

# A Time-Dependent Three-Dimensional Dayside Magnetopause Model Based on Quasi-elastodynamic Theory

Yaxin Gu<sup>1,2</sup>, Yi Wang<sup>1\*</sup>, Fengsi Wei<sup>1</sup>, Xueshang Feng<sup>1</sup>, Andrey Samsonov<sup>2</sup>, Xiaojian Song<sup>3</sup>, Boyi Wang<sup>1</sup>, Pingbing Zuo<sup>1</sup>, Chaowei Jiang<sup>1</sup>, Yalan Chen<sup>1</sup>, Xiaojun Xu<sup>4</sup>, Zilu Zhou<sup>4</sup>

1 Shenzhen Key Laboratory of Numerical Prediction for Space Storm, College of Aerospace Science and Technology, Harbin Institute of Technology, Shenzhen, 518055, China

2 Mullard Space Science Laboratory, University College London, Dorking, RH56NT, United Kingdom

3 Shandong High Technology Research, Shandong, 250100, China

4 State Key Laboratory of Lunar and Planetary Sciences, Macau University of Science and Technology, Macau, 999078, China

*\*Correspondence to: Yi Wang (wingwy@mail.ustc.edu.cn)*

**Abstract.** The interaction between the solar wind and Earth's magnetosphere is a critical area of research in space weather and space physics. Accurate determination of the magnetopause position is essential for understanding magnetospheric dynamics. While numerous magnetopause models have been developed over past decades, most are time-independent, limiting their ability to elucidate the dynamic movement of the magnetopause under varying solar wind conditions. This study introduces the first time-dependent three-dimensional magnetopause model based on quasi-elastodynamic theory, named the POS (Position-Oscillation-Surface wave) model. Unlike existing time-independent models, the POS model physically reflects the dynamic responses of magnetopause position and shape to time-varying solar wind conditions. The predictive accuracy of the POS model was evaluated by using 38,887 observed magnetopause crossing events. The model achieved a root-mean-square error of 0.774 Earth radii ( $R_E$ ), representing a 17.9% improvement over five widely used magnetopause models. Notably, the POS model demonstrated superior accuracy under highly disturbed solar wind conditions (22.1% better) and in higher latitude regions (27.0% better) and flank regions (33.3% better) of the magnetopause. The POS model's remarkable accuracy, concise formulation, and fast computational speed enhance our ability to predict magnetopause position and shape in real-time. This advancement is significant for understanding the physical mechanisms of space weather phenomena and improving

the accuracy of space weather forecasts. Furthermore, this model may provide new insights and methodologies for constructing magnetopause models for other planets.

## 1 Introduction

The magnetopause, the boundary between the interplanetary magnetic field (IMF) and Earth's magnetic field, plays a crucial role in space weather forecasting and understanding solar wind-magnetosphere coupling mechanisms (P. Song, 1996; Russell, 2003; Willis, 1971). It acts as a protective shield against hazardous energetic particles while simultaneously serving as the primary interaction region for solar wind-magnetosphere coupling. The magnetopause exhibits considerable dynamic behaviour due to continuous solar wind variations and various instabilities, even under steady solar wind conditions (Anderson et al., 1968; Eastwood et al., 2015; Song et al., 1988). These dynamics can lead to radiation belt particle loss, field-aligned current intensification, ultra-low frequency wave generation, and solar wind energy conversion into the radiation belts, polar regions, and ionosphere (Archer et al., 2019; Haerendel, 1990; Mann et al., 2012; Mottez, 2016; Plaschke, 2016). Consequently, comprehending the interactions between solar wind and magnetopause is vital for advancing magnetosphere dynamics and improving space weather prediction capabilities (Feng, 2020; Zong et al., 2020).

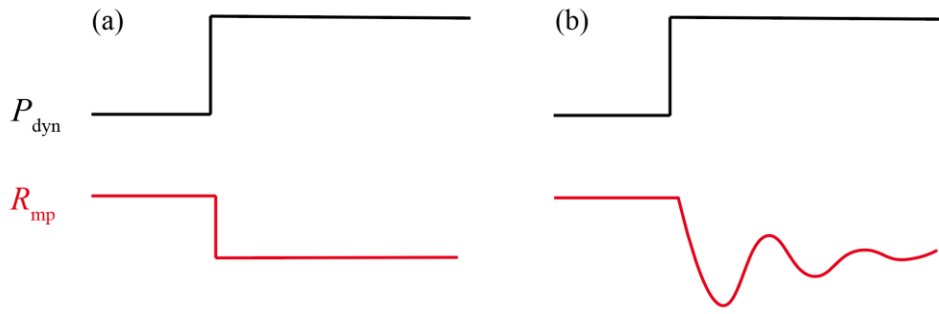
Numerous magnetopause models have been established over the past few decades, generally categorized as physical (or principal) models (Beard, 1960; Ferraro, 1952; Spreiter et al., 1966) and empirical models (Fairfield, 1971; Lin et al., 2010; Shue et al., 1998; Tsyganenko, 1989). Physical models are primarily based on the classic Chapman-Ferraro theory proposed in the 1930s (Chapman and Ferraro, 1930), which states that the magnetopause's equilibrium position is determined by the pressure balance between solar wind dynamic pressure ( $P_{\text{dyn}}$ ) and magnetospheric magnetic pressure ( $P_b$ ). Since the 1960s, the launch of numerous satellites has provided us with a large number of samples of magnetopause crossing events (MCEs), thereby creating the possibility for the establishment of empirical models (Fairfield, 1971; Lin et al., 2010; Petrinec and Russell, 1996; Shue et al., 1998;

53 *Sibeck*, 1991). Many empirical models rely on two key parameters,  $P_{\text{dyn}}$  and IMF  $B_z$ , and some of them  
54 include the Earth's dipole tilt angle ( $\Phi$ ) to calibrate the higher latitude zone. Besides, some empirical  
55 models, proposed from the 1980s, combine physical processes of solar wind-magnetosphere  
56 interactions with satellite observation fitting and involved the impact of magnetospheric currents  
57 system(*Tsyganenko*, 1996; *Tsyganenko*, 1989). Regardless of the assumptions on which these models  
58 are based, all these models have contributed to our understanding of magnetopause movement and its  
59 response to solar wind conditions, in particular, many of them have been widely used in the prediction  
60 of the magnetopause due to their simple form and high prediction accuracy.

61 However, it should be aware that these models primarily describe the average steady-state  
62 characteristics of the magnetosphere. To accurately describe the dynamic coupling process of solar  
63 wind-magnetosphere interaction, it is essential to incorporate time partial derivatives into the dynamic  
64 equations (*Borovsky and Alejandro Valdivia*, 2018; *Petrinec*, 2001; *Smit*, 1968). This approach,  
65 however, complicates the solution of model equations, often necessitating numerical simulations such  
66 as magnetohydrodynamics (MHD) (*Lyon et al.*, 2004; *Merkin and Lyon*, 2010; *Powell et al.*, 1999;  
67 *Raeder et al.*, 2001; *Tóth et al.*, 2005), particle-in-cell (PIC) (*Ashida et al.*, 2014; *Moritaka et al.*, 2012;  
68 *Walker et al.*, 2019), and hybrid simulations (*Ala-Lahti et al.*, 2022; *Gargaté et al.*, 2008; *Omelchenko*  
69 *et al.*, 2021). Numerical simulations are widely used in exploring solar wind-magnetosphere coupling  
70 and can accurately reveal the position of the magnetopause changing with the time-varying solar wind.  
71 *Collado-Vega et al.* (2023) compared the magnetopause predictions obtained by different MHD  
72 models, showing the discrepancies for the standoff position. Their analysis also specifically  
73 addressed the impact of extreme solar wind conditions, which are known to cause space weather  
74 hazards, on the magnetopause. However, the introduction of time partial derivatives makes equations  
75 very difficult to solve. In addition, many prominent numerical simulation models may not include  
76 properly all magnetospheric current systems (e.g. the ring current or the magnetospheric-ionospheric  
77 currents), therefore this may result in systematic errors of the magnetopause prediction (*Samsonov et*  
78 *al.*, 2016). Moreover, numerical models are solved on supercomputers, consuming a significant

79 amount of computing resources and time, rendering them impractical for real-time space weather  
80 forecasting (*Feng, 2020; Lyon et al., 2004; Raeder et al., 2001; Tóth et al., 2005*). This limitation  
81 highlights the need for more efficient, yet accurate, magnetopause models that can capture the dynamic  
82 nature of the magnetopause while remaining computationally feasible for real-time applications. Such  
83 models would significantly enhance our ability to predict and understand space weather phenomena,  
84 bridging the gap between theoretical understanding and practical forecasting capabilities.

