Supplementary Material for "Mutltifractality of Climate Networks"

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1 TDMI Properties

The TDMI measure has the following interesting properties:

- Symmetry: $I\left(R_\lambda(\boldsymbol{x},t), R_\lambda(\boldsymbol{y},t+\tau)\right) = I\left(R_\lambda(\boldsymbol{y},t), R_\lambda(\boldsymbol{x},t-\tau)\right)$
- It measures the shared or redundant information between two time series, and is a generalization of the cross-correlation which can be used to estimate the time delay between processes (Mars and van Arragon, 1982).
- Non-negativeness $I\left(R_\lambda(\boldsymbol{x},t), R_\lambda(\boldsymbol{y},t+\tau)\right) \geq 0$, with equality when both time series have no dependence.
- $I\left(R_\lambda(\boldsymbol{x},t), R_\lambda(\boldsymbol{x},t+\tau)\right)$ is similar to the auto-correlation function, and equals to the entropy of the system at $\tau = 0$.

The rain rate time series is rank transformed and normalized to the unit interval as a data pre-processing step, since mutual information is invariant under it and this can reduce statistical errors in its estimation (Kraskov et al., 2004). Marginal and joint probability densities are computed using histograms with equiquantal binning (Kraskov et al., 2004; Haas et al., 2023; Cellucci et al., 2005).

References

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