This is a review of the mauscript entitled "Dynamically-optimal models of atmospheric motion" by A. G. Voronovich.

The paper presents a new treatment for the discretization of the equations of atmospheric dynamics based on the principle of least action, in which the action is itself discretized. The scheme is a hybrid of Lagrangian-Eulerian dynamics that enables exact numerical maintenance of the conservation laws admitted by the symmetries of Lagrangian via Noether's Theorem. The adoption of the Eulerian grid enables a discretization in terms of tetrahedral (or, in 2d, triangular) finite elements that provide a good treatment of topography. The description of the method in three dimensions is offered, which straight-forwardly covers the validation which is conducted in two dimensions. Finally, the implementation and the 2d example shows the ease with which grid refinement may be accommodated.

The paper is reasonably well organized, and well-written. The technique and its implementation are interesting, and the paper provides a much less ad-hoc representation of the way in which the discretized system can reflect the mimetic properties of the continuum systems–at least in terms of their conservation laws–than more fashionable (and usually Eulerian) treatments that *start* with the conservation laws, rather than with the underlying action principle. For these reasons, this paper should be published, as it makes an important contribution to the literature on the numerics of computational atmospheric dynamics that will have broader appeal to other computational fields.

That said, there are numerous items that should be addressed. Below, these items are split into "technical" and "editorial" considerations. Particularly regarding the former, I hope the author addresses these in the spirit in which they are offered: to help clarify and perhaps generalize the presentation. The editorial comments should largely be accepted so as to correct the grammar. I look forward to seeing this manuscript in its final state, and if a second reading is required, I am happy to provide one.

1 Technical considerations

1. Eq. (1): your transport term is missing a dot product; should be $\vec{v} \cdot \nabla \vec{v}$.Same for entropy transport. Also, for the conservation of mass, second term should read $\nabla \cdot (\rho \vec{v})$. This is a common problem throughout. I'll try to point these out, but please double-check this, as I may miss some. Even though it doesn't cause much confusion, it's an abuse of notation at a minimum, and should be corrected.

2. Eq (7): Rotation term must be a scalar!

3. Equation above line 95: Aren't you missing a $\sum C_{ij}$ term in the surface term? Also, the the integrand $\vec{N}\delta\vec{\xi}$ should be an inner product, but depending on whether you add the $\sum C_{ij}$, and use indices, you may not want a $\cdot$.

4. Eq (9): Again, integrands aren't manifestly scalar because you're missing inner product symbols.

5. Eq (9): I'm being pedantic here, but the integration by parts leads, I think, to a term:

$$\int d\vec{a} \int_{t_0}^{t_1} dt \partial_t [\delta \vec{\xi} \cdot (\partial_t \vec{\xi} + \vec{\xi} \times \vec{\Omega})],$$

so the principle of least action implies that either (a) $\delta\vec{\xi}$ must be orthogonal to $D_t \vec{\xi} \equiv \partial_t \vec{\xi} + \vec{\xi} \times \vec{\Omega}$ for any finite time interval, hence for all time, or that (b) $D_t \vec{\xi}|_{t_0} = D_t \vec{\xi}|_{t_1}$, assuming $D_t \vec{\xi} \neq 0$ at all times. Is this just ignored, or are there potential dynamical implications?

6. Eq (7), Eqs (25): It's often the case that we want to evolve the equations in a background hydrostatic state. Would you modify the Lagrangian to accommodate this by including new canonical variables (the density and pressure/energy fluctuation), or would you modify the Eqs (25) in a sort of ad-hoc way to accomplish this?

7. Eq (16): Shouldn't H on the LHS be $\tilde{H}$, or else the $\tilde{H}$ in the two subsequent equations be just H? Also, there's an inner product symbol missing in the gravity term in Eq (16).

8. $\partial_t \vec{p}$ equation just below Eq (16): When computing the pressure gradient term, I find that:

$$\begin{aligned} -\rho_0 \frac{\partial E}{\partial \xi_i} &= -\rho_0 \frac{\partial E}{\partial \alpha} \frac{\partial \alpha}{\partial \xi_i} \\ &= \rho_0 P \frac{\partial \alpha}{\partial \xi_i} \\ &= \rho_0 P \frac{\partial \alpha}{\partial r_i} \\ &= \rho_0 \frac{\partial (P\alpha)}{\partial r_i} - \rho_0 \alpha \frac{\partial P}{\partial r_i}. \end{aligned}$$

Is this correct? If so, what has happened to the $\rho_0 \frac{\partial (P\alpha)}{\partial r_i}$ term? This will be important for bounded flows.