85       Apart from numerical simulations, very few time-dependent magnetopause models have been  
86 historically developed (*Børve et al., 2011; Freeman et al., 1995; Sato et al., 2022; Smit, 1968*). Figure  
87 1 illustrates the fundamental difference between time-independent and time-dependent models. In  
88 time-independent models, the magnetopause position is directly correlated with instantaneous solar  
89 wind conditions. For example, a step-like increase in solar wind dynamic pressure (such as a shock)  
90 corresponds to an immediate step-like compression of the magnetopause (Figure 1a). However, this  
91 simplification fails to capture the real dynamics of the magnetopause. In reality, the magnetopause  
92 undergoes a more complex process of compression and recovery, exhibiting oscillatory characteristics  
93 in response to abrupt changes in solar wind conditions, as shown in Figure 1b (*Desai et al., 2021;*  
94 *Freeman and Farrugia, 1998; Hu et al., 2005*). Time-dependent models aim to capture these dynamic  
95 processes, providing a more accurate representation of magnetopause behaviour. To describe these  
96 dynamic responses, it is necessary to incorporate time partial derivatives into the governing equations.  
97 However, this inclusion significantly complicates the solution process. Consequently, existing time-  
98 dependent models are predominantly one-dimensional and remain in a preliminary stage of  
99 development.



**Figure 1 The schematic diagram of time-independent (a) and time-dependent (b) magnetopause models.**

*Smit* (1968) conceptualized the magnetopause as a rigid surface and attempted to explain its motion from the perspective of periodic vibration; *Freeman et al.* (1995) investigated the influence of inertial and damping effect on the magnetosphere, employing magnetohydrodynamics to analyse the magnetopause motion; *Børve et al.* (2011) set up a non-adjustable model to analyse the oscillation period of the magnetopause. By investigating the movement of the subsolar point in response to time-varying solar wind, these models are primarily constructed to elucidate specific physical phenomena linked to solar wind-magnetosphere interaction, yet they lack the capability to provide a real-time depiction of the three-dimensional magnetopause position and shape.

Hence, the challenge of constructing time-dependent models lies in balancing the need for accurate dynamic representation with computational feasibility. Although time-dependent models offer a more realistic depiction of magnetopause behaviour, their complexity has limited their development and application, particularly in three-dimensional space. This highlights the necessity for new strategies that can capture time-dependent dynamics while ensuring practical utility for computation, especially for real-time space weather forecasting and related magnetospheric researches. Previously, our work revealed the quasi-elastodynamic processes involved in the interaction between solar wind and magnetosphere (*Gu et al.*, 2023). It suggests that the dynamic behaviour of each point on the magnetopause can be viewed as an equilibrium position (P), radial global oscillations around

119 equilibrium position (O), and surface wave-like structure around the flank regions (S). This work offers  
120 a practical framework for developing a time-dependent three-dimensional magnetopause model.

121 However, our previous work primarily focused on elucidating the quasi-elastic process, with less  
122 emphasis on the outcomes of model predictions (*Gu et al.*, 2023). Key factors influencing  
123 magnetopause dynamics, such as the IMF  $B_z$  and Earth's dipole tilt angle ( $\Phi$ ), were not incorporated.  
124 Additionally, the adjustable parameters in the equations were simply chosen and lack of thorough  
125 calibrations. Moreover, both our prior work and most published magnetopause models (*Gu et al.*, 2023;  
126 *Petrinec and Russell*, 1996; *Shue et al.*, 1998) relied on a relatively limited dataset of low-latitude  
127 satellite observations, leading to constraints in accurately representing the higher latitude and flank  
128 regions of the magnetopause. To address these limitations and overcome the inherent shortcomings of  
129 time-independent models, particularly their inability to reflect the dynamic responses of the  
130 magnetopause position and shape to time-varying solar wind conditions, we propose a time-dependent  
131 three-dimensional magnetopause model. This model, which has been tested with the largest dataset of  
132 MCEs to date (38,887 events), demonstrates remarkable prediction accuracy compared to five widely  
133 used magnetopause models. Besides, it offers unparalleled real-time computation speed and a concise  
134 form relative to numerical simulations. We have named this model the Position-Oscillation-Surface  
135 wave (POS) model.

## 136 **2 Dataset and other magnetopause models for comparison**

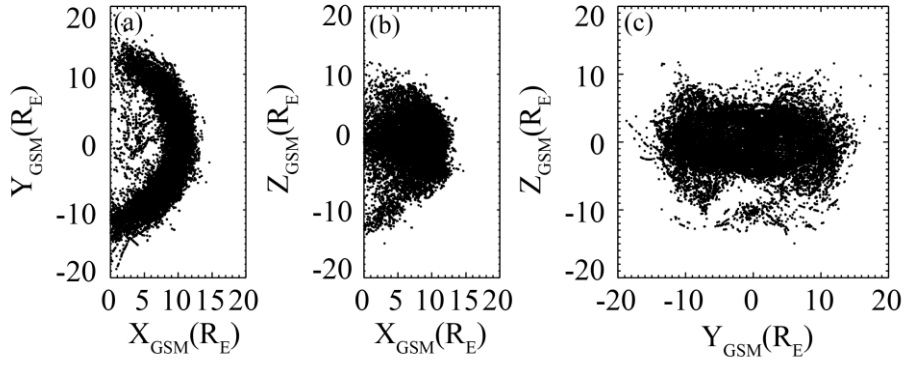
137 The THEMIS (Time History of Events and Macroscale Interactions during Substorms) mission  
138 (*Angelopoulos*, 2008), which consists of five spacecrafts launched into similar elliptical, near-  
139 equatorial orbits in 2007, has significantly enhanced our ability to observe the magnetosphere. The  
140 mission provides high-resolution ( $\sim 3$  s) magnetic field measurements through the THEMIS/Flux Gate  
141 Magnetometer (FGM) (*Auster et al.*, 2008) and plasma data from the THEMIS/electrostatic analyser  
142 (ESA) (*McFadden et al.*, 2008). The Cluster II mission (*Escoubet et al.*, 2001), involving four identical  
143 spacecraft launched in 2000, also offers high-resolution ( $\sim 4$  s) magnetic field measurements using the

144 CLUSTER/Flux Gate Magnetometer (FGM) (*Balogh et al.*, 1997) and particle data and moments from  
 145 the Cluster Ion Spectrometry Hot Ion Analyser (CIS-HIA) (*RÈMe et al.*, 1997).

146 The WIND spacecraft, launched into orbit around Earth in 1994 and relocated to Lagrange L1  
 147 point after 2004, provides continuous, high-quality in-situ solar wind observations. This study utilizes  
 148 high-resolution ( $\sim 3$  s) plasma data from the WIND/3D Plasma Analyzer (3DP) (*Lin et al.*, 1995) and  
 149 magnetic field data from the Magnetic Field Investigation (MFI) (*Lepping et al.*, 1995) for upstream  
 150 solar wind observations. For this study, we have compiled a dataset consisting of 51,590 THEMIS  
 151 MCEs and 38,321 Cluster MCEs. After excluding redundant crossings (i.e., those occurring  
 152 simultaneously on the same satellite), invalid data (i.e., crossing without valid upstream solar wind  
 153 observations), and nightside MCEs (where  $X_{\text{GSM}} < 0 R_E$ ), a total of 38,018 THEMIS MCEs and 869  
 154 CLUSTER MCEs (see Figure 2) are selected for this study. The time shift ( $\delta t$ ) between WIND to the  
 155 satellite MCE is determined by comparing the time of each crossing ( $t_1$ ) with the probable arrival time  
 156 of corresponding solar wind observation from WIND ( $t_0 + \delta t$ ), satisfying  $(t_0 + \delta t) - t_1 < 300$  s. The  
 157 300s threshold is set as the potential error window for the time shift from L1 to the magnetopause.  $\delta t$   
 158 is calculated as  $(L1 - r) / \langle v_x \rangle$ , L1 ( $L1 = 235 R_E$ ) is the distance from the Earth to the L1 point,  $r$  denotes  
 159 the radial position of the magnetopause, and  $\langle v_x \rangle$  is the 1-hour sliding average of the solar wind  
 160 velocity in the x-component (*Chao et al.*, 2002). A summary of these events is provided in Table 1. The  
 161 distribution of matched solar wind conditions for MCEs is shown in Figure 3. All the data is available  
 162 in the CDAWeb database (<http://cdaweb.gsfc.nasa.gov/>), and the time resolution of the magnetic field  
 163 and plasma data used in the study is interpolated into 3 seconds, set in GSM coordinates.

