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Nonparametric estimation of age-depth models from sedimentological and stratigraphic information

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Abstract. Age-depth models are fundamental tools used in all geohistorical disciplines. They assign stratigraphic positions to ages (e.g., in drill cores or outcrops), which is necessary to estimate rates of past environmental change and establish timing of events in sedimentary sequences. Methods to estimate age-depth models commonly use simplified parametric assumptions on the uncertainties of ages of tie points. The distribution of time between tie points is estimated using simplistic assumptions on the formation of the stratigraphic record, for example that sediment accumulates in discrete events that follow a Poisson process. In general, age-depth models are a crude simplification that fail to provide a comprehensive implementation of all empirical data or expert knowledge (e.g., from sedimentary structures such as erosional surfaces or from basin models). In other words, many information sources that can potentially provide geochronologic information remain un- or underused.

Here, we present two non-parametric methods to estimate age-depth models from complex sedimentological and stratigraphic data. The methods are complementary as they use different sources of information (sedimentation rates and observed tracer values), are implemented in the admtools package for R Software and allow the user to specify any error model and distribution of uncertainties. As use cases of the methods, we

1. construct age-depth models for the Late Devonian Steinbruch Schmidt section in Germany and use it to estimate the timing of the Frasnian-Famennian boundary and the duration of the Upper Kellwasser event.
2. use measurements of extra-terrestrial ^{3}He from ODP site 960 (Maud Rise, Weddell Sea) to construct age-depth models for the Paleocene–Eocene thermal maximum (PETM).

The first case study suggests that the Upper Kellwasser event lasted 89 kyr (IQR: 84 to 97 kyr) and places the Frasnian-Famennian boundary at 371.834 ± 0.101 Ma (2 σ), whereas the second case study provides a duration of 41 to 48 kyr for the PETM recovery interval. These examples show how information from a variety of sedimentological and stratigraphic sources can be combined to estimate age-depth relationships that accurately reflect uncertainties of both available data and expert knowledge.

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1 Introduction

Age-depth models are a fundamental tool in all geohistorical disciplines where samples can not be dated directly. They assign ages to sampling positions (e.g. stratigraphic heights, core depths), allowing to determine the timing of past events and reconstruct rates of past change. Although age-depth models are rarely the focus of standalone publications, their role for the interpretation of historical data becomes obvious when they are revised, often altering our interpretations of past changes. For example, Malmgren et al. (1983) showed that evolution of planktic foraminifera shows short intervals with rapid changes in body size, arguing that this is a common mode of evolution. Revising the age-depth model, MacLeod (1991) showed that the intervals of rapid change coincide with stratigraphic condensation. After accounting for this effect, a random walk was the most likely explanation for the change in body size. This example demonstrates that age-depth models can change our interpretation of how evolution acts on geological time scales fundamentally (Bookstein, 1987; Gould and Eldredge, 1972).

Because of their importance, age-depth models are applied across multiple spatial and temporal scales and environments (terrestrial, marine, lacustrine), ranging from decadal-scale chronologies in modern lakes to the global geological time scale covering hundreds of million of years (Gradstein, 2020; Lacourse and Gajewski, 2020; Cerda et al., 2019). As a result, a wide range of scientific communities engage in the development of age-depth models. This leads to a plethora of available methods to estimate age-depth models, such as `Bchron` (Haslett and Parnell, 2008), `CLAM` (Blaauw, 2010), `Oxcal` (Ramsey, 2009, 2008), or `Bacon` (Blaauw and Christen, 2011), with variable methodological complexity, ranging from simple interpolation procedures to elaborate Bayesian methods.

Every method to estimate age-depth models makes assumptions on sediment accumulation. For example, the `P_sequence` model in `OxCal` assumes that sediment accumulates in discrete events that follow a Poisson distribution (Ramsey, 2008), meaning the events are independent of each other and the waiting time between them is exponentially distributed. They argue that this is a reasonable assumption for the slow accumulation of individual grains or deposition of regular layers. Similarly, `Bchron` (Haslett and Parnell, 2008) assumes that the number of changes in sedimentation rate (the slope of the age-depth model) follows a Poisson distribution. Methods originally developed based on knowledge of specific timescales or environments are often applied outside of that domain, which potentially leads to violating their assumptions. For example, `Bchron` (originally developed for Quaternary records) and the derived "modified `Bchron`" (Trayler et al., 2019) were used to improve the global Devonian time scale (Harrigan et al., 2021; De Vleeschouwer and Parnell, 2014), constrain the timing of the Late Paleozoic Ice Age in the Parana Basin (Cagliari et al., 2023), and date saltmarsh sediment to reconstruct late Holocene sea-level changes (Parnell and Gehrels, 2015). However, it is not clear that in all such cases the assumptions of these age-depth modeling procedures are suitable representations, and therefore applicable, for such a wide range of temporal and spatial scales and depositional environments. These assumptions are often made because of their mathematical convenience, and not because of their empirical realism.

The stratigraphic record might be complex, but different sub-disciplines of stratigraphy offer ways to constrain its structure. Astrochronology can provide estimates on accumulation rates by matching proxy records with orbital signals (Meyers, 2019; Li et al., 2018). Sequence stratigraphy provides qualitative predictions on changes in sedimentation within a sequence.

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Forward models allow us to examine the effect of different assumptions about sedimentation, which can be used to constrain biasing effects of stratigraphic architectures (Hohmann et al., 2024). The majority of methods to estimate age-depth models are not able to incorporate complex stratigraphic information into their estimates, or are constrained to simplistic assumptions (e.g. additional breakpoints in sedimentation rates (Trayler et al., 2023)).

Here, we present two nonparametric methods, FAM (**F**lux - **A**ssumption **M**atching) and ICON (**I**ntegrated **CON**ndensation), to estimate age-depth models from complex stratigraphic and sedimentological data. ICON estimates age-depth models from arbitrarily complex data on sedimentation rates observed in a stratigraphic column. This knowledge can, for example, be derived from astrochronology (`eCoco` by Li et al. (2018) or `eTimeOpt` by Meyers (2019)), sequence stratigraphic interpretations, or expert knowledge. FAM estimates age-depth models and sedimentation rates by comparing observed tracer values (extraterrestrial helium-3, pollen, or radiogenic tracers such as ^{210}Pb) with assumptions of their fluxes in the time domain. Both methods are nonparametric in the sense that they do not make assumptions on the processes or probability distributions that govern sediment accumulation and the structure of the stratigraphic record. They are assumption explicit in the sense that they assume the law of superpositon holds, but all other assumptions must be provided by the user. This ensures that model assumptions are specified verbatim and clearly communicated, and the estimated age-depth models are data-driven rather than assumption-driven. Both methods are implemented in the R package `admtools` (Hohmann, 2024a), which is available on CRAN (The Comprehensive R Archive Network), and is developed as an open source project on GitHub.

We illustrate the methods using two empirical cases. First, we examine how the propagation of uncertainties of sedimentation rates estimated using cyclostratigraphy influence the duration of the Late Devonian Kellwasser Event and the age of the Frasnian-Famennian boundary (Da Silva et al., 2020). Second, we determine how variable fluxes of extraterrestrial ^{3}He change the interpretation of PETM recovery time at ODP Site 690 (Farley and Eltgroth, 2003). These cases were chosen because they are characterized by both environmental upheaval and varying sedimentation rates. Interpreting the environmental upheaval requires good age-depth models, which are challenging to estimate due to the changes in sedimentation rate.

The examples show that the developed methods are able to incorporate complex sedimentological and stratigraphic data into the estimation of age-depth relationships and their uncertainties, resulting in age-depth models with empirically realistic uncertainties.

The target groups of this manuscript are researchers using stratigraphy at any point of their work, especially those investigating processes in the time domain, such as material or element fluxes, the nature and rate of biological evolution, as well as the pacing of environmental change. Secondly, the text is addressed to the community interested in developing and promoting methods of age-depth estimation. The documentation of the `admtools` package contains extensive worked examples and the supplementary material for this article provides literate code, which can serve as a starting point for users to develop their own applications. Thus, to use the methods described here, users are encouraged to run the code along with studying the presented examples.

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2 Model development

2.1 Assumptions

We assume that:

1. sediment accumulation is uninterrupted, i.e. is strictly positive (but can be arbitrarily low), and
2. the law of superpositon holds, meaning strata found lower in a section are older than those found higher up.

With these assumptions, age-depth models are strictly monotonous functions between the time and stratigraphic domain, where strict monotonicity reflects the law of superposition.

2.2 Preliminaries

We distinguish between time domain D_T (time dimension, SI units seconds or derived units such as years) and stratigraphic domain D_L (length dimensions, SI units meter). Throughout the manuscript, indices of T (or L) indicate that a function is defined in the time (or stratigraphic) domain. We use time t (increasing towards today) and height h (increasing upwards) for equations, as this ensures that the direction of integration is always from older to younger strata. Using age (increasing away from today) or depth (common when working with drillcores) would prevent that. Technically we work with time-height models, but we will still refer to them as age-depth models as the name is well established. Conversions from time to age or height to depth can be made by using time before a reference point, and height below a reference point. Any combination of age or time and depth or height would be correct and is simply a matter of choice of reference frame and scientific community. For example, cores are typically described in terms of depth, whereas sections are commonly measured upwards and expressed in height.

