# Dissipation ratio and eddy diffusivity of turbulent and salt finger mixing derived from microstructure measurements

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Abstract. Eddy diffusivity is usually estimated by using the Osborn relation assuming a constant dissipation ratio of 0.2. In this study, we examine dissipation ratios and eddy diffusivities of turbulent mixing and salt finger mixing based on microstructure datasets. We find the dissipation ratio of turbulence Γ<sup>T</sup> is highly variable with a median value clearly greater than 0.2, which shows strong seasonal variation and decreases slightly with depth in the western equatorial Pacific, but obviously increases in vertical with depth in the midlatitude Atlantic. Γ<sup>T</sup> is jointly modulated by the Ozmidov scale to the
Thorpe scale ratio R<sub>OT</sub> and the buoyancy Reynolds number Re<sub>b</sub>, namely Γ<sup>T</sup> ∝ R<sub>OT</sub> <sup>-4/3</sup>·Re<sub>b</sub> <sup>1/2</sup>. The eddy diffusivity based on observed Γ<sup>T</sup> is larger than that estimated with 0.2, and presents a much stronger bottom enhancement. The eddy diffusivities of heat and salt for salt finger are calculated by two "analogical" Osborn equations; and their corresponding "effective" dissipation ratios Γ<sub>θ</sub><sup>F</sup> and Γ<sub>S</sub><sup>F</sup> are explored. Γ<sub>θ</sub><sup>F</sup> scatters over two orders of magnitude with a median value of 0.47, and is mostly linearly correlated with Γ<sub>S</sub><sup>F</sup> as Γ<sub>S</sub><sup>F</sup>≈5Γ<sub>θ</sub><sup>F</sup>. The density flux ratio for salt finger decreases sharply with density ratio R<sub>θ</sub> smaller than 2.4 but regrows to a larger value with R<sub>θ</sub> exceeding 2.4. The salt finger-induced eddy diffusivities become morealso increase with depth, with some being comparable orto even stronger than the mean turbulent diffusivities with depthones. This study highlights the influences of variable dissipation ratios and different mixing types on eddy diffusivity estimates, and should help further improvement of mixing estimate and parameterization.

#### 1 Introduction

25 Microscale turbulence in the ocean is patchy and intermittent. Compared with molecular diffusion, it mixes materials in a larger scale with a higher efficiency, playing a leading role in re-distributing heat (Pujiana et al., 2018), dissolved gases (Sabine et al., 2004), pollutants (Kukulka et al., 2016), nutrients and plankton (Whitt et al., 2017), thus shaping ocean general circulations and influencing bio-chemical processes in the ocean (Wunsch and Ferrari, 2004). These effects impact global environment and climate change (Jackson et al., 2008).

Due to these significant effects of microscale mixing, the outputs of ocean general circulation and climate models are deeply affected by the mixing intensity and variation (Jayne, 2009). Since the grid size is too coarse to resolve microscale processes, mixing parameterizations are mostly used as a proxy of turbulence effects in such models (Klymak and Legg, 2010). The proposing, verification and development of mixing parameterizations heavily rely on our perceptions of mixing intensity and spatiotemporal variation observed in the real ocean. Therefore, to accurately estimate eddy diffusivity based on observations has always been an unremitting pursuit of researchers. On one hand, many parameterization methods are developed and widely used to infer eddy diffusivity (e.g., GHP scaling, Gregg et al., 2003; MG scaling, MacKinnon and Gregg, 2003; and the Thorpe scale method, Dillon, 1982), thanks to abundant accumulation of traditional hydrographic observations. These methods yield a mediocre estimate based on fine-scale profiles of temperature and/or velocity with resolution significantly larger than microscale, and may have applicability problems induced by different mechanisms and hydrologic conditions (Mater et al., 2015). On the other hand, microstructure measurements provide a much more accurate estimate of turbulence behaviors (St. Laurent et al., 2012), although the amount of data is relatively small. With the development of observation technology and the advancement of instruments, microstructure data is experiencing a rapid growth. However, neither parameterizations nor microstructure measurements can directly provide eddy diffusivity values; what they infer is the dissipation rate of turbulent kinetic energy (TKE)  $\varepsilon$ . Assuming mixing is driven by turbulence, the eddy diffusivity of density is then estimated by the conventional Osborn relation,  $K_{\rho} = \frac{R_f}{1-R_f} \cdot \frac{\varepsilon}{N^2}$  with  $R_f/(1-R_f) = \Gamma^T = 0.2$  (e.g., St. Laurent et al., 2012), where  $R_f$  is flux Richardson Number,  $N^2$  is buoyancy frequency squared, and  $\Gamma^T$  is the dissipation ratio of turbulence.

