

Reviewer #2

1 General comments

This paper derives a novel continuum-mechanical model for ice melange dynamics. In recent years, ice melange has been recognized to buttress glacier flow and affect calving fluxes, among other things, which makes it important to include in ice sheet models. However, its properties resemble those of a granular medium, which makes its representation in conventional ice sheet models challenging. This study provides an important step towards including these effects. My comments below focus mostly on clarifying the derivation of the model, which I think is the most important part of the paper.

Thank you for your comments, which have improved the clarity of the paper.

2 Specific comments

a) Line 55: Before jumping right into the derivation of the model, it would be very helpful for the readers if the authors briefly summarized the assumptions upon which the model is going to be based and where the main differences to the traditional SSA model are going to be. Providing these pointers early will help the reader to understand what is going on later.

Thank you. We have added a couple of paragraphs to the beginning of Section 2 as suggested.

b) I would also suggest a sketch of the geometry. Especially later when the integrations are performed, readers are expected to be familiar with the vertical integration of momentum balance equations and which boundary conditions are applied, which is probably not true for everyone.

We have added a figure, as also suggested by Reviewer #1.

c) The model is stated incompletely: please write out which boundary conditions are assumed, that is, where is the surface of the melange, where is the base, what is the inflow boundary condition, what is the outflow, what are the stresses/velocities there, this sort of thing.

All of these things are described in Sections 2.1 and 2.2. However, to improve clarity we included a schematic and also added two paragraphs to the beginning of Section 2 in order to provide some model background prior to getting into the details.

d) Line 59: "Tectonic stress" – I hadn't come across this term before. In the continuum-mechanical literature usually deviatoric stresses are used which are defined in a similar fashion — also in your equation (5) below. Could you add a short note here explaining how these stresses differ from deviatoric stresses and why you use them?

Tectonic (i.e., resistive) stresses are commonly used in glaciology to simplify the depth-integration. We now state that and reference van der Veen and Whillans (1989).

e) Generally, there is really no need to introduce strain rates and velocities until after equation (7), which is a more natural place to start talking about how stresses are related to strain rates. Consider re-ordering for clarity.

We prefer to leave this as is in order to reduce the number of places where we have to introduce more variables. This way we are laying out some of the basic variables in the model before getting into the derivation.

f) Line 64: "inertial number": can you provide the definition here? Most readers will probably not be familiar with this number

Done (also in response to Reviewer #1).

g) Line 66: please typeset $\partial\sigma_{ij}/\partial x_j = \rho g \delta_{iz}$ as a separate equation – this makes it easier for people scanning the text without reading every word to follow the maths

Done.

h) Line 71: Integrating from where to where (boundary conditions!)?

From z to the surface, now stated in the text.

i) Line 59 & equation (3): R_{ij} seem to refer to both vertically-averaged and non-vertically averaged stresses. Maybe change the latter to r_{ij} ?

Due to our model assumptions, there are no vertical gradients in flow. The only terms that vary with depth are the static pressure, isometric pressure, and full stress tensor (i.e., not tectonic or deviatoric stresses). We have revised the text to make this more clear, and included bars over the stress tensor when specifically talking about depth-averaged values. We have already used different variables (lowercase vs uppercase) when describing the non-vertically averaged and vertically averaged pressures.

j) g' is a slightly confusing choice for the granular fluidity, as g is also standard gravity and g' is sometimes used for $(1 - \rho_i/\rho_w)g$. Could you use a different letter?

We understand the possible confusion, but g' is used by the granular mechanics literature to describe the granular fluidity, and we prefer to be consistent with those studies.

k) equation (17): is it possible for the denominator to go to zero or become negative? Are you applying a regularization here when solving numerically?

Yes, this is correct. We now state here that the NSMA requires a regularization. We later discuss our regularization scheme, which ensures that the granular fluidity does not go to 0 (see Section 2.2).

l) line 153: this is a good point to mention that you consider your analysis in a moving reference frame that is fixed at the glacier terminus

Done. We also now explain this in the new introductory paragraphs at the top of Section 2.

m) line 164-165: where exactly is the regularization applied?

It is applied when inserting μ , which depends on ϵ_e , into the granular fluidity equations. We added a sentence explaining this.

n) line 169: "The value of μ_w is related" → "The value of μ_w in (19) is related"

Done.

o) from the text it wasn't clear to me whether μ_w in equation (19) must come from solution of the y -component of the fluidity equation which makes solution of (19) more onerous?

It comes from consideration of the shear-dominated regime, so yes, it requires solving the transverse velocity profiles. But really the way to think of this is that we have a system of coupled equations. The longitudinal stress balance equation depends on U and μ_w , and the transverse velocity profiles depend on U and μ_w . Both equations have to be solved at the same time. See beginning of Section 2.3.

We have also added a couple of introductory paragraphs to Section 2, which we hope will clarify the model set-up and the equations that we solve, including the calculation of μ_w .

p) Line 284-296: I couldn't quite follow this argument, and I don't think the results in questions are shown in the paper. Maybe add a sketch to explain your argument?

This paragraph was revised in response to comments by Reviewer #1. Essentially, increasing the ice thickness (and associated fluxes) from 600 to 800 m produced a 100% increase in the buttressing force. The force required to keep a tall iceberg from capsizing, which can be used to estimate the force required to prevent large-scale calving events, scales with the thickness squared. Increasing the thickness from 600 to 800 m would require a 77% increase in buttressing force to prevent iceberg capsizing of full-glacier-thickness icebergs. Thus, the modeled buttressing force increases more rapidly with glacier thickness than the force that would be required to prevent capsizing.

q) Line 297-301: Point out somewhere here that melting enters into the model through 'B in equation (25)

Thank you for this suggestion. At the beginning of Section 3.1.2 we now indicate how each of the forcings/parameters enter into the model equations.

r) Line 298-300: "[...] mélange extent is sensitive to melt due to its indirect effect on lateral shear stresses": would that be the second term on the right hand side of equation (19) which depends on H^2 (please add reference in manuscript)? If not, how does this dependence come about?

Essentially yes, but due to the model's complexity we cannot show this analytically and prefer to leave the text as is. As you point out, that term describes the lateral shear stress and it does depend on H^2 . However, the shear stress also depends on μ_w , which is not constant (except in the quasi-static limit). For example, for the same calving flux, thinner ice mélange results in faster velocities (e.g., see Equation 22), and faster velocities result in higher values of μ_w .