Let $T : D_L \to D_T$ be the function that maps a stratigraphic position to its time of deposition, and $H : D_T \to D_L$ the function that maps a point in time to the stratigraphic position formed at said time. Both T and L provide a description of the age-depth model. By definition, they are inverses of each other:

$$T = H^{-1} \text{ and } H = T^{-1} \tag{1}$$

Throughout this article, we mean instantaneous sedimentation rate when we speak of sedimentation rates. We distinguish between the sedimentation rate in the time domain s_T and the sedimentation rate observed in the stratigraphic domain s_L. The sedimentation rate observed at a stratigraphic position h is the rate with which said position was formed, yielding the following two relations:

$$s_T(T(h)) = s_L(h) \text{ and } s_T(t) = s_L(H(t)) \tag{2}$$

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If H is differentiable, we know that

$$\frac{dH}{dt} = s_T$$

Due to the law of superposition (strict monotonicity), sedimentation rates in both domains are strictly positive. The amount of sediment accumulated in the time interval $[t_1, t_2]$ is

$$H(t_2) - H(t_1) = \int_{t_1}^{t_2} s_T(x)\, dx \quad (3)$$

2.3 Estimating age-depth models from sedimentation rates

Sedimentation rates determine how fast new material accumulates, and how much time is recorded per increment of stratigraphic thickness. Accumulating this information form a reference point can be used to construct age-depth models. Conversely, if an age-depth model is given, its slope is the sedimentation rate (Trayler et al., 2023; Hohmann, 2021). Here, we formalize how age-depth models can be constructed from arbitrary sedimentation rates in the stratigraphic domain. We refer to this method as ICON, standing for **I**ntegrated **CON**densation for reasons explained below.

2.3.1 Model formulation

By the inverse rule, we get

$$\frac{dT}{dh} = \frac{1}{s_L} \quad (4)$$

(see e.g., Hohmann (2021)). Accordingly, the amount of time recorded in the stratigraphic interval $[h_1, h_2]$ is given by

$$T(h_2) - T(h_1) = \int_{h_1}^{h_2} \frac{1}{s_L(x)} dx \quad (5)$$

We refer to the inverted sedimentation rate in the stratigraphic domain

$$c(h) := \frac{1}{s_L(h)} \quad (6)$$

as (stratigraphic or sedimentary) *condensation*, which is used here as time recorded in sediment and does not imply low accumulation rates. Using condensation instead of sedimentation rate in the stratigraphic domain has two advantages. First, it has the correct dimension and units (time per length, years per meter) to represent the amount of time represented in the rock record. This allows to directly determine the amount of time represented in a section by integrating over condensation. Second,

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it reduces the ambiguity that comes with dealing sedimentation rates in both in the time and stratigraphic domain. While sedimentation rates in the time domain can in general be zero or negative under sedimentary stasis or erosion, condensation must always be positive as we can only observe intervals with net positive sediment accumulation in the rock record.

Given two tie points (t_i, h_i) , (t_{i+1}, h_{i+1}) and condensation $c(h)$, define the dimensionless normalization constant

$$C_i := \frac{t_{i+1} - t_i}{\int_{h_i}^{h_{i+1}} c(x)\,\mathrm{d}x} \tag{7}$$

and introduce the *tie-point corrected condensation*:

$$\hat{c}^i(h) = c(h) \cdot C_i \tag{8}$$

This correction ensures that between tie points, the time represented by condensation matches the time elapsed between the tie points. Then the age-depth model is given by

$$T(h) = t_i + \int_{h_i}^{h} \hat{c}^i(x)\,\mathrm{d}x \quad \text{for } h \in [h_i, h_{i+1}] \tag{9}$$

This is simply adding the time elapsed between h_i and h to the known time at the lower tie point. Specifically, $T(h_i) = t_i$ and $T(h_{i+1}) = t_{i+1}$. For multiple tie points, the age-depth model is given by the closed expression

$$T(h) = \sum_i \mathbf{1}_{H_i} \cdot \left(t_i + \int_{h_i}^{h} \hat{c}^i(x)\mathrm{d}x \right) \tag{10}$$

where $\mathbf{1}_A$ is the indicator function on the set A and $H_i = (h_i, h_{i+1}]$ is the i-th stratigraphic interval. For this expression to be valid below and above the highest tie point, two minor adjustments are necessary. First, below the lowest tie point (t_0, h_0), the direction of integration needs to be reversed to makes sure the integral represents a positive amount of time, leading to the expression

$$T(h) = t_0 - \int_{h}^{h_0} \hat{c}^i(x)\mathrm{d}x \tag{11}$$

Second, above and below the highest tie point, normalization is not necessary as there is no possible mismatch between the time represented by the sedimentation rate and the time elapsed between tie points (because there are no two tie points). Skipping normalization is achieved by setting the normalization constants in these intervals to 1. This means between tie points, information on sedimentation rates or condensation is adjusted to be congruent with the timing of the tie points and only contributes information about the relative distribution of time within the section, while it is taken at face value below/above the lowest/highest tie point.

By the monotonicity of the integral, T as defined above is monotonous, and H is uniquely defined. With this, the sedimentation rate in the time domain can be determined using

$$s_T(t) = s_L(H(t)) \tag{12}$$

2.4 Estimating age-depth models from tracer values

Assume there is a constant influx of a tracer into the sediment with time. Then observing elevated tracer values in a section indicates stratigraphic condensation and low sedimentation rates, while reduced tracer values in the section indicate stratigraphic dilution and high sedimentation rates. Comparing tracer values with assumptions on their flux can thus be used to constrain the time preserved in the stratigraphic record, and construct age-depth models. Using extraterrestrial ^{3}He as tracer, this approach has for example been employed to constrain the timing of the Paleocene-Eocene Thermal Maximum (PETM) (Farley and Eltgroth, 2003) and the Cretaceous-Paleogene (K-Pg) boundary extinction (Mukhopadhyay et al., 2001) as well as to examine small-scale fluctuations of sedimentation rate in limestone marl alternations (Blard et al., 2023). Jarochowska et al. (2020) used three independent constant flux tracers to construct relative age-depth models for the late Silurian Lau Carbon Isotope Excursion to correct rates of redox proxy- and isotope changes for increasing sedimentation rates in a shallowing-upward succession in Gotland, Sweden. Similarly, Appleby and Oldfield (1978) compared observed ^{210}Pb values in cores with ^{210}Pb concentrations predicted from constant flux and exponential decay to estimate age-depth relationships in young sediments, an approach termed the CRS model (Abril-Hernández, 2023). What unifies these approaches is that they compare the assumed tracer flux into the sediment in the time domain with tracer values observed in the stratigraphic domain to construct age models.
Here, we provide the general mathematical framework for the construction of age-depth relationships from comparisons of arbitrarily complex tracer fluxes with observed tracer values in a section. We refer to this method as FAM, standing for **F**lux **A**ssumption **M**atching.

2.4.1 Model formulation

Assume we have have observations of a tracer f_L in the stratigraphic domain (dimension X/L , where X is the unit in which the tracer is measured) and some knowledge on tracer fluxes with time $\tilde{f}_T$ (dimensions X/T). Let (t_i, h_i) and (t_{i+1}, h_{i+1}) be two tie points. Define the dimensionless constant

$$C_i = \frac{\int_{h_i}^{h_{i+1}} f_L(x)\mathrm{d}x}{\int_{t_i}^{t_{i+1}} \tilde{f}_T(x)\mathrm{d}x} \tag{13}$$

and define the empirically calibrated assumption on tracer flux in the time domain

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$$f_T^i := C_i \tilde{f}_T \tag{14}$$

This normalization ensures the amount of tracer assumed to be embedded in the sediment between t_i and t_{i+1} matches the amount of tracer observed between h_i and h_{i+1} in the section.