However, there are two inadequacies in the application of the Osborn relation. First, the value of  $\Gamma^T$  should be carefully inspected. In the frame of steady, homogeneous turbulence, a balance between TKE production (P), buoyancy flux (B) and dissipation can be reached,  $P+B-\varepsilon=0$ . And  $\Gamma^T$  is the ratio of the buoyancy flux to the dissipation,  $B/\varepsilon$ , which describes the relative proportion of how much TKE is converted to potential energy and irreversibly dissipated to heat. Combining limited measurements with theoretical prediction, Osborn (1980) took the critical value of  $R_f$  as  $R/\le 0.15$ , resulting in  $K_\rho < 0.2\varepsilon/N^2$ . Following that,  $\Gamma^T$  is usually taken as a constant of 0.2. Eddy diffusivities of heat  $(K_\theta)$ , salt  $(K_S)$  and density are equal for turbulent mixing, so these diffusivity values can be easily determined by the Osborn relation as long as  $\Gamma^T$  is accurately measured.  $\Gamma^T \approx 0.2$  is confirmed to be reasonable by some observations (Gregg et al., 2018); however, besides findings from laboratory experiments and direct numerical simulations (Barry et al., 2001; Jackson and Rehmann, 2003; Shih et al., 2005; Salehipour et al., 2016), there are considerable and accumulating observational evidence indicating  $\Gamma^T$  is significantly variable in both space and time, with a variation range covering several orders of magnitude, typically from  $10^{-2}$  to  $10^1$  (Moum, 1996; Smyth et al., 2001; Mashayek et al., 2017; Ijichi and Hibiya, 2018; Monismith et al., 2018; Vladoiu et al., 2021; Li et al., 2023).

Observations conducted in different regions showed the statistical feature of  $\Gamma^T$  is significantly distinct from region to region, and the repeated measurements at some locations suggested  $\Gamma^T$  is obviously greater than 0.2 (Ijichi and Hibiya, 2018), indicating taking  $\Gamma^T$ =0.2 could significantly underestimate eddy diffusivity in these regions. Besides, microstructure

measurements from both upper layer and the whole water column suggested  $\Gamma^T$  generally increases with depth, by as much as an order of magnitude (Ijichi and Hibiya, 2018; Li et al., 2023). Thus, taking  $\Gamma^T$  as a constant also leads to an underestimate of eddy diffusivity in the deep layer. These underestimated eddy diffusivities may be a part of the answer to "the missing mixing" puzzle (Wunsch and Ferrari, 2004). Some studies do show that the magnitude and pattern of  $\Gamma^T$  plays a key role in regulating global ocean general circulation (Mashayek et al., 2017; Cimoli et al., 2019). Moreover,  $\Gamma^T$  is reported to be modulated by turbulence features and is closely correlated with several parameters describing turbulence state, such as turbulence "age"  $R_{OT}$  (the ratio of the Ozmidov scale to the Thorpe scale; Ijichi and Hibiya, 2018) and turbulence "intensity"  $Re_b$  (buoyancy Reynolds number; Mashayek et al., 2017). However, different correlations between  $\Gamma^T$  and these parameters are found in different regions. Taking  $Re_b$  as an example, different studies concluded that their relation could be negatively correlated (Monismith et al., 2018), nonmonotonically correlated (Mashayek et al., 2017), or uncorrelated (Ijichi and Hibiya, 2018). In a word, taking  $\Gamma^T$  as a constant of 0.2 brings a large bias into eddy diffusivity estimate, yet our limited understanding prevents us from assigning a reasonable value for  $\Gamma^T$ .