164 **Table 1 Summary of collected 89,911 satellite MCEs and dataset used in this paper**

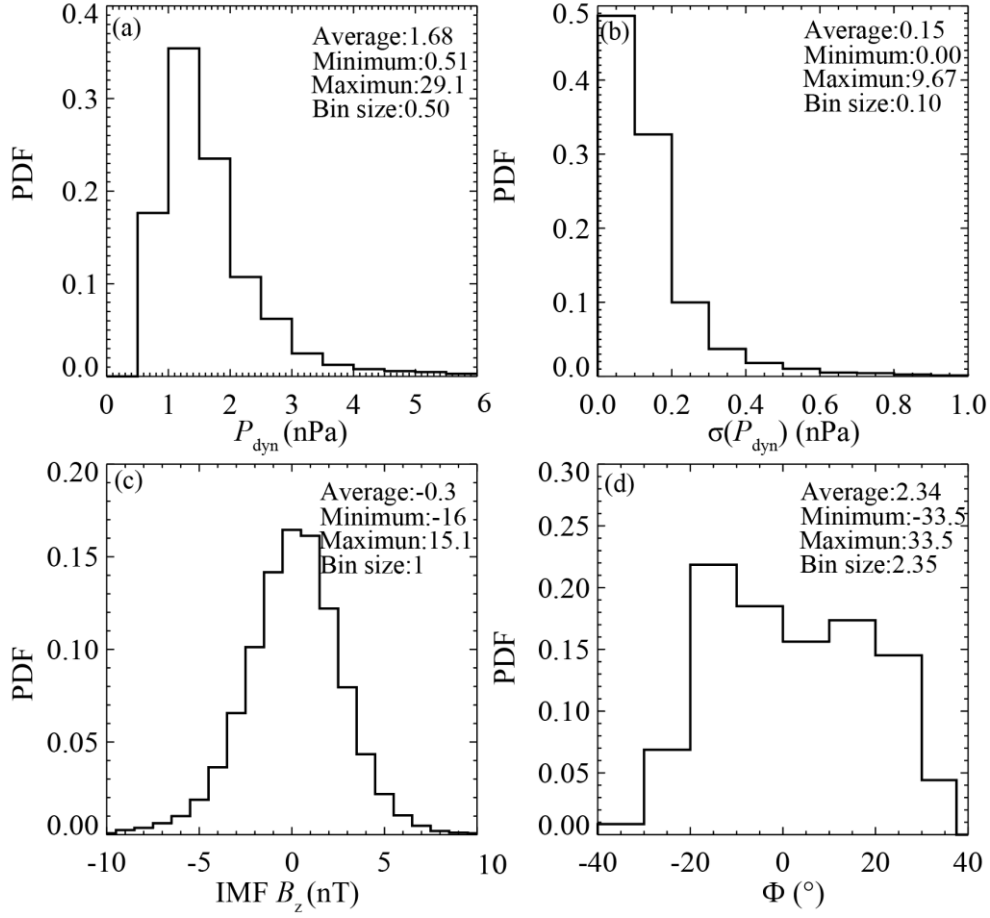
Dataset	Satellite	Time interval	Number of datasets
<i>Song et al.</i> (2021)	THEMIS	2007-2022	17,647
<i>Staples et al.</i> (2020a)	THEMIS	2007-2016	33,943
<i>Grimmich</i> (2024)	CLUSTER	2001-2020	38,321
In this paper	THEMIS/CLUSTER	2004-2022	38,887



**Figure 2 Projections of 38,887 MCEs in GSM coordinate: (a) X-Y plane, (b) X-Z plane, (c) Y-Z plane.**

In previous research on time-independent magnetopause models, the physical models (*Beard*, 1960; *Ferraro*, 1952; *Spreiter et al.*, 1966), although theoretically grounded, usually oversimplify intricate solar wind-magnetosphere interactions to facilitate calculations, usually without demonstrating apparent higher prediction accuracy compared to widely-used empirical models (*Chao et al.*, 2002; *Lin et al.*, 2010; *Petrinec and Russell*, 1996; *Shue et al.*, 1998; *Shue et al.*, 1997).. Hence, this article concentrates on comparing several notable time-independent empirical models renowned for their superior prediction accuracy (*Chao et al.*, 2002; *Lin et al.*, 2010; *Petrinec and Russell*, 1996; *Shue et al.*, 1998; *Shue et al.*, 1997).





**Figure 3 Probability density function of the upstream solar wind observation in: (a) solar wind dynamic pressure  $P_{\text{dyn}}$ , (b) standard deviation of dynamic pressure ( $P_{\text{dyn}}$ ), (c) interplanetary magnetic field  $B_z$  component (IMF  $B_z$ ) and (d) dipole tilt angle ( $\Phi$ ).**

Empirical models are typically constructed using satellite observations of MCEs. While these models vary in their use of satellite datasets, parameters considered, coordinate systems employed, and functions applied, most are parameterized using the dynamic pressure ( $P_{\text{dyn}}$ ) and the interplanetary magnetic field  $B_z$  component (IMF  $B_z$ ). For example, *Petrinec and Russell* (1996) (hereafter PR96) employed an ellipsoidal function to construct a magnetopause model, while *Shue et al.* (1997) (hereafter S97) developed a flexible function incorporating two variables: the subsolar magnetopause position ( $R_0$ ) and the tail flaring angle ( $\alpha$ ). This function has gained widespread use as a foundational approach to describing magnetopause shape. For instance, *Shue et al.* (1998) (hereafter S98) accounted

187 for the saturation effect of IMF  $B_z$  on  $R_0$ , and *Chao et al.* (2002) (hereafter C02) extended their model  
 188 for application under normal and extreme solar wind conditions.

189 Nevertheless, these models primarily rely on low-latitude satellite observations and may not  
 190 adequately capture the distinctive characteristics of the magnetopause in the higher latitude region.  
 191 Besides, they are constructed with  $P_{\text{dyn}}$  and IMF  $B_z$ , while it is found that the dipole tilt angle  $\Phi$  is of  
 192 great significance in modelling magnetopause, especially in the higher latitude region. *Formisano et*  
 193 *al.* (1979) constructed an average magnetopause size and shape for two dipole tilt angle values ( $\Phi$ ).  
 194 *Boardsen et al.* (2000) developed a higher latitude magnetopause model parameterized by not only  
 195  $P_{\text{dyn}}$  and IMF  $B_z$  but also the dipole tilt angle ( $\Phi$ ), recognizing its significant influence on the shape of  
 196 the higher latitude magnetopause. While this model is specifically designed for higher latitude regions,  
 197 it is not as effective in accurately calculating the magnetopause at low latitudes compared to other  
 198 models due to inherent limitations.

199 The above models are generally developed under the assumption of axial symmetry, while the  
 200 actual magnetopause shape is asymmetric in both the Y and Z directions, so they are essentially 2D or  
 201 2.5D models. To describe the 3D structure of the magnetopause, *Lin et al.* (2010) (hereafter L10)  
 202 developed a three-dimensional magnetopause model parameterized by  $P_{\text{dyn}}$ , thermal pressure ( $P_t$ ), IMF  
 203  $B_z$ , and  $\Phi$ . The coordinate systems employed in these empirical models are typically in aberrated  
 204 coordinates which accounts for Earth's orbital motion (*Chao et al.*, 2002; *Petrinec and Russell*, 1996;  
 205 *Shue et al.*, 1998; *Shue et al.*, 1997), or the corrected coordinates which compensates for both Earth's  
 206 orbital motion and deviations in solar wind velocity from the Sun-Earth line (*Boardsen et al.*, 2000;  
 207 *Lin et al.*, 2010). A summary of the five widely used magnetopause models is presented in Table 2.