In the absence of taphonomic and diagenetic effects, the total volume of tracer observed in a stratigraphic interval I_L is identical to the total volume of tracer incorporated into the sediment over to time interval I_T during which I_L was formed (see Hohmann (2021)). Based on this, we know that

$$\int_{t_i}^{t} f_T^i(x)\mathrm{d}x = \int_{h_i}^{h} f_L(x)\mathrm{d}x \tag{15}$$

holds, because tracer volume placed in the sediment over $[t_i, t]$ is identical to tracer volume observed in $[h_i, h]$. The age-depth model is given by all t and h for which this relationship holds (the graph of the relation). Solving the above equation for t (or h) yields a representation of T (or H). Introducing the short notations

$$\Theta_{t_i}(t) := \int_{t_i}^{t} f_T(x)\mathrm{d}x \quad \text{and} \quad \Lambda_{h_i}(h) := \int_{h_i}^{h} f_L(x)\mathrm{d}x \tag{16}$$

T can be written explicitly as

$$T(h) = \Theta_{t_i}^{-1} \circ \Lambda_{h_i}(h) \tag{17}$$

The age-depth model in the presence of multiple tie points can be generated by stitching together multiple stratigraphic intervals, leading to the representation

$$T(h) = \sum_i \mathbf{1}_{H_i} \left(\Theta_{t_i}^{-1} \circ \Lambda_{h_i}(h)\right) \tag{18}$$

where $H_i = (h_i, h_{i+1}]$ is the i-th stratigraphic interval. To expand this approach to heights above/below the highest/lowest tie point, some minor adjustments are required. First, below the lowest tie point the direction of integration is reversed to ensure the integrals return positive tracer volumes. Second, above/below the highest/lowest tie point, the normalization coefficient C_i is not necessary, as there can not be a mismatch between observed and assumed tracer fluxes between tie points because there is no second tie point which could generate such a mismatch. Computationally, this is solved by setting the normalization coefficient to 1 above or below the highest or lowest tie point, respectively.

https://doi.org/10.5194/egusphere-2024-2857
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There is one relevant edge case where normalization above or below the highest tie point (t_N, h_N) or lowest tie point (t_0, h_0) is required. This is when the integrals over both $\tilde{f}_T$ and f_L on the unbounded intervals (h_N, ∞) and (t_N, ∞) (resp. $(-\infty, h_0)$ and $(-\infty, t_0)$) are finite. This is for example the case when estimating age-depth relationships based on radiogenic tracers such as ^{210}Pb. Here, the observed tracer values drop to 0 down core due to exponential decay. The assumed tracer flux in the time domain can vary, but will eventually drop to 0 because of the exponential decay of the tracer, leading to an f_L that is defined on an unbounded interval, but with a finite tracer volume (Abril-Hernández, 2023).

2.4.2 Estimating sedimentation rates

Based on the representation of T , the inverse function and the composition rule, we get

$$s_L(h) = \frac{f_T(T(h))}{f_L(h)} \tag{19}$$

for the instantaneous sedimentation rates observable in the stratigraphic domain, and

$$s_T(t) = \frac{f_T(t)}{f_L(H(t))} \tag{20}$$

for the instantaneous sedimentation rate observable in the time domain. Sedimentation rates are given by the ratio of assumed tracer flux to observed tracer flux. For constant assumed tracer flux, this provides a mathematical equivalent to the intuition that elevated observed tracer values indicate low sedimentation rates, while reduced observed tracer values correspond to high sedimentation rates.

2.4.3 Special cases

In general, the equations arising from FAM need to be solved numerically by combining integration with root-finding procedures. Here, we give two examples where the age-depth relationships from FAM can be written as analytical expressions, and use these examples to demonstrate how FAM generalizes existing methodology for estimating age-depth relationships.

2.4.4 Constant tracer flux in the time domain: the cFAM model

Constant tracer fluxes have occasionally be used to estimate sedimentation rates and age models (Mukhopadhyay et al., 2001, Farley and Eltgroth (2003) Jarochowska et al. (2020) Blard et al. (2023)), but this is not a well-established approach. Here, we derive closed expressions for age-depth models and sedimentation rates derived under the assumption of a constant tracer flux, a method we refer to as cFAM (**c**onstant **F**lux **A**ssumption **M**atching).

Constant tracer influx in the time domain implies $\tilde{f}_T(t) = c$ for all t and some flux value c. Between two tie points (t_i, h_i), (t_{i+1}, h_{i+1}), we get

$$C_i = \frac{\int_{h_i}^{h_{i+1}} f_L(x) \mathrm{d}x}{c\,(t_{i+1} - t_i)} \tag{21}$$

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for the normalization constant, so the empirically calibrated assumption on tracer flux is

$$f_T(t) = \frac{\int_{h_i}^{h_{i+1}} f_L(x)\mathrm{d}x}{t_{i+1} - t_i} \tag{22}$$

As a result, the age model between the tie points under cFAM is

$$T(h) = t_i + \frac{(t_{i+1} - t_i)\int_{h_i}^{h} f_L(x)\mathrm{d}x}{\int_{h_i}^{h_{i+1}} f_L(x)\mathrm{d}x} \tag{23}$$

Note here that the value of the flux c does not appear in the equation, as it is cancelled out by the normalization. Only relative changes in observed tracer values contribute to the age-depth model between tie points. Above the highest tie point (t_N, h_N) and below the lowest tie point (t_0, h_0), we get

$$T(h) = t_N + \frac{1}{c}\int_{h_N}^{h} f_L(x)\,dx \quad \text{and} \quad T(h) = t_0 - \frac{1}{c}\int_{h}^{h_0} f_L(x)\,\mathrm{d}x \tag{24}$$

For the sedimentation rates, note that f_T is independent of t, so we get for the sedimentation rate in the stratigraphic domain

$$s_L(h) = \frac{\int_{h_i}^{h_{i+1}} f_L(x)\mathrm{d}x}{(t_{i+1} - t_i) f_L(h)} \tag{25}$$

between tie points and

$$s_L(h) = \frac{c}{f_L(h)} \tag{26}$$

above/below the highest tie point.

Jarochowska et al. (2020) used an auxiliary time scale where $t = 0$ corresponds to the bottom of the section and $t = 1$ corresponds to the top of the section, allowing them to correct rates for variations in sedimentation rates in the absence of absolute age constraints. This is equivalent to introducing two artificial tie points $(0, h_0)$ and $(1, h_1)$, where h_0 and h_1 are the bottom and the top of the section, respectively. In this case, cFAM reduces to cFAM-at (where the at stands for **a**uxiliary **t**ime). In this case, the age model is given by

$$T(h) = \frac{\int_{h_0}^{h} f_L(x)\mathrm{d}x}{\int_{h_0}^{h_1} f_L(x)\mathrm{d}x} \tag{27}$$

and sedimentation rates by

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$$s_L(h) = \frac{\int_{h_i}^{h_{i+1}} f_L(x)\mathrm{d}x}{f_L(h)} \tag{28}$$

Note that both expressions are solely dependent on empirical data measured in the section.

2.4.5 Radiogenic tracers

Here, we show that when FAM is combined with the assumption that tracer flux follows an exponential decay, it reduces to the CRS (constant rate of supply) model by Appleby and Oldfield (1978) for dating sediments using ^{210}Pb, a common dating tool for recent (100 to 150 years) aquatic sediments. We use the derived expressions for sedimentation rate to derive an estimator of instantaneous sedimentation rates in the stratigraphic domain for the CRS model.

For this section, we use age a and depth d instead of time t and height h, as they are more fitting to the context of dating recent core material. Assuming the tracer decays exponentially with time, we get

$$\tilde{f}_T(a) = c\exp(-\lambda a) \tag{29}$$

for tracer flux in the time domain, here representing the amount of preserved tracer of age a. Let $f_L(d)$ be the tracer content at depth. Then the normalization constant is

$$C = \lambda \int\limits_0^\infty f_L(x)\mathrm{d}x \tag{30}$$

and we get

$$f_T(a) = f_T(a)\lambda \int\limits_0^\infty f_L(x)\mathrm{d}x \tag{31}$$

for the empirically calibrated tracer flux in the time domain. Note here that tracer fluxes were normalized, although they are defined on an unbounded interval. Then the age-depth relationship is the solution to the equation

$$\int\limits_0^a \exp(-\lambda x)\mathrm{d}x = \frac{\int_0^d f_L(x)\mathrm{d}x}{\lambda\int_0^\infty f_T(x)\mathrm{d}x} \tag{32}$$

Solving for a, we get

$$A(d) = -\frac{1}{\lambda}\ln\left(1 - \frac{\int_0^d f_L(x)\mathrm{d}x}{\int_0^\infty f_L(x)\mathrm{d}x}\right) \tag{33}$$

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for the age of the core as a function of depth, matching the formulations of the CRS model given by Appleby and Oldfield (1978) and Abril-Hernández (2023). The sedimentation rate at depth d can then directly be estimated via

$$s_L(d) = \frac{\lambda \int_0^\infty f_T(x)\mathrm{d}x \left(1 - \frac{\int_0^d f_L(x)\mathrm{d}x}{\int_0^\infty f_L(x)\mathrm{d}x}\right)}{f_L(d)} \tag{34}$$

Note that FAM allows to incorporate arbitrary assumptions on how tracer flux changes with time into the estimation of a ^{210}Pb chronology, allowing to incorporate expert knowledge on fluctuations in ^{210}Pb flux. While there is an analytical solution for the CRS model available, the integral equations defining the more general cases need to be solved numerically.

2.5 Randomization

The estimation of age-depth models as described above is purely deterministic. Here, we show that it expands to the probabilistic estimation of age-depth models without additional assumptions on the nature of the underlying probability distributions.

For ICON, we first assume tie points are deterministic and in strict temporal and stratigraphic order ($t_i < t_{i+1}$ and $h_i < h_{i+1}$), and consider the sedimentation rate a stochastic process. Based on the law of superposition, the condensation is a strictly positive stochastic process, which we assume to be regular enough to be integrated. With this, all involved integrals are well-defined, the normalization factor is not zero, and T is a strictly increasing stochastic process. Given the strict order holds almost surely, this construction remains valid when the tie points are randomized. Identical arguments hold for FAM: When observed and assumed tracer fluxes are almost surely positive and integrable, Θ and Λ are both strictly increasing, and the arising integral equations can be solved uniquely. Summarizing, both the construction of ICON and FAM immediately expand from the deterministic to the probabilistic case without any assumptions on the involved probability distributions.

