

a third are filtered out, resulting in 2652 land grid points. Monthly precipitation data can contain zero values and in some cases very small negative numerical residuals. Therefore, for each ESM, a cut-off for quasi-zero is introduced by replacing zero and negative values with half of the smallest non-negative precipitation value found in the entire dataset. Data from five scenarios that represent combinations of Shared Socioeconomic Pathways (SSPs) and Representative Concentration Pathways (RCPs) are used, namely SSP1-1.9 (notation indicating the combination of SSP1 and RCP1.9), SPP1-2.6, SPP2-4.5, SPP3-7.0, and SSP5-8.5, and the historical simulations are considered (O'Neill et al., 2016). We refer to these SSP–RCP combinations as SSPs or scenarios. Not all 24 models provide temperature and precipitation data for each SSP (see Table A1 in Appendix A).

For each ESM, the emulator is trained independently, based on a single-ensemble member across all available SSPs. The historical simulation and the SSP5-8.5 scenario of the remaining ESM ensemble members are used for evaluating the emulator performance and are referred to as validation runs. When generating emulations from actual ESM data, we generate a single precipitation realisation for each available temperature field. Therefore, the number of emulations exactly equals the number of ESM runs. A special focus is put on the three models with the highest number of validation runs: ACCESS-ESM1-5 (Ziehn et al., 2019), CanESM5 (Swart et al., 2019), and MPI-ESM1-2-LR (Schupfner et al., 2021). These three models offer at least 30 ensemble members each, which allows us to compare ensemble statistics and, in particular, extreme event distributions. As an example, ACCESS-ESM1-5 has 40 ensemble members (see Table A1). We calibrate on ensemble member “r1i1p1f1” across scenarios to then generate 39 precipitation emulations across scenarios based on the gridded temperatures from the remaining 39 ensemble members.

MESMER-M-TP has been designed as a module that can be coupled to existing temperature emulators. To additionally evaluate the emulator performance and the propagation of uncertainties in this context, the trained emulator is coupled to emulated monthly temperatures of the historical simulation and the SSP5-8.5 scenario. The emulated temperature dataset was specifically generated for this study and is described in Appendix C1. We generate an ensemble of 100 temperature and precipitation realisations per model and scenario.

3.2 Calibration

The methodological framework described in Sect. 2 offers hyperparameters (see Sect. 2.5) for both the temperature-driven precipitation response module and the residual variability module. As part of the temperature-driven precipitation response, $P_{s,m}$ is reconstructed from information in the n -closest temperature time series, $\{T_{r,m}\}_{r \in S_{s,m}}$, with $|S_{s,m}| = n$. For simplicity and comparability, we assume that n is con-

stant across models, months, and grid points. Therefore, $S_{s,m}$ only depends on the spatial location and reduces to S_s . The choice of n is a trade-off between model complexity (for higher n , the PCA has more coefficients and takes longer to compute) and prioritising local modes of variability over large-scale/global relationships. We find that across months and models, the strongest correlations between the variability in temperature and the variability in precipitation occur in almost 80 % of the cases within the closest 150 grid points. Thus, we set $n = 150$, such that we can derive precipitation based on the 150 closest temperature locations. We have tested the approach for a variety of n and find that, across grid points and months, results for $n \in [75, 400]$ are comparable, while introducing larger n is too computationally intensive. We also tested using a single global decomposition by setting $n = 2652$, which leads to good results in some areas (e.g. North America) and performs poorly in other regions (e.g. Southeast Asia). As the set of $\{T_{r,m}\}_{r \in S_s}$ are highly correlated, we apply a PCA transformation prior to using them as independent variables for the GLM (see Sect. 2.3.2). The amount of explained variance in each PC decreases rapidly over the first five PCs and strives towards zero with an increasing component number. To include as much information as possible, while not inflating the model, we set $p = 8$. It is possible to include higher-order terms in the model, that is, to add $\mathbf{X}_{s,m}^2$ as a predictor or allow for interaction terms. We found that the model performance improved when we allow for first-order interaction terms between the trend in the first PC and all other PCs. The physical interpretation begins so that the relative importance of the PCs varies with the trend in local temperatures. Including additional terms had little effect on the model performance. Therefore, the calibrated model equation for the trend contribution to precipitation reads as follows:

$$f_{s,m} = e^{\beta_{s,m}^0} \quad \text{intercept} \\ \times e^{\beta_{s,m}^1 \times \text{PCA}_{s,m}^{0,\text{trend}} + \beta_{s,m}^2 \times \text{PCA}_{s,m}^{0,\text{var}} + \sum_{p=1}^7 \beta_{s,m}^{p+2} \times \text{PCA}_{s,m}^p} \quad \text{first order} \\ \times e^{\text{PCA}_{s,m}^{0,\text{trend}} \times (\beta_{s,m}^{10} \times \text{PCA}_{s,m}^{0,\text{trend}} + \beta_{s,m}^{11} \times \text{PCA}_{s,m}^{0,\text{var}} + \sum_{p=1}^7 \beta_{s,m}^{p+11} \times \text{PCA}_{s,m}^p)} \quad \text{interaction.} \quad (10)$$

Last, we set the parameters of the residual variability module. We apply a PCA on the precipitation residuals in order to resolve spatial correlations and treat the PCs independently. We keep 98 % of the variability in the original residual signals. The bandwidth of the KDE was chosen via k -fold cross-validation and was mostly constant across months and models. To reduce computational complexity, we have set $h_m = 0.1$ as a global parameter.

3.3 Validation

The validation framework consists of two steps: (1) evaluating the emulator's performance when it emulates precipita-