208 **Table 2 Summary of five widely used magnetopause models and POS model**

Model Name	Number of (higher latitude) MCEs used	Time range of MCEs used	Dimensions
PR96	1,147	1979-1980	2D/2.5D

S97	553	1978-1986	2D/2.5D
S98	553	1978-1986	2D/2.5D
C02	552	1978-1986	2D/2.5D
L10	1,226 (1,482)	1994-2008	3D
POS	31,562 (7,325)	2004-2022	3D

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### 3 The POS Model

In our previous work (Gu *et al.*, 2023), we modelled the compression-recovery process of the magnetopause as a quasi-elastodynamic phenomenon. In this framework, the dynamic pressure,  $P_{\text{dyn}} = n_{\text{sw}} m_p v_x^2$ , serves as the driving force on the system, where  $n_{\text{sw}}$ ,  $m_p$ , and  $v_x$  are the number density, proton mass, and the x component of the solar wind velocity in the GSM coordinates, respectively. The system's restoring force is described by,  $P_b = B^2 / 2\mu_0$ , where  $B$  is the total magnetic field at magnetopause and  $\mu_0$  is the vacuum permeability. After accounting for damping and non-ideal effects,  $P_{\text{damp}}$ , meanwhile neglecting the complex coupling interactions, the momentum equation for the magnetosheath in a unit cylinder can be represented by equation (1).

$$M_{\text{msh}} a_{\text{msh}} = P_b - P_{\text{dyn}} - P_{\text{damp}} \quad (1)$$

Given that the derivation process of the foundational formula is the same as our previous work, and this paper is focused on model predictions rather than physical processes, we will refrain from reiterating it here. The relationship depicting the temporal evolution of the magnetopause position ( $r$ ) are introduced in equation (2).

$$n_{\text{sw}} m_p r \ddot{r} = \frac{(\lambda B_d(r, \theta, \varphi) + B_c(r))^2}{2\mu_0} - n_{\text{sw}} m_p v_x^2 \cos^2 \alpha - k \Sigma_p B_p^2 \dot{r} - \eta \dot{r} / r \quad (2)$$

Where  $(r, \theta, \varphi)$  represents the corresponding spherical coordinates,  $r$  is the radial distance,  $\theta$  is the latitude angle between  $[-90^\circ, 90^\circ]$  and  $\varphi$  is the longitude angle adjusted to  $[-180^\circ, 180^\circ]$  with  $0^\circ$  oriented towards the Sun for simplicity, and all the vectors have been projected to the normal direction of magnetopause (Gu, et al. 2023). This simplified equation enables us to capture the fundamental

228 dynamics of the magnetopause's response to solar wind fluctuations while ensuring computational  
 229 efficiency. The first term on the right side of equation (2) signifies the restoring force  $\mathbf{P}_b$ . Here,  $\mathbf{B}_d$   
 230 denotes the Earth's dipole field,  $\lambda$  is the magnetospheric compressibility coefficient, and  $\mathbf{B}_c$  accounts  
 231 for contributions from various magnetospheric currents. The second term on the right side represents  
 232 the driving force  $\mathbf{P}_{\text{dyn}}$ , where  $\alpha$  denotes the angle between the x-direction and the normal direction of  
 233 the magnetopause. The third term on the right side of equation (2) characterizes a position-dependent  
 234 dragging effect estimated from the ionosphere, while the fourth term illustrates a global non-ideal  
 235 viscous effect.  $\mathbf{B}_p$  is the estimated ionospheric magnetic field in the polar region,  $\Sigma_p$  stands for the  
 236 equivalent Pederson conductivity,  $k$  serves as a position-dependent mapping factor, and  $\eta$  represents  
 237 the viscous coefficient. The final two terms contribute to the damping and non-ideal effects of the  
 238 system, denoted as  $\mathbf{P}_{\text{damp}}$ . Equation (2) provides a foundation for developing a time-dependent  
 239 magnetopause model that can reflect the system's dynamic behaviour more accurately compared to  
 240 conventional time-independent models. We will introduce the key parameters in detail in the following  
 241 sections. All the parameters are in SI units.

242 In this study, we incorporate the impact of IMF  $\mathbf{B}_z$  and  $\Phi$  in the magnetospheric magnetic  
 243 pressure. To determine the equation's final fitting coefficients, we conducted 1,000 independent  
 244 iterations, each involving a random selection of 5,000 MCEs from our dataset of 38,887 MCEs.

### 245 **3.1 The magnetospheric compressibility coefficient ( $\lambda$ )**

246 The magnetospheric compressibility coefficient,  $\lambda$ , measures the magnetosphere's response to  
 247 solar wind pressure, specifically the ratio of the magnetospheric magnetic field to the pure dipolar  
 248 magnetic field (*Schild, 1969; Spreiter et al., 1966*). This coefficient is one of the most critical  
 249 parameters directly affecting the position of the magnetopause. Typically,  $\lambda$  has a value of 2.44 at the  
 250 subsolar point, but it changes as the magnetopause shifts and varies with latitude and longitude,  
 251 suggesting a more complex formulation (*Chen et al., 2023; Shue et al., 2011*). *Mead and Beard (1964)*  
 252 used a self-consistent method, discovered an inward concave structure at the higher latitude

253 magnetopause, which is influenced by the inclination angle of the Earth's dipole. Their work also  
 254 determined the surface shape of the magnetopause when the solar wind flow is perpendicular to the  
 255 dipole axis ( $\Phi = 0^\circ$ ), providing an expression for  $\lambda$  as a function of the angles  $\theta$  and  $\varphi$ .

256 Several models (*Boardsen et al.*, 2000; *Formisano et al.*, 1979; *Lin et al.*, 2010) have been  
 257 developed to investigate the influence of  $\Phi$  (the angle between Earth's magnetic axis and the solar  
 258 wind direction) on the magnetopause's position and shape, offering valuable insights into the  
 259 magnetosphere's three-dimensional structure. These models predict an asymmetric response of the  
 260 magnetosphere to variations in  $\Phi$ . *Boardsen et al.* (2000) quantified the effects of  $\Phi$  on the higher  
 261 latitude magnetopause using MCEs data from the northern hemisphere. Their work revealed how the  
 262 dipole tilt angle influences the magnetopause structure in polar regions, which are particularly sensitive  
 263 to changes in the orientation of Earth's magnetic field relative to the solar wind. *Lin et al.* (2010) further  
 264 demonstrated that an increase in  $\Phi$  causes a slight shift in the centres of the magnetopause cross-  
 265 sections, moving them towards the negative Z direction in the subsolar region and towards the positive  
 266 Z direction in the tail region. According to *Olson* (1969), who provided a detailed representation of  $\lambda$   
 267 for various tilt angles ( $\Phi = 0^\circ, 10^\circ, 20^\circ, 30^\circ$ ) on a  $15^\circ \times 15^\circ$  grid of  $\theta$  and  $\varphi$  values, the dipole tilt angle  
 268 significantly alters the fundamental behaviour of  $\lambda$ , with varying effects on  $\theta$  and  $\varphi$ . This influence is  
 269 more pronounced in higher latitude regions ( $\theta > 30^\circ$ ). Building on previous work by *Gu et al.* (2023) ,  
 270 in which  $\lambda = 2.44 - 0.4\theta^2 + \varphi^2$  , and incorporating the effect of  $\Phi$  at different positions of the  
 271 magnetopause  $\lambda(\theta, \varphi)$ , we derive a more precise expression for  $\lambda$ , specifically tailored to our model, as  
 272 presented in Equation (3):

$$\begin{aligned}
 A &= \tanh[5.568(|\theta| - 0.5325)] + 1.0 \\
 \lambda(\theta, \varphi, \Phi) &= 2.44 - (0.4 + 0.3A)(\theta + 0.2A\Phi)^2 + (1.0 - 0.5A|\Phi|)\varphi^2
 \end{aligned} \tag{3}$$