2.6 Implementation

Both estimation procedures for age-depth models are implemented in the R package `admtools` (Hohmann, 2024a). The implementation uses the R internal procedure `integrate` to numerically determine the integrals arising in FAM and ICON, and `uniroot` to solve the function inversions in FAM (qua, Brent (2002)). The two methods are implemented in the functions `sed_rate_to_multiadm` (ICON) and `strat_cont_to_multiadm` (FAM). Both produce an object of type `multiadm`, representing a collection of age-depth models (sample paths of T), each of which is a possible scenario for the examined section. These objects can be reused, e.g. for plotting, transforming data, determining uncertainties of stratigraphic positions or timing, or for transformation of other objects (e.g., time series) between the stratigraphic and the time domain.

Timing and positions of tie points can follow arbitrary probability distributions as long as they are strictly ordered. It is the user's responsibility to ensure strict order. In the most general case, tie points are coded as functions that take no inputs and, upon each evaluation, return one sample drawn from the distribution of tie point times/heights. Effectively, these functions are user-defined random sample generators. While they are not dependent on any input parameters, they can wrap around complex empirical data that determines uncertainties. Details on how these functions are coded can be found in the package vignettes (long form documentation with worked examples) or the project webpage. To simplify the definition of tie points, wrappers for common use cases for tie points are provided (uniform or normally distributed tie points or deterministic tie points). Note that not only the timing, but also the stratigraphic position of tie points can be randomized (e.g., when the age information is associated with a bed, or its stratigraphic position was not recorded precisely).

The stochastic processes representing sedimentation rates or tracer fluxes can be passed to the estimation procedures as function factories that take no arguments. A function factory is a function that itself returns a function. The function factory represents the stochastic process. Each time it is evaluated, it returns a function that represents a sample path. This function can be evaluated at specific points to return the sedimentation rates/tracer values at said points. In our case, the function factories can be thought of as very complex random number generators. Instead of returning a random number or vector of random numbers, they return a random function, effectively making them infinite-dimensional random number generators. Similar to the way tie points are coded, they can depend on user data without having to pass said data to the estimation procedures. To simplify the definition of function factories, wrappers for the most common use cases are provided. For tracer fluxes, these includes cases for constant, linear, and quadratic fluxes, empirical measurements of tracer means and standard deviations (see example on the PETM). For sedimentation rates, options to turn arrays of sedimentation rate estimates are provided, including ways to directly take outputs from the `eTimeOpt` function from the `astrochron` package and turn them into a function factory (Meyers, 2014, Meyers (2019)).

Computation times vary, but are typically below one minute (see examples below). Computation time is dependent on the irregularity of the input functions (sedimentation rates, tracer fluxes), which determines how fast the numeric integration is. Functions with rapidly changing function values are difficult to integrate numerically, with discontinuous functions being the most challenging ones. For fixed input functions, computation time scales linearly with the number of stratigraphic positions where the age-depth models are determined, and the number of age-depth models estimated within one `multiadm` object.

The package implements unit tests for both the estimation procedures and the underlying logic of the age-depth transformation that are run each time a part of the code is changed. Systematic testing of code is considered best practice in software development, and improves the quality of scientific software (Hunter-Zinck et al., 2021, Nanthaamornphong and Carver (2017)). Unit tests for FAM and ICON test edge cases where analytical solutions are known in advance (one, two, or multiple tie points, constant sedimentation rate, constant tracer fluxes, etc.). Multiple vignettes (short form articles that provide a more complex use case) for both procedures are available after installation via the command `browseVignettes("admtools")` , and can also be browsed on the packages webpage (https://mindthegap-erc.github.io/admtools/)

Code adheres to the FAIR4RS (FAIR for research software) principles (Barker et al., 2022), which are based on the FAIR principles for scientific data management and stewardship (Wilkinson et al., 2016). Code development is performed on Github. Each minor release (based on semantic versioning) is published on CRAN (the Comprehensive R Archive Network), assigned a doi and archived on Zenodo.

The package is fully open source, all code can be inspected on GitHub. Contributors are invited to make enhancement request, improve documentation and bug reports, contribute code, and improve integration with the existing geoscientific

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software landscape. Contributing guidelines for the package are specified in the `CONTRIBUTING.md` file in the root of the directory.

3 Examples

We apply the newly developed methods to two existing studies, the Late Devonian Mass Extinction and the Paleocene-Eocene Thermal maximum. We use these examples to examine how the ability to incorporate added uncertainty changes age estimates. For details on computational reproducibility, see the README file in Hohmann (2024b). All analyses were performed with `admtools` version 0.3.1 (Hohmann, 2024a)

3.1 Duration and timing of the Late Devonian mass extinction

The Late Devonian mass extinction at the Frasnian-Famennian boundary is considered one of the “big five” Phanerozoic mass extinctions (Muscente et al., 2018; Raup and Sepkoski, 1982), and a multitude of causes have been discussed, including climate warming or cooling (Thompson and Newton, 1988), volcanism (Racki et al., 2018), extraterrestrial impacts (Claeys et al., 1992) , or changes in the weathering cycle (Averbuch et al., 2005). In many locations, the sedimentologic expression of this perturbation is expressed by two dark, organic-rich lithologies referred to as the Upper and the Lower Kellwasser beds, each being associated with an anoxic event called the Upper and the Lower Kellwasser Event (Carmichael et al., 2019).

Da Silva et al. (2020) combined cyclostratigrapic methods with U-Pb dates from Percival et al. (2018) to constrain the absolute timing and duration of the Late Devonian Kellwasser events and the Frasnian-Famennian boundary in the Steinbruch Schmidt section, Germany. For the cyclostratigraphic analysis (code and data available in da Silva (2024)), they used the `eTimeOpt` function from the `astrochron` package for R Software (Meyers, 2014, 2019; R Core Team, 2023). `eTimeOpt` uses a moving window approach to find sedimentation rates that lead to the highest concentration of power of the precession and eccentricity frequencies and best expression of short eccentricity or precession amplitude modulation in the analyzed proxy record. The method returns r^2_{opt}, a measure of fit between the proxy signal and the predicted patterns for a range of heights and sedimentation rates. Da Silva et al. (2020) used the `eTimeOptTrack` function of the `astrochron` package to extract sedimentation rate estimates from `eTimeOpt` results, yielding the sedimentation rates at which r^2_{opt} is maximal. These deterministic sedimentation rates change with stratigraphic position, and were used to constrain the duration of the anoxic events and the timing of the Frasnian-Famennian boundary. However, simply using the sedimentation rate with the best fit (highest r^2_{opt}) neglects uncertainties in the estimation of sedimentation rates, as a wide range of sedimentation rates can potentially provide a good fit to a given signal (Figure 1). Here, we use ICON to examine how the propagation of uncertainties of sedimentation rates changes the duration and timing estimates for the Kellwasser event and the Frasnian-Famennian boundary.

Da Silva et al. (2020) analyzed magnetic susceptibility (MS), log Ti concentration, and $\delta^{13}C$ values, with similar results for all three proxies. For this example, we focus solely on MS. Data preprocessing and `eTimeOpt` analysis was kept identical to da Silva (2024) for comparability, see Da Silva et al. (2020) for details. Here, we show the results of `eTimeOpt` testing for precession amplitude modulation. Results for short eccentricity amplitude modulation are similar and shown in the Appendix

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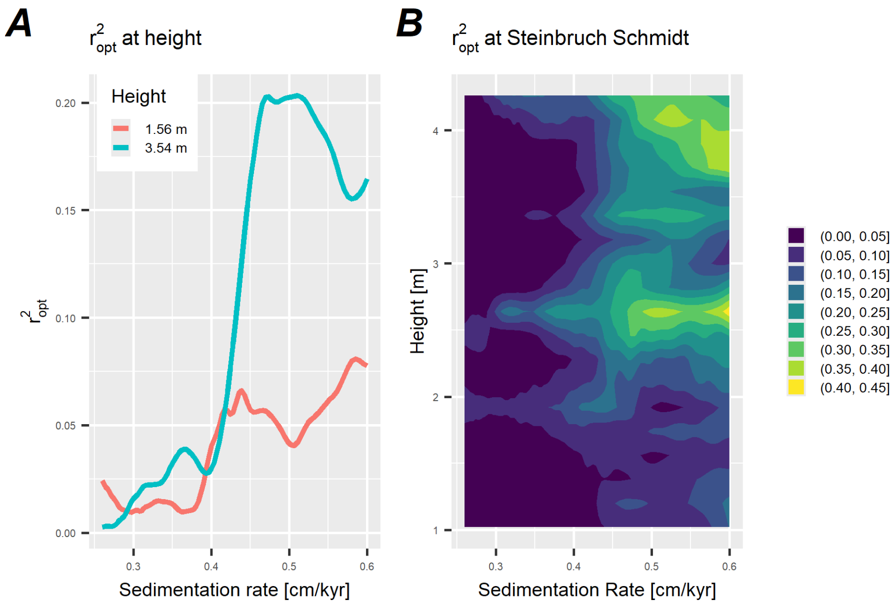

Figure 1. r^2_{opt} values from `eTimeOpt` testing for precession amplitude modulation at 3.54 m and 1.56 m (A) and throughout the whole section (B) in the Steinbruch Schmidt section. The heights shown in A roughly corresponding to the locations of the Upper Kellwasser bed and the location of the dated ash bed.