The other inadequacy involves the driving mechanism of mixing. Although turbulent mixing dominates ocean mixing, there are considerable mixing events caused by the release of potential energy due to unstable temperature or salinity stratification (while the density stratification is stable), that is, double diffusion (Schmitt, 1994). Double diffusion has two manifestations, salt finger and diffusive convection. The former is associated with warmer, salter water overlying colder, fresher water; and the latter corresponds to the opposite scenario. Due to their unique requirements of vertical structures for temperature and salinity, diffusive convection is mostly prominent in the polar and subpolar regions, while salt finger prevails in the tropics and sub-tropical regions (van der Boog et al., 2021); and salt finger is our focus in this study. For the importance of salt finger mixing, analysis of global thermohaline staircase indicated salt finger only contributes a small fraction of the required energy to sustain mixing (van der Boog et al., 2021); however, not all salt finger events present staircases (St. Laurent and Schmitt, 1999), and the regional effects of salt finger mixing can be much profound (Fine et al., 2022). Some studies suggested salt finger mixing is significant when turbulent mixing is weak, while others suggested salt finger and turbulence can co-exist and interact with each other (Ashin et al., 2023). Unlike turbulent mixing, salt finger mixing, supplied by the release of potential energy, acts to strengthen the density stratification with a negative value of  $K_0$ . With P being negligible, the balance between B and  $\varepsilon$  leads to  $R_f(1-R_f)=-1$ , and hence  $K_g=-\varepsilon/N^2$  is applied to salt finger (McDougall, 1988). Therefore, if the mixing mechanism is not identified clearly, the conventional Osborn relation can estimate neither the correct sign nor the accurate magnitude of eddy diffusivity of density for salt finger mixing. Besides, the eddy diffusivities of heat, salt and density for salt finger mixing are inequivalent, namely  $K_{\theta} < K_{S}$  (Schmitt et al., 2005). Therefore,  $K_{\theta}$  and  $K_{S}$  for salt finger mixing cannot be estimated by the Osborn relation; and they can be calculated by a different manner involving the dissipation ratio  $\Gamma^{\rm F}$  (note that  $\Gamma^{\rm F}$  for salt finger is equivalent to  $-K_{\theta}/K_{\theta}$  instead of  $R_{\theta}/(1-R_{\theta})$ ; St. Laurent and Schmitt, 1999), density ratio  $R_{\varrho}$  (describing the relative contributions of temperature and salt to density) and density flux ratio r (the ratio of vertical heat flux to vertical salt flux) (see Section 2.3).

To overcome the shortcomings mentioned above, we turn to open microstructure datasets (Section 2), to first identify salt finger mixing from turbulent mixing (Section 3). Then, we explore the variability of  $\Gamma^{T}$  for turbulent mixing (Section 4.1), and examine  $\Gamma^{F}$  and the relation between  $R_{\rho}$  and r for salt finger mixing (Section 4.2). We also derive diffusivities  $K_{\rho}$ ,  $K_{\theta}$  and  $K_{S}$  and analyze them for both turbulent mixing and salt finger mixing (Section 5). A summary is given in Section 6.

#### 00 2 Data and Methods

#### 2.1 Data

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We first thank the Climate Process Team for publicly sharing the "Microstructure Database" (MacKinnon et al., 2017). The data used in this study are selected from the shared microstructure sampling projects covering global oceans. Since the calculation of dissipation ratio requires the dissipation rate of thermal variance ( $\chi_\theta$ ), and the vertical gradients of temperature  $\theta$  and salinity S are needed, we chose all five projects that provide this variable  $\chi_\theta$  and are in the form of vertical profiles. Besides  $\chi_\theta$ ,  $\theta$  and S, we also use  $\varepsilon$ , temperature  $\theta$  and salinity S, which have all been standardized to the same vertical grid for each project. The locations, operating period, etc. of the five projects are given in Table 1 and Fig. 1. The MIXET projects are performed in the western equatorial Pacific, while the BBTRE and NATRE are conducted in the Atlantic between 40°S and 40°N. Salt finger is always active in the mid-to-low latitudes of the Atlantic, while its occurrence in the Pacific shows strong temporal variation (Oyabu et al., 2023). These data provide a great opportunity to investigate the spatial-temporal variation of dissipation ratio and eddy diffusivity induced by turbulent mixing and salt finger mixing.

Table 1. Information on the projects used in this study.