### 3.2 Contributions from various magnetospheric currents ( $B_c$ )

Previous studies on the impact of magnetospheric currents on the position of the magnetopause led to the development of a static magnetopause current model, where the magnetic field of magnetopause surface current and tail current were fitted using polynomials to reveal the relationship between variations in the magnetospheric magnetic field and changes in magnetopause position ,e.g.  $B_{\text{surf}}(r, \theta, \phi)$  and  $B_{\text{tail}}(r, \theta, \phi)$  (Choe and Beard, 1974a, b; Matsuoka *et al.*, 1995). In our earlier study (Gu *et al.*, 2023), the magnetic field of the current system, denoted as  $B_{c0}(r)$ , did not account for variations in  $\theta$  and  $\phi$ . This limitation is addressed in the present study. The fundamental form of  $B_{c0}(r, \theta, \phi)$ , incorporating these angular dependencies, is introduced in Equation (4). The current system exhibits asymmetry effects, consistent with other models such as T96 and T01 (Tsyganenko, 1996; Tsyganenko, 2001). These models also incorporate dawn-dusk asymmetry in the magnetospheric current, reflecting the influence of these angular dependencies.

$$B_{c0}(r) = [-401904 / (\frac{r}{R_E})^4 + 65489 / (\frac{r}{R_E})^3 + 1500 / (\frac{r}{R_E})^2 - 40][1 + 0.4 \sin(2\theta)^2][1.0 - 0.1 \sin(\phi)] \times 10^{-9} \quad (4)$$

In our previous work,  $B_c(r)$  was defined as a piecewise function of  $P_{\text{dyn}}$ , which could yield discontinuous and non-physical results at the transition points (Gu *et al.*, 2023). To address this limitation, we now consider the impact of  $P_{\text{dyn}}$  in a continuous form, eliminating the piecewise dependence. Furthermore, the impact of the IMF  $B_z$  on the magnetopause position is directionally dependent, with a southward IMF triggering dayside magnetic reconnection, an essential process already incorporated in most existing models (Aubry *et al.*, 1970; Dungey, 1961; Fairfield, 1971). To quantify this effect, we adopt a hyperbolic tangent function, similar to that in Shue *et al.* (1998). Finally, by considering the combined effects of both IMF  $B_z$  and  $P_{\text{dyn}}$ ,  $B_c$  is expressed as in Equation (5):

$$\begin{aligned} B_c &= B_{c0}(r, \theta, \phi) f(B_z) f(P_{\text{dyn}}) \\ &= B_{c0}(r) [1.0 + 0.2 \tanh(-0.5(B_z + 2.5))] [1 + 0.1 P_{\text{dyn}}] \end{aligned} \quad (5)$$

This formulation of  $B_c$  provides a more refined and physically accurate depiction of the influence of the magnetospheric current system on magnetopause dynamics. By incorporating the dependence

of  $\mathbf{B}_c$  on IMF  $\mathbf{B}_z$  and  $\mathbf{P}_{\text{dyn}}$ , and specific locations on the magnetopause, the expression captures the complex spatial variations in the magnetospheric current system that contribute to magnetic pressure, making our model fully three-dimensional.

### 3.3 The damping items

The damping terms in our model consist of a position-dependent dragging effect from the ionosphere  $\mathbf{F}_d = k\Sigma_p \mathbf{B}_p^2 \dot{\mathbf{r}}_{\text{mp}}$  and a global non-ideal viscous effect  $\mathbf{F}_N = \eta \dot{\mathbf{r}}_{\text{mp}} / \mathbf{r}_{\text{mp}}$ , consistent with our previous work (Chen and Wolf, 1999; Gu et al., 2023; Wang and Chen, 2008). We set  $\mathbf{B}_p = 3 \times 10^{-5}$  T to represent the approximate ionospheric magnetic field in the polar region, while  $\Sigma_p = 3.4$  S serves as the equivalent Pedersen conductivity. The viscous coefficient is artificially set to  $\eta = 2 \times 10^{-8}$ . As defined in Equation (6), the position-dependent mapping factor  $k$  is empirically calibrated based on the magnetopause location  $(r, \theta, \phi)$ , increasing when the magnetopause compresses and decreasing with increasing latitude and longitude.

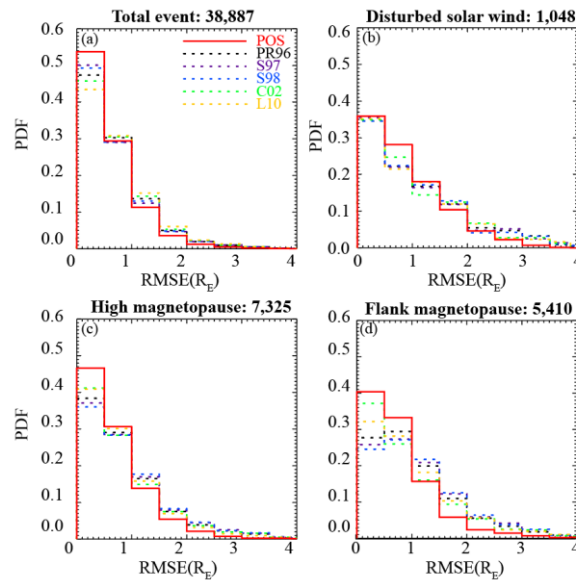
$$k = [196(0.05 + e^{-0.05(r/R_E)^2}) - 3.2|\theta| - 1.6|\phi|] \times 10^{-7} \quad (6)$$

## 4 Result

By substituting the relevant parameters into equation (2) and assuming the initial shape of the magnetopause as a paraboloid,  $x = -0.03(y^2 + z^2) + R_0$ , where  $R_0$  is determined by the pressure balance at the subsolar point, the position of each point on the magnetopause can be computed instantaneously on a personal computer. The prediction accuracy of the POS model and other notable time-independent models mentioned earlier is evaluated and compared with MCEs at the real THEMIS location. We use the root-mean-square error (RMSE), denoted as  $\Delta$ , to quantify the prediction accuracy by comparing the model's calculations with MCEs observations. A dataset of 38,887 MCEs observed by the THEMIS and CLUSTER satellites is used for testing. To evaluate the performance of the POS model relative to other models, we calculate the ratio  $\delta(\Delta)/\Delta_{\text{POS}}$ , where  $\delta(\Delta)$  represents the difference in RMSE between

321 a previous model and the POS model, and  $\Delta_{\text{POS}}$  is the RMSE of the POS model. This comparison is  
 322 conducted from various perspectives. The probability density distributions of RMSE for each model  
 323 are illustrated in Figure 4.

324 It can be seen that all models are capable of adequately predicting magnetopause positions, with  
 325 the majority ( $> 70\%$ ) showing RMSE within  $1 R_E$ . Our model demonstrates superior accuracy, with  
 326 80% of its prediction errors falling below  $1 R_E$ . Predicting the magnetopause under disturbed solar  
 327 wind conditions is more challenging, while the POS model shows improved performance in such  
 328 conditions, with 60% of predictions remaining within  $1 R_E$ . Given the inherent asymmetry of the  
 329 magnetopause, we evaluated the models' performance in both the flank region ( $|\phi| \geq 60^\circ$ ) and the higher  
 330 latitude region ( $|\theta| \geq 30^\circ$ ). The POS model consistently outperforms the others in both regions,  
 331 especially in the flank region. Notably, as a time-dependent three-dimensional model, the POS model  
 332 seldom produces poor predictions, with RMSE exceeding  $3 R_E$  in only rare cases.



333

334 **Figure 4. Distribution of models' RMSE in total(a) and in disturbed solar wind(b); (c) and (d) is the**  
 335 **prediction ability in higher latitude magnetopause ( $|\theta| \geq 30^\circ$ ) and in magnetopause flank region ( $|\phi| \geq 60^\circ$ )**



## 4.1 Time-dependent feature

The models' prediction accuracy is listed in Table 3, it can be seen that all evaluated models exhibit remarkable predictive capabilities, with  $\Delta < 1 R_E$ , aligning closely with other statistical results found in the literature (*Staples et al.*, 2020b). While it should be noted that the  $\Delta$  values calculated for other models in this study may slightly differ from those reported in their original papers. This discrepancy arises due to our use of a significantly larger MCE dataset for comparison.