(Figures 3, A1, A2) . `eTimeOpt` results for r^2_{opt} were extracted using the `get_data_from_eTimeOpt` function of the `admtools` package. Interpreting `eTimeOpt` results probabilistically has two challenges:

1. At fixed heights, it provides r^2_{opt} values as a function of sedimentation rate. However, it is unclear how r^2_{opt} values can be meaningfully translated into a probability that the sedimentation rate falls within a certain interval.
2. The correlation structure of sedimentation rates is unclear: Given we know sedimentation rates at one point in the section, how would that influence our estimates further up or down section?

To address this, we introduce the following two assumption into this example. First, the sedimentation rates are determined at random heights following a Poisson process with rate λ, an assumption borrowed from `BChron` (Haslett and Parnell, 2008). This means the number of height points follows a Poisson distribution with an expected value of $\lambda \cdot l$ (where l is the lenght of the section), and the locations of the points are independent and identically distributed according to a uniform distribution. Second, at each stratigraphic position, sedimentation rates follow a distribution with probability density function proportional to r^2_{opt}. As a result, sedimentation rates with a higher r^2_{opt} are more probable than those with a low r^2_{opt}. If a range of sedimentation rates has comparable r^2_{opt} values, they are all equally probable, alleviating the problem with `eTimeOptTrack` that only the sedimentation rate with the highest r^2_{opt} is selected (Figure 1). Third, between the selected points, sedimentation rate changes linearly. These assumptions are incorporated by the `sed_rate_from_matrix` function, which takes the outputs of the `get_data_from_eTimeOpt` function and returns a sedimentation rate factory as required by ICON.

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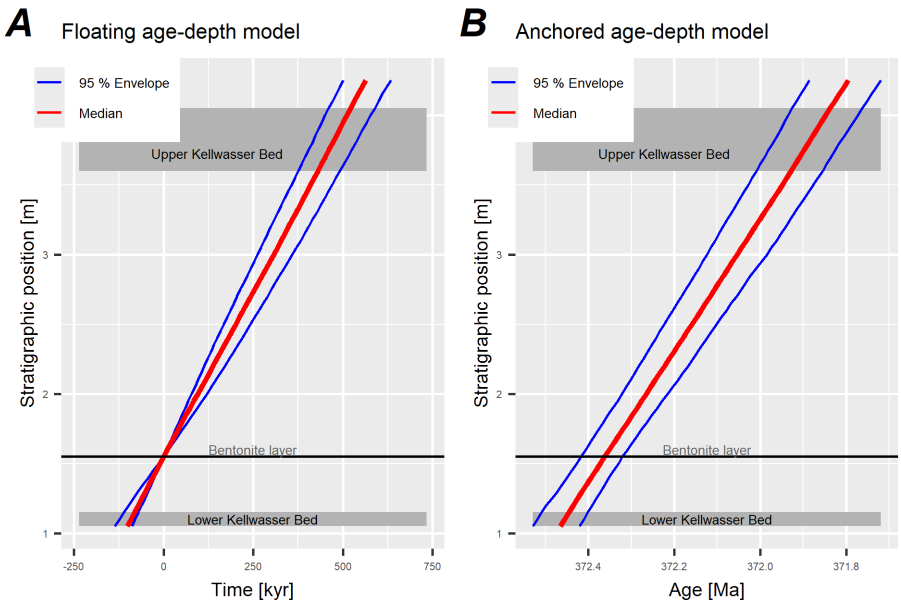

Figure 2. Floating age-depth model (A) and anchored age-depth model (B) for the Steinbruch Schmidt section based on `eTimeOpt` results testing for precession amplitude modulation. In the floating age-depth model, time is measured relative to the bentonite layer dated by Percival et al. (2018), and all uncertainties in the age-depth model arise from the uncertainties of the sedimentation rates estimated using `eTimeOpt`. The anchored age-depth model is constructed by combining the absolute age of Percival et al. (2018) for the bentonite layer with the sedimentation rates estimated from eTimeOpt. Uncertainties result from both the uncertainties in U-Pb dating and estimates of sedimentation rates using `eTimeOpt`.

To distinguish between uncertainties introduced by the U-Pb date and those from `eTimeOpt`'s estimated sedimentation rates, we construct two age-depth models: One floating age-depth model that measures time relative to the dated bentonite layer (Figure 2 A), and one anchored in absolute time, where the bentonite layer is assigned the age determined by Percival et al. (2018) (372.36 plus minus 0.053 Ma, considering only measurement uncertainties) (Figure 2 B). For this example, we choose a rate parameter $\lambda = 3$ for the sedimentation rate, and use 1000 Monte Carlo samples to generate age-depth models, and report results rounded to the next kyr.

3.1.1 Results

The floating time scale displays that age uncertainty increases away from the tie point (Figure 2 A), resulting in the distinct sausage-shape described by De Vleeschouwer and Parnell (2014). Median time contained in the 2.5 m between the bentonite layer and the Frasnian-Famennian boundary is 513 kyr with an interquartile range (IQR) of 56 kyr and a standard deviation (SD) of 43 kyr based on the uncertainty of the sedimentation rate (Figure 3). For the anchored age-depth model, the 95 % envelope is almost parallel (Figure 2 B). This is because the uncertainties arising from the U-Pb date and the sedimentation rate estimates are independent as a result sub-additive: It is unlikely that both yield too low (resp. high) values simultaneously, so

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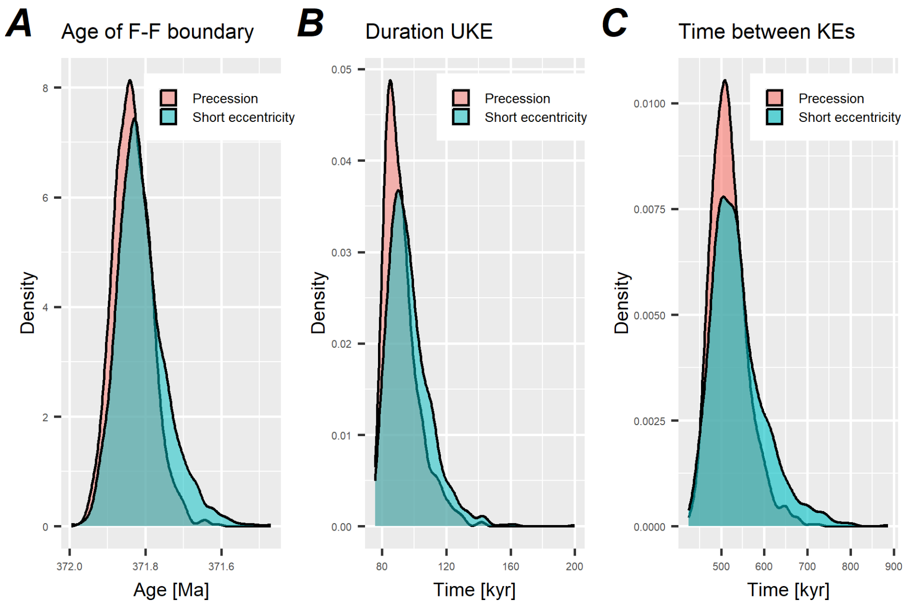

Figure 3. Durations of the Frasnian-Famennian boundary (A), Upper Kellwasser Event (B), and time elapsed between the Kellwasser Events (C) based on testing for short eccentricity modulation and precession amplitude modulation.

their combined uncertainty is lower than the sum of their uncertainties. This indicates that over the observed, stratigraphically short interval, the error introduced by uncertainty in sedimentation rates is negligible relative to the uncertainty of absolute dates. Note that the uncertainty of sedimentation rates is limited by the maximum and minimum sedimentation rates passed to `eTimeOpt` and the averaging within the sliding window, therefore by the window size. In our case the sedimentation rates were constrained a priori to 0.1 to 0.6 cm/kyr as in Da Silva et al. (2020). If a wider range of sedimentation rates were analyzed, sedimentation rates would contribute to more uncertainty to the age-depth model.