Project	Location	Period	Profile Number of Profiles	Vertical Resolution (m)
MIXET1	156°E, 0°-2°N	04.20-05.14, 2012	51	1
MIXET2	156°E, 0°-5°N	10.25-11.18, 2012	101	1
BBTRE96	10°-30°W, 12°-26°S	01.22-02.27, 1996	74	0.5
BBTRE97	15°-40°W, 10°-26°S	03.13-04.18, 1997	89	0.5
NATRE	20°-30°W, 24°-27° <mark>S</mark> N	03.25-04.22, 1992	150	0.5

MIXET: MIXing in the Equatorial Thermocline (Waterhouse et al., 2014; Richards et al., 2015)

BBTRE: Brazil Basin Tracer Release Experiment (Polzin et al., 1997)

115 NATRE: North Atlantic Tracer Release Experiment (St. Laurent and Schmitt, 1999; Polzin and Ferrari, 2004)

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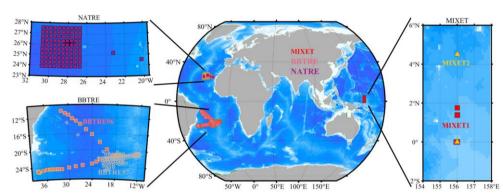


Figure 1: Station locations of the projects used in this study.

#### 2.2 Identifying turbulent mixing and salt finger mixing

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The profiles are divided into half-overlapped patches for further analysis. Following St. Laurent and Schmitt (1999), we choose 10 times of the vertical resolution as patch size, that is, 10 m (5 m) for projects with vertical resolution of about 1 m (0.5 m). We first examine if and which type of double diffusion is favorable for each patch in thermodynamical sense by the Turner angle, Tu=atan<sup>-1</sup>( $\alpha\theta_z$ - $\beta S_z$ - $\alpha\theta_z$ + $\beta S_z$ ) (Ruddick, 1983). Here,  $\alpha$  and  $\beta$  are the thermal expansion and saline contraction coefficients, respectively;  $\theta_z$  and  $S_z$  are the vertical gradients of the original temperature and salinity profiles, respectively; and "atan-1" is the four-quadrant inverse tangent. Tu varies between -180° and 180°, dividing water column into four thermodynamical regimes: doubly stable (|Tu|<45°), salt finger favorable (45°<Tu<90°), diffusive convection favorable (-90°<Tu<-45°), and gravitationally unstable (|Tu|>90°) (Ruddick 1983). Tu is related to density ratio  $R_\rho$  by  $R_\rho$ =-tan(Tu+45°). We exclude weak double diffusion signals (45°<Tu<60° for salt finger favorable and -60°<Tu<-45° for diffusive convection favorable) for further identification.

Besides the specific thermodynamical precondition, distinct statistical features are presented when double diffusion-induced mixing is dominant. First, Re<sub>b</sub> is found to be no greater than O(10) for active double diffusion (Inoue et al., 2007), and salt finger is rare for Re<sub>b</sub> between 10 and 10<sup>4</sup> (St. Laurent and Schmitt, 1999). Re<sub>b</sub> is defined as Re<sub>b</sub>= $\varepsilon/\nu N^2$ , where  $\nu$  is molecular viscosity coefficient. Moreover, double diffusion generally corresponds to elevated  $\chi_{\theta}$  (St. Laurent and Schmitt, 1999; Inoue et al., 2007), and the magnitude of  $\chi_{\theta}$  is significantly larger than  $\varepsilon$  when double diffusion prevails and turbulence is absent (Nagai et al., 2015). Therefore, we use Re<sub>b</sub><25 and  $|\chi_{\theta}|/|\varepsilon| \ge 7$  as additional criteria for the identification of double diffusion.

5 For doubly stable and gravitationally unstable water column, since their thermodynamical condition excludes the existence of double diffusion, we assume the mixing within the column is uniquely induced by turbulence only. The most prominent difference between turbulence patches with |Tu|<45° and those with |Tu|>90° is that Re<sub>b</sub> of the former is significantly smaller

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than that of the latter. And  $|Tu| > 90^{\circ}$  generally means the presence of overturns. Therefore, the former patches are grouped as "weak turbulence", and the latter are "energetic turbulence".