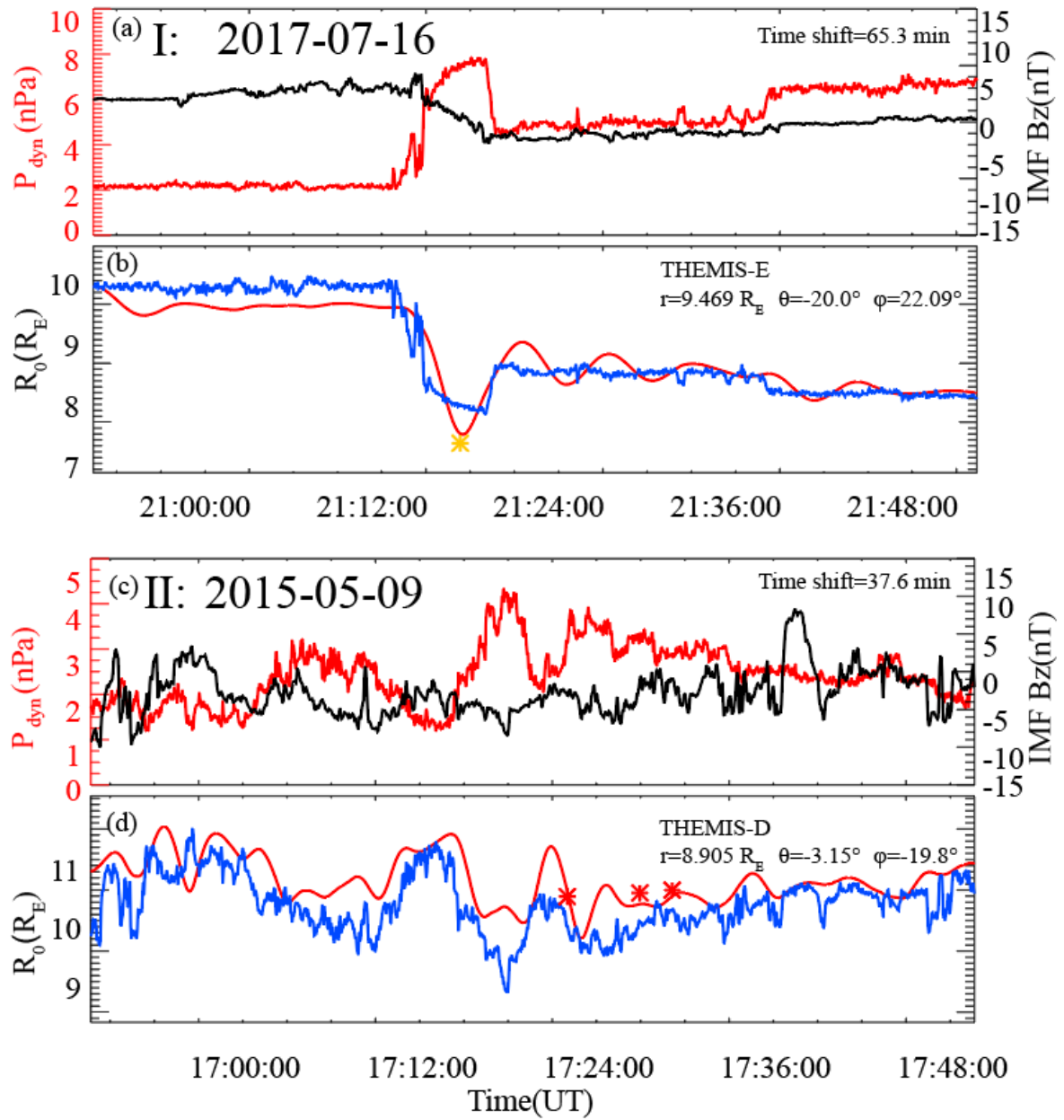
**Table 3 Models' prediction accuracy for all MCEs and in disturbed solar wind.**

Model Name	Total (38,887 MCEs)		[ $\sigma(\mathbf{P}_{\text{dyn}})/\langle \mathbf{P}_{\text{dyn}} \rangle > 100\%$ (1,048 MCEs)	
	$\Delta(R_E)$	$\delta(\Delta) / \Delta_{\text{POS}}$	$\Delta(R_E)$	$\delta(\Delta) / \Delta_{\text{POS}}$
PR96	0.899	+16.1%	1.389	+23.6%
S97	0.884	+14.2%	1.383	+23.0%
S98	0.894	+15.5%	1.388	+23.5%
C02	0.926	+19.6%	1.325	+17.9%
L10	0.960	+24.0%	1.377	+22.5%
POS	0.774	Average:17.9%	1.124	Average:22.1%

Notably, the POS model demonstrates superior predictive performance, with an average improvement of 17.9% over the other models. Additionally, time-independent models have inherent limitations in capturing the dynamic response of the magnetosphere to solar wind fluctuations, particularly when the magnetopause standoff distance is not in phase with  $\mathbf{P}_{\text{dyn}}$  (*Archer et al.*, 2019). In cases of highly disturbed upstream solar wind, where ratio of standard deviation of  $\mathbf{P}_{\text{dyn}}$  to average  $\mathbf{P}_{\text{dyn}}$  ( $\sigma(\mathbf{P}_{\text{dyn}})/\langle \mathbf{P}_{\text{dyn}} \rangle$ ) exceeding 100%, the POS model shows an even greater improvement in predictive accuracy, with a 22.1% enhancement compared to other models. These results suggest that by incorporating time-dependent effects into magnetopause modelling, particularly during periods of solar wind disturbance, the POS model can more effectively capture the non-linear and out-of-phase responses of the magnetopause to rapidly changing solar wind conditions. This results also indicate

353 that time-dependent model represents an obvious advancement in predicting and understanding  
354 magnetospheric dynamics across a wide range of solar wind conditions.

355 The magnetopause is rarely static, exhibiting continuous motion under varying solar wind  
356 conditions and displaying complex dynamics during both intense disturbances and gentle changes. A  
357 notable feature of these dynamics is the periodic oscillation within the Pc5 frequency range (2-7 mHz),  
358 often termed the “magic frequency” in magnetospheric physics (*Plaschke et al.*, 2009a; *Plaschke et*  
359 *al.*, 2009b; *Samson et al.*, 1992). The magnetopause oscillations can be driven by quasi-periodic solar  
360 wind dynamic fluctuations, or explained by magnetospheric cavity mode and Kruskal-Schwarzschild  
361 mode (*Archer et al.*, 2013; *Kepko and Spence*, 2003; *Kivelson et al.*, 1984; *Kruskal and Schwarzschild*,  
362 1954). Our previous research indicates that the oscillations of the magnetopause ought to have  
363 eigenfrequencies ( $f_0$ ) which are determined by the restoring force ( $\mathbf{P}_B$ ), the external driving force ( $\mathbf{P}_{\text{dyn}}$ )  
364 as well as the damping force ( $\mathbf{P}_{\text{damp}}$ ) (*David Halliday*, 2021; *Freeman et al.*, 1995; *Gu et al.*,  
365 2023). Magnetopause will responses to solar wind with phase difference ranging from 0 to 180 degrees,  
366 depending on the driving frequency of the solar wind ( $f_{\text{drive}}$ ). The magnetopause behaves as a low-pass  
367 filter, effectively screening out very high-frequency solar wind fluctuations (e.g.,  $f_{\text{drive}} > 15f_0$ , where  $f_0$   
368 is the eigenfrequency of the magnetopause). This filtering effect results in smoother predictions of  
369 magnetopause behaviour which could be found in Figure 5. For relatively high fluctuations (e.g.,  $15f_0 >$   
370  $f_{\text{drive}} > 2f_0$ ), the phase difference between the solar wind and magnetopause approaches 180 degrees,  
371 indicating an anti-phase response. At resonance ( $f_{\text{drive}} \approx f_0$ ), the magnetopause exhibits a 90-degree  
372 phase lag relative to the solar wind forcing. Conversely, the magnetopause only behaves in-phase with  
373 the solar wind under low-frequency fluctuations ( $f_{\text{drive}} < 0.5f_0$ ), which is the scenario typically revealed  
374 by time-independent models.



