal direction' \rightarrow 'periodic in the horizontal direction'.