Based on Tu,  $Re_b$  and  $|\chi_\theta|/|\epsilon|$ , we classify the dominant mixing mechanisms into four types: weak turbulence ( $|Tu|<45^\circ$  with small  $Re_b$ ), energetic turbulence ( $|Tu|>90^\circ$ \_with large  $Re_b$ ), salt finger ( $60^\circ < Tu < 90^\circ$ ,  $Re_b < 25$  and  $|\chi_\theta|/|\epsilon| \ge 7$ ), and diffusive convection ( $-90^\circ < Tu < -60^\circ$ ,  $Re_b < 25$  and  $|\chi_\theta|/|\epsilon| \ge 7$ ). Diffusive convection prevails mostly in the polar and subpolar regions (van der Boog et al., 2021); thus, it is rarely identified in this study (Section 3). As a result, diffusive convection is excluded from further analysis.

## 145 2.3 Estimating dissipation ratios and eddy diffusivities for turbulent mixing and salt finger mixing

Assuming steady and homogenous state, the production dissipation balances for TKE (Osborn, 1980) and thermal variance (Osborn and Cox, 1972) are valid for both turbulence and salt finger (St. Laurent and Schmitt, 1999; Inoue et al., 2007),

$$\left(1 - R_{\mathcal{F}}\right)K_{\rho}N^2 - R_{\mathcal{F}}\varepsilon = 0,\tag{1}$$

$$2K_{\theta}\theta_{z}^{2} - \chi_{\theta} = 0. \tag{2}$$

150 Define a general form of dissipation ratio  $\Gamma$  as  $\frac{\chi_{a}N^{2}}{2r\theta_{a}^{2}}$  (Oakey, 1985), combining (1) and (2) yields

$$\Gamma = \left(\frac{R_{\mathcal{F}}}{1 - R_{\mathcal{F}}}\right) \frac{K_{\mathcal{B}}}{K_{\mathcal{F}}} = \left(\frac{R_{\mathcal{F}}}{1 - R_{\mathcal{F}}}\right) \left(\frac{R_{\mathcal{F}} - 1}{R_{\mathcal{F}}}\right) \left(\frac{r}{R_{\mathcal{F}}}\right) \left(\frac{r}{r - 1}\right) = \frac{\chi_{\mathcal{B}} N^2}{2e\theta_{\mathcal{F}}^2}$$

$$(3)$$

where density flux ratio  $r = \frac{\alpha K_B \theta_Z}{\beta K_S S_Z} = \frac{K_B}{K_S} \cdot R_{\rho}$ .

For turbulent mixing,  $\Gamma^{\mp} = R_f (1 - R_f)$ ; and the eddy diffusivities of heat, salinity and density for turbulent mixing are  $K_{\theta}^{\mp} = K_{\theta}^{\mp} = \Gamma^{\mp} \frac{e}{N^2}$ . Here, we use superscripts "T" and "F" to indicate turbulent mixing and salt finger mixing, respectively.

For salt finger mixing, with R<sub>p</sub>(1 R<sub>p</sub>)= 1, the eddy diffusivity of density can still be derived from the Osborn relation as K<sup>E</sup><sub>p</sub> = - <sup>s</sup>/<sub>N<sup>2</sup></sub> (McDougall, 1988), which is five times of the conventional Osborn relation estimate and has a negative sign. However, the eddy diffusivities of heat and salinity for salt finger mixing are more complex (Schmitt et al., 2005),

$$K_{\theta}^{E} = \Gamma_{\theta}^{E} \frac{e}{N^{2}} = \left(\frac{R_{p}-1}{R_{x}}\right) \left(\frac{e^{E}}{1-e^{E}}\right) \frac{e}{N^{2}} \tag{4}$$

$$K_{\mathcal{S}}^{\mathbf{F}} = \Gamma_{\mathcal{S}}^{\mathbf{F}} \frac{\varepsilon}{N^{2}} = \frac{R_{p}-1}{1-p^{2}} \frac{\varepsilon}{N^{2}}. \tag{5}$$

Note that (4) is actually  $K_{\theta}^{F} = \chi_{\theta}/2\theta_{\pi}^{2}$ , but in a form analogous to the Osborn relation. And these "analogical" Osborn relations for salt finger indicate the "effective" dissipation ratios for heat and salt are in different forms; but both are deeply related to the density flux ratio  $r^{F}$  and  $R_{\rho}$ ,  $r^{F}$  can be derived as  $R_{\rho} \frac{\chi_{\theta}N^{2}}{2\pi\theta_{\pi}^{2}} / \left(R_{\rho} \frac{\chi_{\theta}N^{2}}{2\pi\theta_{\pi}^{2}} + R_{\rho} - 1\right)$ , and then used to infer  $K_{\theta}^{F}$  and  $K_{S}^{F}$ .