**Figure 5. Case study for the overall oscillation of magnetopause using time-independent model S98 and time-dependent POS model to predict its position. (a), (c) The corresponding upstream solar wind dynamic pressure  $P_{\text{dyn}}$  (red line) and interplanetary magnetic field  $B_z$  component (black line) observed by Wind with a time shift of 65.3 min and 37.6 min, respectively; (b), (d) The predictions of S98 (blue) and POS (red) model's prediction based the input solar wind and the asterisks represent the subsolar positions of MCEs projected from THEMIS.**

The time-dependent POS model demonstrates the capability to depict these magnetosphere oscillations and the phase difference accurately. Figure 5 presents two specific cases illustrating the

POS model's time-dependent performance compared with subsolar MCEs projected from THEMIS. In Case I, both models initially predict the magnetopause position at  $\sim 11.5 R_E$  before a pressure pulse in solar wind. The POS model uniquely predicts four oscillations around its equilibrium position ( $\sim 10 R_E$ ) before the magnetopause reaches a new pressure balance. This dynamic behaviour cannot be physically captured by any time-independent models. In Case II, the POS model accurately captures the oscillations around 21:24 UT-21:33 UT, which are not all in-phase with the solar wind dynamic pressure ( $P_{\text{dyn}}$ ). Notably, the POS model depicts anti-phase responses observed in the second and third crossings, while the S98 model shows a reverse trend in motion that deviates more from observations. These results suggest that by incorporating time-dependent effects into magnetopause modelling, particularly during periods of solar wind disturbance, the POS model can more effectively capture the non-linear and out-of-phase responses of the magnetopause to rapidly changing solar wind conditions.

#### 4.2 Three-dimensional characteristic

The POS model developed here incorporates the asymmetrical effects of dipole tilt angles, latitude, and longitude differences, as integrated into equations (2) and (3). The model's parameters were comprehensively calibrated, allowing it to more accurately depict the three-dimensional shape of the magnetopause. To assess its validity across different magnetopause regions, extensive tests were performed, with results presented in Table 4. In the higher latitude magnetopause ( $|\theta| \geq 30^\circ$ ), a region where many models face challenges, the POS model, alongside the L10 model, demonstrates superior performance, showing an impressive 27.0% improvement in accuracy compared to other models. Similarly, in the flank regions ( $|\phi| \geq 60^\circ$ ), where surface waves and other magnetospheric fluctuations complicate position and shape determination, the POS model maintains its high accuracy, with a 33.3% improvement over other models. These results suggest that the POS model offers a more accurate and comprehensive representation of the magnetopause across its entire structure, outperforming other models in both higher latitude and flank regions.

**Table 4 Models' prediction accuracy for higher latitude and flank regions**

Model name	$ \theta  \geq 30^\circ$ (7,325 MCEs)		$ \phi  \geq 60^\circ$ (5,410 MCEs)	
	$\langle \Delta \rangle (R_E)$	$\delta (\Delta)/\Delta_{POS}$	$\Delta (R_E)$	$\delta (\Delta)/\Delta_{POS}$
PR96	1.149	+27.8%	1.315	+31.8%
S97	1.180	+31.3%	1.388	+39.1%
S98	1.195	+32.9%	1.403	+40.6%
C02	1.130	+25.7%	1.268	+27.1%
L10	1.053	+17.1%	1.278	+28.0%
POS	0.899	Average:27.0%	0.998	Average:33.3%

409

410 Surface waves are a distinct feature of the magnetopause, originating from various factors  
411 including solar wind and bow shock dynamics, as well as instabilities within the magnetopause and  
412 magnetosphere under specific conditions. Several localized physical processes have been identified as  
413 potential drivers of these surface waves, including the Kelvin-Helmholtz instability, magnetic  
414 reconnection and flux transfer event (Agapitov *et al.*, 2009; Archer *et al.*, 2021; Hartinger *et al.*, 2013).  
415 It is also found tailward-moving surface wavelet could be driven by disturbed solar wind (large  $\sigma$   
416  $(P_{dyn})/\langle P_{dyn} \rangle$ ) (Sibeck *et al.*, 1989). Our previous study has revealed a distinct mechanism for the  
417 formation of surface wave-like structures in the magnetopause (Gu *et al.*, 2023). The interplay between  
418 dynamic pressure ( $P_{dyn}$ ), magnetic pressure ( $P_b$ ), and damping pressure ( $P_{damp}$ ) results in different  
419 oscillation periods at various points on the magnetopause. These variations create a time lag within the  
420 magnetopause structure, manifesting as a surface wave-like pattern. Figure 6 shows a surface wave-  
421 like structure predicted by the POS model during relatively disturbed upstream solar wind conditions.  
422 The POS model's predictions are compared with those of the C02 model, which has been demonstrated  
423 as the most effective time-independent model in the flank region according to our evaluation. Figure  
424 6 presents a specific case study illustrating the POS model's time-dependent performance compared  
425 with THEMIS MCEs positions mapped to X-Y plane. Figure 6 (a) displays the solar wind dynamic  
426 pressure and the north-south component of the interplanetary magnetic field. The radial positions of

427 the different points in magnetopause in the XY plane ( $Z=0$ ), as calculated by our model—being fully  
428 three-dimensional—exhibits an asymmetric flank region, are traced in Figure 6(b). Positive and  
429 negative flanks respond differently to variations in the solar wind, with discrepancies becoming more  
430 pronounced at higher  $\phi$ . In this specific example, the dusk region is more significantly disturbed than  
431 the dawn region. Notably, the magnetopause shapes calculated in Figures 6(c)-(e) reveal surface wave-  
432 like structures evolving over time. THEMIS MCEs observed in the flank region corroborate this  
433 predicted surface wave-like structure, indicating that the magnetopause position predicted by the POS  
434 model is more accurate than that predicted by the C02 model.

435 The POS model's predictions are compared with those of the C02 model, which has been  
436 demonstrated as the most effective time-independent model in the flank region according to our  
437 evaluation. Figure 6 illustrates a surface wave-like structure predicted by the POS model during  
438 relatively disturbed upstream solar wind conditions. The predicted surface wave-like structure is  
439 corroborated by THEMIS MCEs in the flank region, where the actual magnetopause position is closer  
440 to Earth than predicted by C02 model.