For the Frasnian-Famennian boundary, our anchored age-depth model yields a median age of 371.837 Ma, with an IQR of 67 kyr (1st and 3rd quartile is 371.803 and 371.867 Ma, respectively). The age distribution is approximately normal, resulting in an age estimate of 371.834 $\pm$ 0.101 Myr in the standard 2σ representation (Figure 3). This is 36 kyr older than the age estimated by Da Silva et al. (2020), with age uncertainty reduced by 6 % (108 kyr vs. 101 kyr). Our uncertainty is lower than that listed by Da Silva et al. (2020), as they use multiple proxies, estimate one deterministic sedimentation rate per proxy, and combine these estimates to arrive at their final uncertainty. We use a single proxy (MS) to estimate uncertain sedimentation rates, and propagate these uncertainties into the age estimate. Combining uncertain sedimentation rate estimates from multiple proxies would most likely give uncertainties comparable or larger than 108 kyr, as adding sources of uncertainties can only increase the uncertainty of the final estimate. Becker et al. (2020) in Gradstein (2020) lists an age of 371.1 Myr $\pm$ 1.1 Myr (2σ) for the Frasnian-Famennian boundary. Both our and Da Silva et al. (2020) mean age estimates are elevated compared to this (by 734

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and 770 kyr, respectively), but with reduced uncertainties (108 and 101 kyr, respectively) relative to the global geological time scale.

Median duration of the Upper Kellwasser Event, stratigraphically expressed as black shale intervals, is 92 kyr (1st and 3rd quartile: 84 to 97 kyr, IQR 13 kyr) and is positively skewed, matching the duration estimate of approx. 90 kyr by Da Silva et al. (2020). The median time elapsed between the Kellwasser events, measured from the top of the Lower Kellwasser Event to the bottom of the Upper Kellwasser Event, is 513 kyr (1st and 3rd quartile: 489 and 542 kyr, IQR: 54 kyr), and is also positively skewed (Figure 3). Note that duration estimates are identical whether they are derived from the floating or the anchored age-depth model, and depend solely on sedimentation rates. This is because the uncertainties of the tie point cancel out when calculating durations, allowing us to obtain duration estimates with uncertainties below those of the of the U-Pb dates by Percival et al. (2018). This shows that even in the absence of absolute ages, floating age-depth models are a powerful tool to determine durations and relative timing of events in deep time.

Neither the duration of the Lower Kellwasser Event not the time from the onset of the Lower Kellwasser Event to the onset of the Upper Kellwasser event could be estimated with the applied approach. This is because the moving window approach in `eTimeOpt` does not provide sedimentation rate estimates for the lowest and highest parts of the section, of which the former contains the onset of the Lower Kellwasser Event. These durations could be estimated by using a narrower window in `eTimeOpt`, however this would make the results not comparable directly to those of Da Silva et al. (2020).

Wichern et al. (2024) provide an estimated duration of the Lower Kellwasser Event of approx. 250 kyr at the nearby Winsenberg section. Combined with our estimates of 514 kyr between end of the Lower Kellwasser Event and onset of the Upper Kellwasser Event, the estimated time between the onset of both events is approximately 764 kyr, and the total duration of the Kellwasser crisis is approximately 853 kyr, slightly shorter than that estimated by Wichern et al. (2024).

3.1.2 Robustness of Age-Depth Models for the Paleocene-Eocene Thermal maximum

The Paleocene-Eocene Thermal Maximum (PETM) is a short interval of global carbon cycle perturbation associated with climate warming approximately 56 Myr ago (Vahlenkamp et al., 2020; Sluijs et al., 2007). Multiple causes have been proposed, including organic matter oxidation, enhanced volcanism, dissociation of gas hydrates, and methane release from vent systems (Dickens et al., 1995; Kurtz et al., 2003; Frieling et al., 2016; Storey et al., 2007). The PETM is a potential geological analogue for anthropogenic climate change (Carmichael et al., 2017; Haywood et al., 2011), making it crucial to understand the timing of its onset and recovery.

The Ocean Drilling Program (ODP) Site 690 contains both onset, peak, and recovery intervals of the PETM, making it a prime candidate to examine timing and pacing of the event. Multiple disparate age-depth models for this site have been proposed. Röhl et al. (2000), Norris and Röhl (1999) and Röhl et al. (2007) used cyclostratigraphy and correlation to other sites to determine age constraints, whereas Farley and Eltgroth (2003) used measurements of extraterrestrial ^3He coupled with the assumption of a constant extraterrestrial ^3He flux into the sediment to estimate sedimentation rates and an age-depth model. While the variability of ^3He fluxes over geologically short timescales is low, it is known that the ^3He flux can vary by about

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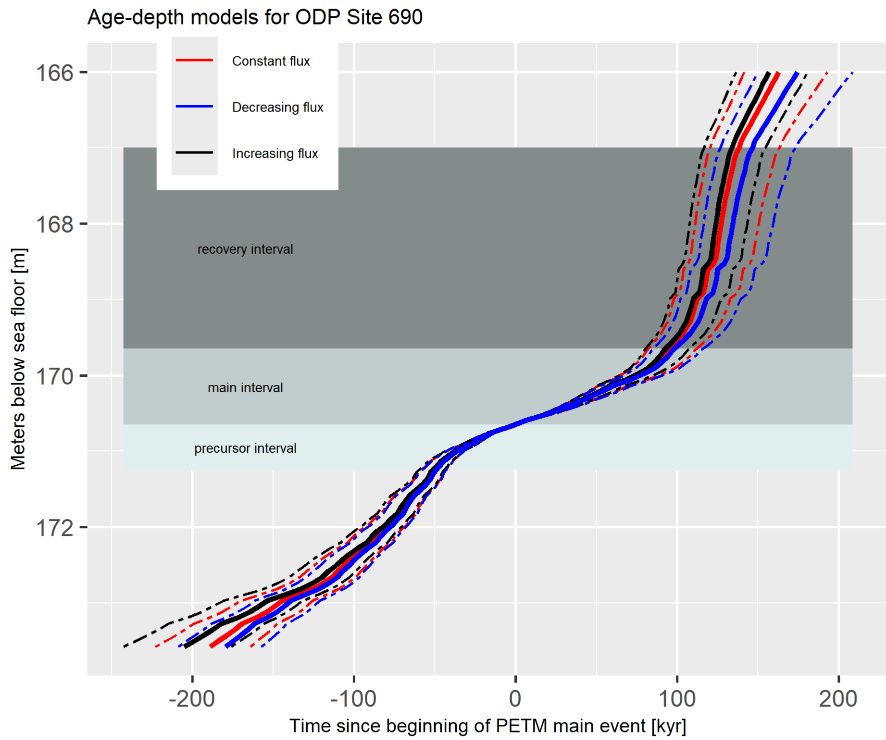

Figure 4. The three age-depth models derived under the assumption of constant flux (black line), increasing flux (red line) and decreasing flux (blue line). Thick lines are median age over 1000 replicates, dashed lines are the 95 % envelope of the ages.

450 an order of magnitude over geologically long time scales (Farley, 2001, Takayanagi and Ozima (1987)). Here, we use FAM to examine how robust the age-depth model for the PETM proposed by Farley and Eltgroth (2003) is to variability in ^{3}He fluxes.

As stratigraphic points of reference, we use the PETM pre-, main- and recovery intervals as defined by Farley and Eltgroth (2003), and use the raw data published in Farley and Eltgroth (2003). We measure time relative to the beginning of the main interval to construct a floating age-depth model for the section, matching the time scale shown in Fig. 4 of Farley and Eltgroth
455 (2003) . While this does not give an absolute age of the PETM, it provides absolute durations of the PETM intervals defined above. For the constant flux value, we use the value of 0.69 pcc ($1pcc = 10^{-12}cm^3$ of He at STP (standard temperature and pressure)) per cm^2 per kyr calculated by Farley and Eltgroth (2003) . We assume this value is a mean value, and the actual flux follows a normal distribution with this mean and a standard deviation of 0.05, reflecting the 15 % 2σ uncertainty of the flux estimate given by Farley and Eltgroth (2003) . For the increase and decrease, we assume that over an interval of 1 Myr, the flux
460 values increases or decreases by a factor of two. We place the beginning of the PETM main interval in the middle of this time interval. We ran the analysis using 1000 Monte Carlo simulations, and give results rounded to the next kyr.