9. Eq (17): Inner product symbols on first and last terms in brackets.

10. Just below Eq (19): I wonder if you want to reference Sec 4 to point reader here to where the changes required in 2d would occur (e.g., the factor of $1/4 \rightarrow 1/3$)?

11. Eq (20): Inner products on LHS and RHS.

12. line 164 and 166: you don't need the vector symbols on the $\vec{p}_i$, since you're referencing by momentum component, right? Also, just to be clear, you don't retain the correction (right-most) term in the equation on line 166 in the tests, right?

13. Eq (23): The $\tilde{H}$ should have a τ subscript, correct?

14. Eq (23): This will sound like a silly question, but it's important for your discussion of implementation on line 266: to be clear, each vertex is represented in the sum in Eq (23) only once, correct? Also, to be clearer still: there is no restriction on the number of tetrahedra/triangles that may share a given vertex, is there?

15. line 192: Isn't 'reassignment' really just Lagrangian remapping?

16. Eq (26) & (28), et seq: This formulation is the well-known 'barycentric approximation', isn't it? If so, I think this should be stated, and a reference provided.

17. Eq (27): If this is the barycentric approximation, then the condition on the τ_i should be $1 \geq \tau_i \geq 0$ (including the 1 and 0), and there's also a condition $\sum_i \tau_i = 1$ that should be provided.

18. Eq (29): But this is only a single equation for three τ_i. Do you substitute $\vec{R} = \vec{a}_i, i = 1, 3$ and solve the linear system then? Can you please add this? Also, I don't see clearly how solutions to Eq (29) select the tetrahedra from which the fluid particle came. Is the ordering you gave in Eq (27) used somehow to determine this? Normally, the barycentric approximation is used to interpolate within a given tetrahedron, but we have to know already which tetrahedron we're in. Can you please clarify this? [Note: a schematic illustration might help.]

19. line 218: 'the topological structure of the initial and shifted points ~~doesn't~~ ⟨does not⟩ change': But, since you're remapping to an Eulerian grid, why does this matter?

20. line 278: One of the key aspects of your formulation and implementation is the potential for vectorization which will lead to significant performance gains on-node. And, if the vertices are unique, and domain decomposition is done via the individual tetrahedra, it begs the question as to how you handle the data exchange for "coarse-grain" parallelization. These questions certainly interest me, and are critical considerations for dynamical cores. I know that this issue is not the main focus of this paper, but should you perhaps say something here or in a following (sub-)section about performance considerations, even if it's speculative at this point? Depending on what you decide, some mention of performance (potential, or actual numbers) might be added to the abstract, and to the conclusion.

21. Section 4: This is a nice test problem. But the intent should be to validate your numerical formulation and its implementation. Given this, and assuming there is no (quasi-)analytic solution to this test, it seems crucial to compare your solution, at least casually/observationally, with that from a validated numerical solver, unsatisfying as this might be. Perhaps the Editor can make a decision about this, if it's too onerous a thing to do.

22. line 354-356: In the atmosphere, as for other geophysical systems, irreversible mixing is a key element for mesoscale finer scale statistical behavior. This is determined by not only the existence of entropic behavior, but on the manner in which dissipation is handled (e.g., there are significant differences between those that act mainly at low vs. higher wavenumbers). And, whatever the dissipation mechanism, the amount must be accounted for in the energy/entropy. Can the term provided on line 361 ensure this?

2 Editorial considerations

Below, the angle brackets, ⟨XXX⟩, indicate that the text 'XXX' should be added. The strikeout, ~~XXX~~ means that the 'XXX' should be removed. The arrow, →, suggests an emendation.