441

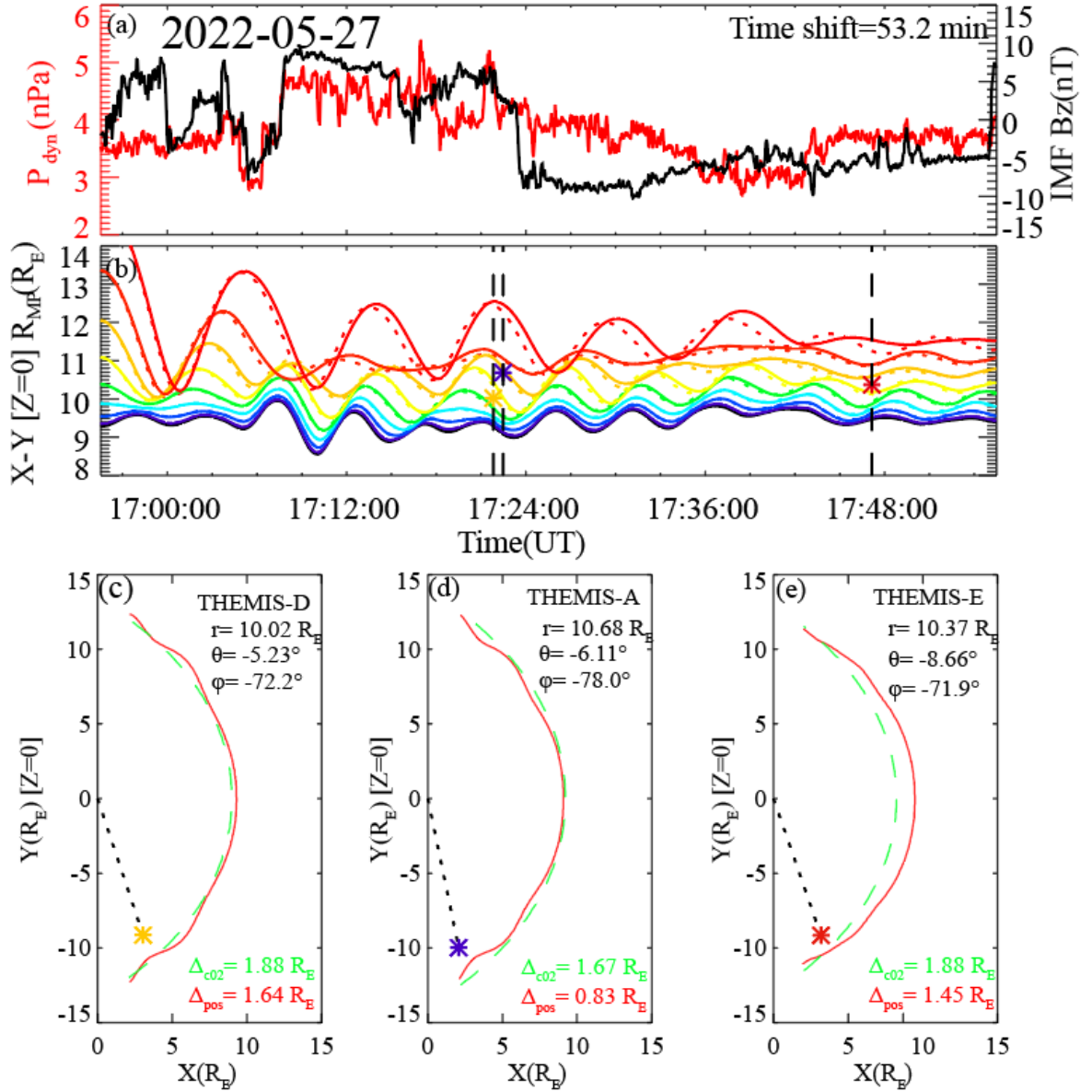


Figure 6. A surface wave-like structure in X-Y magnetopause flank region. (a) The corresponding solar wind dynamic pressure (red) and IMF  $B_z$  component (black), (b) The red, orange, yellow, green, blue, purple and black colours represent the initial magnetopause positions at  $\phi = \pm 80^\circ$ ,  $\pm 70^\circ$ ,  $\pm 60^\circ$ ,  $\pm 50^\circ$ ,  $\pm 40^\circ$ ,  $\pm 30^\circ$ ,  $\pm 20^\circ$ ,  $\pm 10^\circ$ ,  $0^\circ$ , respectively (dot line is the corresponding negative value of  $\phi$ ). The asterisk in purple (THEMIS-A), yellow (THEMIS-D) and red (THEMIS-E) indicate the satellite observation of MCEs projected onto the X-Y plane; (c) (d) (e) The shape of magnetopause in the X-Y plane at different time predicted by POS model (red dash line) and C02 model (green dot line), the asterisks represent the THEMIS MCEs positions mapped to X-Y plane.

## 5 Discussion and Conclusion

Accurately calculating the position of the magnetopause is essential for space weather forecasting and understanding the underlying physical mechanisms involved in the solar wind- magnetosphere interaction. In this work, we developed the POS model, the first time-dependent three-dimensional magnetopause model based on quasi-elastodynamic theory. By incorporating key solar wind parameters such as  $P_{\text{dyn}}$ , IMF  $B_z$  and  $\Phi$ , this model effectively depicts magnetopause dynamics. The POS model offers a new approach to describing magnetopause position, overall oscillation, and surface wave-like structures as interconnected phenomena. Its time-dependent feature excels in capturing dynamic processes, particularly under highly disturbed solar wind conditions. The three-dimensional nature allows for accurate depiction of the overall magnetopause shape, with notable precision in higher latitude regions and flank areas. This capability addresses limitations in existing models and provides a more comprehensive picture of magnetopause dynamics from a different perspective. However, there are still limitations and areas for improvement that future research should address:

(1) Adapting to extreme solar wind conditions: Similar to the force-deformation relationship of a spring that requires a specific range of applicability, the POS model has not been specifically optimized for extreme solar wind conditions (e.g.,  $P_{\text{dyn}} < 0.5$  nPa and  $P_{\text{dyn}} > 40$  nPa). When the solar wind dynamic pressure is low, the quasi-elastic process between the solar wind and the magnetopause exhibits stronger damping characteristics, while at very high solar wind dynamic pressures, the magnetopause shows increased rigidity. Future iterations could incorporate more suitable damping coefficients and include  $P_{\text{dyn}}$  in the magnetospheric compressibility coefficient to broaden the model's applicability range.

(2) Incorporating additional solar wind factors: Existing research has shown that even under similar solar wind dynamic pressure conditions, changes in solar wind density and velocity have distinct effects on magnetopause position (Samsonov *et al.*, 2020). Additionally, the influence of solar wind temperature, more comprehensive IMF effects (e.g.,  $B_x$  and  $B_y$ ), and other solar wind components



476 (e.g., alpha particles) on magnetopause position are not reflected in the current model. Moreover,  
477 magnetosheath transient effects and other perturbations on the magnetopause position are not  
478 addressed (*Sibeck et al.*, 2022; *Silveira and Sibeck*, 2023; *Silveira et al.*, 2024). Future models could  
479 consider introducing these factors to achieve better predictive results.

480 (3) Better nightside extension: The current POS model is primarily based on the dayside quasi-  
481 elastodynamic theory and is calibrated and validated using dayside MCEs. In the future, the model's  
482 calculation results for the nightside region could be improved by combining the fitting approach of  
483 empirical models with a more flexible curve function calibrated using a larger number of nightside  
484 MCE observations.

485 (4) Cusp region representation: Accurately modelling the magnetopause cusp region, shaped by  
486 Earth's dipole field, remains challenging. While some models approximate this region by fitting two  
487 distinct curves, capturing its shape and position precisely is complex. Improving the representation of  
488 the cusp region will require further analysis of higher latitude satellite data to enhance model accuracy.

489 (5) Parameter fine-tuning: Further refinement of model parameters, potentially through machine  
490 learning techniques or implementing piecewise functions for different regions, could improve the  
491 model's accuracy. However, as noted in the introduction, it's important to balance model complexity  
492 with practicality. Overly complex parameter expressions can lead to increased inconvenience and  
493 higher computational costs. For those seeking the highest possible prediction accuracy, a more  
494 practical approach might involve using numerical simulations.

495 The upcoming SMILE mission (Solar wind Magnetosphere Ionosphere Link Explorer), a joint  
496 mission between the Chinese Academy of Sciences and the European Space Agency, is set to launch  
497 in 2025(*Branduardi-Raymont et al.*, 2018; *Chi and Graziella*, 2018). This mission will provide more  
498 detailed data on magnetopause position and polar cap shape over time, enhancing the ability to validate  
499 and refine existing magnetopause models.

500 In summary, this study introduces the POS model, the first time-dependent three-dimensional  
501 magnetopause model based on quasi-elastodynamic theory. Unlike time-independent models, the POS  
502 model effectively captures the dynamic movement of the magnetopause under varying solar wind  
503 conditions. When compared to five widely used models, the POS model demonstrates superior  
504 predictive accuracy, showing a 17.9% improvement with RMSE=0.774 R<sub>E</sub>. As a time-dependent  
505 model, it demonstrated superior accuracy under highly disturbed solar wind conditions (22.1% better).  
506 Its three-dimensional nature allows for enhanced accuracy in higher latitude regions (27.0% better)  
507 and flank regions (33.3% better) of the magnetopause. Moreover, compared to numerical simulations,  
508 the POS model offers a concise formulation with rapid computational speed, making it feasible for  
509 direct deployment on satellites in the future, where onboard chips could complete calculations, greatly  
510 enhancing satellite intelligence. By providing a more precise and dynamic representation of the  
511 magnetopause, the POS model enhances our ability to predict and analyse space weather events and  
512 may also offer new insights and methodologies for developing magnetopause models for other planets.

513

#### 514 **Code and data availability**

515 The current version of model is available from the project  
516 website: [http://www.spaceweather.org.cn/pos\\_model](http://www.spaceweather.org.cn/pos_model). The exact version of the model used to produce  
517 the results used in this paper is archived on Zenodo: <https://doi.org/10.5281/zenodo.14189153> . The  
518 URL includes the code and the list of MCEs used in this paper. The data of THEMIS satellite can be  
519 obtained from <https://cdaweb.gsfc.nasa.gov/pub/data/themis/> , and the data of WIND satellite can be  
520 obtained from <https://cdaweb.gsfc.nasa.gov/pub/data/wind/> .

#### 521 **Competing interests**

522 The contact author has declared that none of the authors has any competing interests.

523        **Author contribution**

524        Y. W. designed the model. Y.X.G developed the model code and carried them out. Y. X. G and  
525        Y. W. prepared the original manuscript. Y. X. G and X. J. S. prepared the MCE list. F. S. W., X. S. F.,  
526        A. S., X. J. S., B. Y. W., P. B. Z., C. W. J., Y. L. C, X. J. X. and Z. L. Z. discussed the scientific results,  
527        reviewed and revised the manuscript.

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534        ZDSYS20210702140800001.

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