All three age-depth models show clear signs of sedimentary condensation and dilution over the PETM interval, as expressed by their changing slopes (Figure 4). Both the pre and the main interval are strongly condensed relative to the recovery interval

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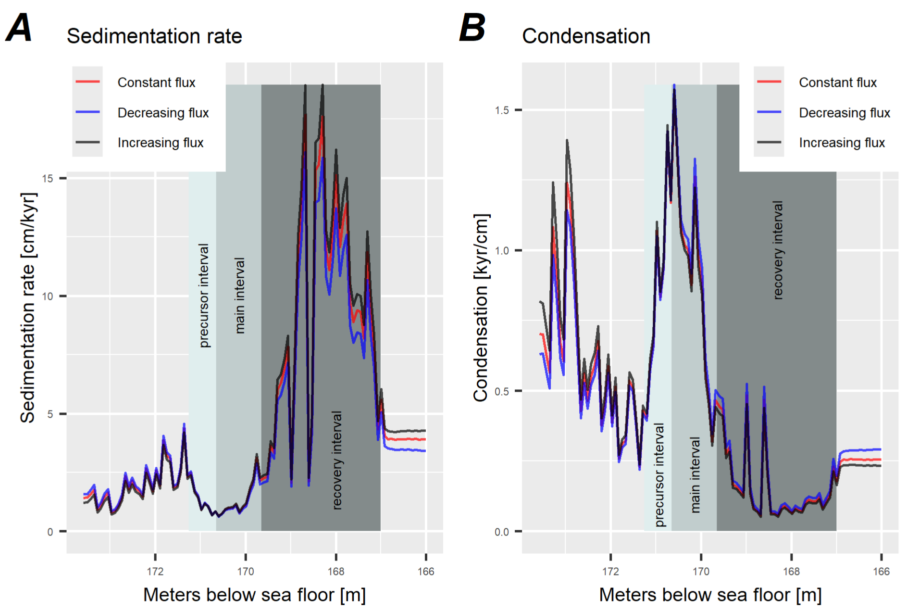

Figure 5. Median sedimentation rate and condensation in the stratigraphic domain for the three scenarios defined

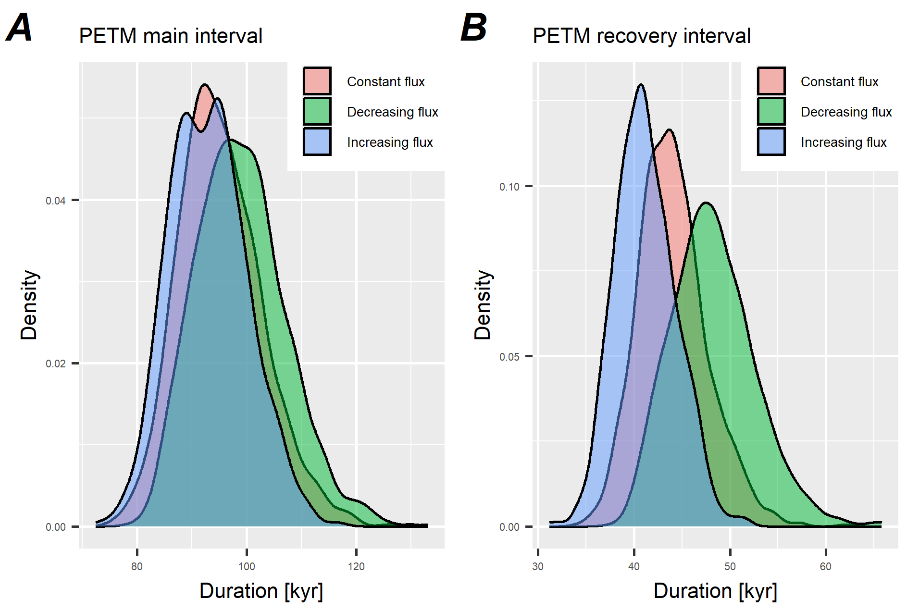

Figure 6. Density of the estimated duration of the PETM main and recovery interval

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(, Figure 5). This is clearly displayed by the recovery interval, which is 2.65 times thicker than the main interval (2.65 m vs. 1 m), but more than 50 % shorter than the main interval when measured in time (median values: 55, 53, and 51 % increasing, constant, and decreasing flux scenario). Median duration of the PETM main interval is 93, 94, and 98 kyr for the increasing, constant, and decreasing flux scenarios, with comparable interquartile ranges (10, 10, and 11 kyr) (Figure 6 A). The recovery interval is short compared to the main interval, with a median duration of 41, 44, and 48 kyr for the increasing, constant, and decreasing flux scenario, and comparable interquartile ranges (4, 5, and 4 kyr, respectively) (Figure 6 B).

Over the 4.25 m of the examined PETM intervals, sedimentation rates vary by a factor of more than 25 for all three scenarios. The range of sedimentation rates is 0.6 to 18.9 cm/ky for the increasing, 0.6 to 18 cm/ kyr for the constant, and 0.6 to 16.0 cm/kyr for the decreasing flux scenario, with the lowest values at the transition of the pre to the main interval, and highest values in the recovery interval (Figure 5 A). Conversely, condensation ranges from 0.06 kyr/cm and 1.58 kyr/cm across the scenarios (Figure 5 B). Note that the sedimentation rate estimates are elevated by a factor of more than 5 in the recovery interval compared to cyclostratigraphic analyses (Röhl et al., 2007, Röhl et al. (2000)).

While there is no large offset in sedimentation rates and age-depth models between the scenarios, the varying fluxes generate systematic deviations from the constant flux scenario. Under increasing (decreasing) flux, sedimentation rates in the top of the examined interval are systematically higher (lower), leading to shortened (extended) durations of the recovery interval. For the bottom of the section, the effect is reversed, where increasing (decreasing) flux leads to extended (shortened) durations of the pre-interval. Overall, results are consistent with those of Farley and Eltgroth (2003) and show that they are robust with respect to variations in ^3He flux. Under increasing and decreasing fluxes, median PETM main interval durations differ by only 3 % from the constant flux scenario.

We conclude that variability of ^3He fluxes by a factor of two on the timescale below 1 Myr only have a weak effect on the age-depth model at ODP site 690 and the derived estimates of PETM duration. Using tracer fluxes to estimate age-depth models is a powerful tool to estimate high-resolution age-depth models and identify local variations in condensation and sedimentation rates.

3.2 Discussion

We have introduced two methods to estimate age-depth models from complex stratigraphic or sedimentological data. ICON used information on upsection change in sedimentation rate to estimate age-depth models, while FAM compares observed tracer values with assumptions on past tracer fluxed to constrain age-depth relationships. As examples, we have applied the new methods to constrain the timing and duration of the Frasnian-Famennian boundary and the Upper Kellwasser Event with sedimentation rates constraints from cyclostratigraphy (Da Silva et al., 2020), and examined the robustness of age-depth models for the Paleocene-Eocene Thermal Maximum (PETM) to variations in tracer fluxes (Farley and Eltgroth, 2003).

3.2.1 Comparison with other methods

Bayesian approaches such as `Bchron`, modified `Bchron`, `Bacon` and `OxCal` are common among the current methods to estimate age-depth models (Blaauw and Christen, 2011; Trayler et al., 2019; Haslett and Parnell, 2008) (Table 1). Trachsel and

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Telford (2017) pointed out that Bayesian approaches perform well compared to "classic" approaches such as CLAM. FAM and ICON are not Bayesian, as they do not rely on prior or posterior distributions. Bayesian methods can become computationally expensive when parameter spaces are high-dimensional. As stratigraphic data gets more complex, the number of parameters and computation time increases. For example, the computations accompanying Trayler et al. (2023) publication of `astroBayes` will take "several days or weeks" to compute on a laptop. In contrast, the computation time for the examples show here is typically within minutes.

By imposing monotonicity constraints on sample paths, Bayesian methods can resolve age reversals and reduce age uncertainties at the tie points. In ICON and FAM, users decide how to resolve age reversals, and the uncertainties of age-depth models at tie points cannot be improved upon by the method. In this hierarchical design, information from between tie points cannot reduce uncertainties of the tie points. Information on sedimentation rates and tracer fluxes are typically either expressed on an ordinal scale or associated with high uncertainty. The hierarchical design reflects the idea that an absolute age (e.g. from U-Pb dating) can not be improved upon by information about the relative distribution of time (e.g. from changes in sedimentation rates) between tie points. Especially for changes in sedimentation rates estimated from cyclostratigraphy, this prevents circular reasoning, as the usage of external age constraints is recommended with cyclostratigraphic analysis (Sinnesael et al., 2019).

Assumptions on sediment accumulation employed by current methods to estimate age-depth models are often made for mathematical convenience (e.g. assume a Poisson structure). In fact, we do not know if sedimentary events follow a Poisson distribution across all timescales and depositional environments (e.g., Schumer and Jerolmack (2009)). Such assumptions do not reflect our understanding of the structure of the stratigraphic record and should rather be considered simplified and unspecific error models that reflect various degrees of pessimism about the the structure of the stratigraphic record. ICON and FAM improve upon this by explicitly relying on stratigraphic and sedimentological information to estimate age-depth models.

3.2.2 Limitations of this approach

In `admtools` v0.3.0 (Hohmann, 2024a), neither ICON nor FAM can incorporate information on gaps in the fossil record. Mathematically, it it possible to include gaps into ICON, and will be implemented in later versions of the package. For FAM, incorporation of gaps is possible in the edge case of constant assumed tracer flux. For more complex assumed tracer fluxes, it is not obvious to see whether gaps can be incorporated, as they remove parts of the signal the observed tracer is matched to. However, even with the option to incorporate hiatuses, hiatus durations and locations are notoriously challenging to estimate and justify empirically. Often times, the existence of hiatuses is invoked to explain a mismatch between signals of different sections. Hohmann et al. (2024) used forward simulations of carbonate platforms to show that gap duration and locations are functions of external drivers of stratigraphic architectures, and Meyers and Sageman (2004) used evolutive harmonic analysis to identify relatively short hiatuses (1 - 100 kyr) in a simulation study. These are just two examples showing that gaps locations and durations are not random, and can be identified with careful data analysis and forward models. Note that in a case where there are unidentified gaps in a section, the reconstructed sedimentation rate will always be lower than the "true" sedimentation rate, leading to a Sadler-type effect (Sadler, 1981).