1. Please double-check the font size requirement; I found it to be too small.
2. line 30: 'to achieve this within ⟨the⟩ Lagrangian/Hamiltonian approach': add the 'the'.
3. line 36: 'Salmon and Simth 1994' →'Salmon and Smith 1994'. Please check elsewhere.
4. line 38: 'calculate ⟨the⟩ state of the atmosphere'
5. line 39: 'one approximately calculates' →'one calculates approximately'
6. line 41: 'Such an approach seems to be more consistent': More consistent than what? The first approach on line 30?
7. line 43: 'approximation of ⟨the⟩ corresponding density.'
8. line 47: 'dynamic⟨s⟩ of a continuous atmosphere'; add the 's'.
9. line 47: 'a combination of ⟨the⟩ finite-element method'
10. line 49: 'calculated in a noncanonical coordinates, what leads' →'calculated in noncanonical coordinates, which leads': remove 'a', and change 'what' to 'which'.
11. line 52: 'of the action in ⟨a⟩ spatial domain': add the 'a'
12. line 52: 'only assuming a continuous dependence' →'by assuming only a continuous dependence'

13. line 53: 'one can discretize ⟨the⟩ calculation'
14. line 53: 'not only in spatial but in time domain as well': Awkward. Suggest: 'not only in a spatial domain, but in the time domain as well'
15. line 61: 'that precede ⟨the⟩ transition'
16. line 62: 'with the following ⟨(Eulerian)⟩ equations'. This, to distinguish your Eqs (25) later on.
17. line 67: 'we denote~~d~~ pressure with ⟨a⟩ capital letter). In ~~the~~ Lagrangian'
18. Eq (3): The first determinant should have an exponent of -1, instead of 1.
19. line 85: 'demonstrate that ~~equation~~ Eq. (2)': don't need 'equation', since you have 'Eq.'.
20. "Decorations" like the vector symbol aren't showing up all that well. This may a problem with my printer, or it may be a math font problem (or both). Any way you can check this?
21. line 102: 'is considered ~~is~~ ⟨as⟩ a function'
22. line 139: is 'mode' supposed to be 'model'?
23. line 148: 'accuracy of ⟨the⟩ square of the ratio of the linear size of ⟨the⟩ terahedra'
24. line 152: 'contribution from all ~~tetrahedrons~~ ⟨tetrahedra⟩'
25. line 154: $\vec{p}\partial_t\vec{\xi}$ should be $\vec{p}\cdot\partial_t\vec{\xi}$, right?
26. line 178: 'parenthesis' →'parentheses'. Also, there are two summations, and two sets of parentheses; to which are you referring. I get it, but it's not clear.
27. line 187-188: 'corresponding conservation laws; conservation will be exact; however' →'corresponding conservation laws. In this case conservation will be exact; however'
28. Section 3: This section is a significant fraction of the paper length. If there is a way to **at least** provide subsections, this would likely aid the reader, and look less dense. I would suggest that subsections could be added without any re-structuring in at least 2 places: You might provide a subsection called 'Lagrangian reassignment and local Hamiltonians' starting on line 192 that goes until line 265, or you can separate lines 192-229 into one subsection, and lines 230-265 into another. I suggest that lines 266-278 should logically form its own subsection on implementation.
29. line 193: 'to reassign values of momenta $\vec{p}_k$⟨,⟩ and'
30. line 217: 'existence and uniqueness of such ⟨a⟩ solution follows'

31. Eq (30: place comma after equation.
32. line 239: 'After⟨wards,⟩ the LHS...'
33. line 260: 'the issue of ⟨an⟩ absorbing layer'
34. line 271: 'all the ~~tetrahedrons~~ ⟨tetrahedra⟩ are...'
35. line 275: 'that at a particular area' →'that in a particular region'
36. line 285-286: 'In the 2D case ~~tetrahedral~~ ⟨tetrahedra⟩ are replaced'
37. line 317: 'The form of ⟨the⟩ Hamiltonian'; 'the action ~~don't~~ ⟨does not⟩ change'
38. line 321: '(see Eq. (38), in this case' →'(see Eq. (38); in this case'
39. line 326: 'in ⟨the⟩ vertical, an 200 s in time'
40. line 329: 'and ~~the~~ ⟨a⟩ fourth-order'
41. line 332-333:'accounted ~~in~~ ⟨for⟩. It was found that mass was conserved to ~~a~~ machine precision; conservation of energy ~~wasn't~~ ⟨was not⟩'
42. line 347: 'motion of the ~~continuous~~ ⟨continuum⟩ atmosphere'