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Table 1. Comparison of different methods to estimate age-depth models

Method	Bayesian	Information source	Developed for time scale	Reference
BChron	yes	tie points	radiocarbon	Haslett and Parnell (2008)
modified BChron	yes	tie points	deep time (borrows assumptions from BChron)	Trayler et al. (2019)
OxCal	yes	tie points, gaps, sed. rates	radiocarbon	Ramsey (2009)
Bacon	yes	tie points	radiocarbon	Blaauw and Christen (2011)
astroBayes	yes	tie points, breaks in sed. rate	deep time	Trayler et al. (2023)
CLAM	no	tie points	radiocarbon	Blaauw (2010)
ICON	no	tie points, sed. rate estimates	any	this publication
FAM	no	tie points, tracer values	any	this publication

An assumption made by all methods to estimated age-depth relationships is that a given stratigraphic position has a unique age, which might be unknown but can be assigned an uncertainty. Dating of organic remains in modern environments (e.g., Dominguez et al. (2016) or Tomašových et al. (2018)) shows that particle ages at a given location can differ significantly as a result of mixing in the surface mixed layer, and thus violate this assumption. This effect might differ across environments, but time-averaging in Holocene deposits can reach values of multiple thousands years (see e.g., Berensmeier et al. (2023)) and thus exceed the age uncertainty of age-depth models by orders of magnitude.

One key feature of FAM and ICON is that they are assumption-explicit in the sense that the user must specify all distributions that contribute to the age-depth model. While this requires additional coding effort, it results in an assumption-explicit age-depth model, where every assumption made is known and documented. As a result, uncertainties in age-depth models are a direct reflection of our understanding of the structure of the stratigraphic record. A second benefit is that age-depth model construction is replicable, even using different computational environments.

3.2.3 Uncertainties in age-depth modelling

For researchers using geohistorical data, age-depth models with low uncertainties are highly desirable. They allow for exact dating of events and provide low uncertainties of rates of past change, opening up the opportunity to draw use past climate perturbations as analogues for future climate change (Trayler et al., 2023). It is easy to construct an age-depth model with seemingly no uncertainties by simply connecting mean ages with a straight line (e.g., Tobin et al. (2012)). However, low uncertainties should not be the target to assess the quality of age-depth models. Trachsel and Telford (2017) found that CLAM underestimates age uncertainties in varved sediments and De Vleeschouwer and Parnell (2014) pointed out that in the Devonian timescale in Gradstein (2012), age uncertainties decreases between absolutely dated tie points - an unexpected and counter-intuitive behavior. Both examples demonstrate that there are empirical and logical lower bounds on uncertainties in age-depth models. We argue that the best age-depth model should not be the one with the lowest uncertainty, but the one that best

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reflects our understanding of and uncertainty about the structure of the stratigraphic record in the environment and timescale of interest. This poses a twofold challenge: First, understanding drivers of the local structure of the record, second, incorporating this information and and the uncertainty associated with it into age-depth models. When such age-depth models are used, timing and rates of past change might be associated with large uncertainties. However, these uncertainties will reflect our understanding of the structure of the stratigraphic record. Both FAM and ICON turn complex information into age-depth models. To make full use these methods, more research into this structure across environments, timescales, and its external controls are required.

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5 Conclusions

We have introduced two new methods, FAM and ICON, to estimate age-depth models from complex stratigraphic and sedimentological data. FAM compares observed tracer values with assumptions of fluxes in the time domain, while ICON uses information on changes in sedimentation rates (e.g., from cyclostratigraphy, sequence stratigraphy, or other expert knowledge) to construct age-depth models. Both methods are non-parametric in the sense that they do not make any a priori model assumptions other than that the law of superposition holds. Users can implement error models that reflect their knowledge about the local drivers of stratigraphic architectures, resulting in age-depth models that are data driven rather than assumption driven.

Code availability. All code used for the examples is available in Hohmann (2024b) and can be accessed under https://doi.org/10.5281/zenodo.13639816. The `admtools` package used for the analysis is available on CRAN (the Comprehensive R Archive Network), with versions being archived on Zenodo (Hohmann, 2024a).

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Appendix A: Supplementary Figures

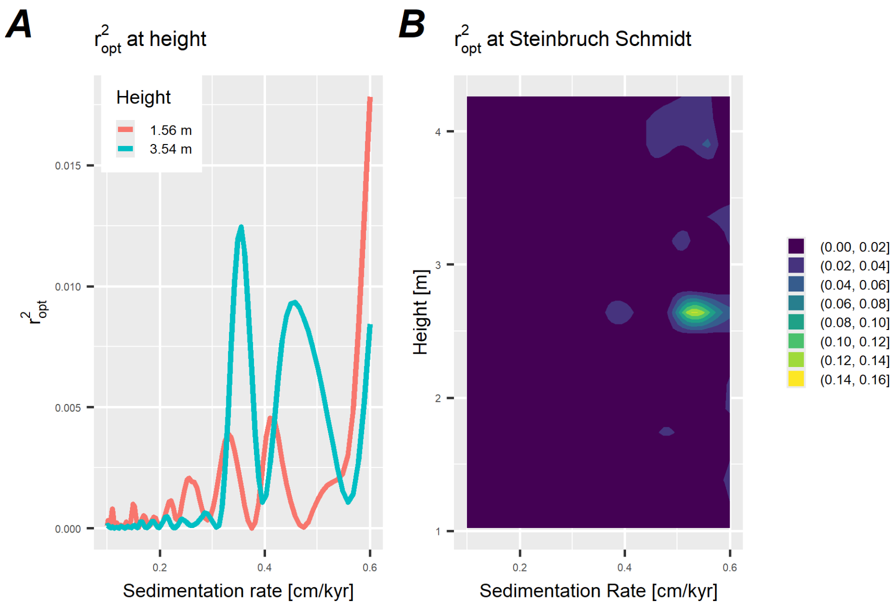

Figure A1. r^2_{opt} values from `eTimeOpt` testing for short eccentricity amplitude modulation at 3.54 m and 1.56 m (A) and throughout the whole section (B) in the Steinbruch Schmidt section. The heights shown in A roughly corresponding to the locations of the Upper Kellwasser Bed and the location of the dated ash bed.

Author contributions. Based on the CRediT taxonomy. **Niklas Hohmann**: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - Original draft, Writing - Review and editing, Visualization. **David De Vleeschouwer**: Conceptualization, Writing - Review and editing. **Sietske Batenburg**: Writing - Review and editing. **Emilia Jarochowska**: Funding acquisition, Project administration, Writing - Review and editing, Validation.

Competing interests. The authors declare no competing interests.

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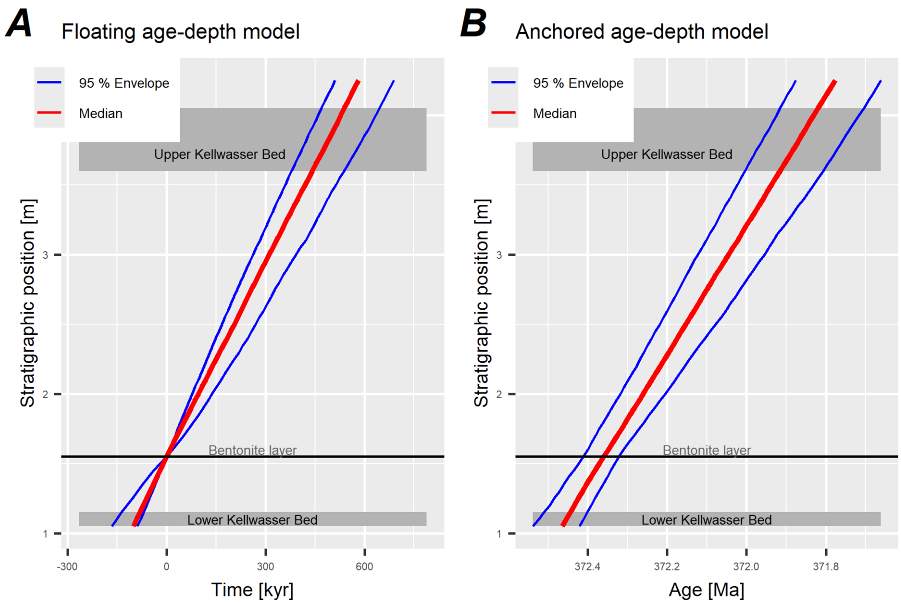

Figure A2. Floating age-depth model (A) and anchored age-depth model (B) for the Steinbruch Schmidt section based on `eTimeOpt` results testing for short eccentricity amplitude modulation. In the floating age-depth model, time is measured relative to the bentonite layer dated by Percival et al. (2018), and all uncertainties in the age-depth model arise from the uncertainties of the sedimentation rates estimated using `eTimeOpt`. The anchored age-depth model is constructed by combining the absolute age of Percival et al. (2018) for the bentonite layer with the sedimentation rates estimated from `eTimeOpt`. Uncertainties result from both the uncertainties in U-Pb dating and estimates of sedimentation rates using `eTimeOpt`.

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