Dissipation ratio  $\Gamma$  is defined as

$$\Gamma = \frac{\chi_{\theta} N^2}{2\varepsilon \theta_z^2} \tag{1}$$

for turbulent mixing and salt-finger mixing (Oakey, 1985). Based on the production-dissipation balances for TKE and thermal variance (Osborn and Cox, 1972; Osborn,1980), and introducing  $R_{\rho}$  and the density flux ratio  $r = \alpha K_{\theta} \theta_{\phi} / \beta K_{\phi} S_z = K_{\theta} / K_{\phi} S_z = K_{\theta} / K_{\phi} S_z = K_{\phi} / K$ 

$$\Gamma = \frac{\chi_T N^2}{2\varepsilon \theta_z^2} = \left(\frac{R_f}{1 - R_f}\right) \frac{K_\theta}{K_\rho} = \left(\frac{R_f}{1 - R_f}\right) \left(\frac{R_\rho - 1}{R_\rho}\right) \left(\frac{r}{r - 1}\right),\tag{2}$$

which is applicable to both turbulent mixing and salt finger mixing (St. Laurent and Schmitt, 1999).

For turbulent mixing only,  $K_S = K_\theta = K_\rho$ . Then, Eq. (2) leads to

$$\Gamma^{\mathrm{T}} = \frac{\chi_T N^2}{2\varepsilon\theta_Z^2} = \frac{R_f}{1 - R_f'} \tag{3}$$

<u>and</u>

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$$K_{\theta}^{\mathrm{T}} = K_{\mathcal{S}}^{\mathrm{T}} = \Gamma^{\mathrm{T}} \frac{\varepsilon}{N^{2}} \tag{4}$$

where superscript "T" indicates turbulent mixing.

However, for salt finger mixing only, with  $\lim_{p\to 0} \frac{R_f}{1-R_f} = -1$  (St. Laurent and Schmitt, 1999), Eq. (2) yields

$$\Gamma^{\mathrm{F}} = \frac{\chi_T N^2}{2\varepsilon\theta_x^2} = -\frac{K_\theta}{K_0} = -\left(\frac{R_\rho - 1}{R_0}\right) \left(\frac{r}{r-1}\right),\tag{5}$$

which cannot be used directly to estimate the salt finger induced eddy diffusivities. And they are estimated separately by introducing  $R_{\varrho}$  and  $r^{\rm F} = R_{\varrho}\Gamma^{\rm F}/(R_{\varrho}\Gamma^{\rm F} + R_{\varrho} - 1)$  (St. Laurent and Schmitt, 1999; Schmitt et al., 2005; Inoue et al., 2007),

$$180 K_{\theta}^{F} = \left(\frac{R_{\rho} - 1}{R_{\rho}}\right) \left(\frac{r}{1 - r}\right) \frac{\varepsilon}{N^{2}} = \Gamma_{\theta}^{F} \frac{\varepsilon}{N^{2}}, K_{S}^{F} = \frac{R_{\rho} - 1}{1 - r} \frac{\varepsilon}{N^{2}} = \Gamma_{S}^{F} \frac{\varepsilon}{N^{2}}. (6)$$

Note that all these equations are written into forms analogical to the Osborn relation for turbulent mixing.  $\Gamma_{\underline{\theta}}^F$  and  $\Gamma_{\underline{\delta}}^F$  are two artificial "mixing efficiencies", which are actually  $\binom{R_{\rho}-1}{R_{\rho}}\binom{r}{1-r}$  and  $\frac{R_{\rho}-1}{1-r}$  before " $\varepsilon/N^2$ " for  $K_{\underline{\theta}}^F$  and  $K_{\underline{\delta}}^F$  estimation.  $\Gamma_{\underline{\theta}}^F$  is the same as  $\Gamma^F$ , while  $\Gamma_{\underline{\delta}}^F$  are further derived based on  $R_{\underline{\theta}}$  and  $r^F$ ,  $\Gamma_{\underline{\delta}}^F = \Gamma^F \cdot R_{\underline{\theta}}/r^F$ . Investigating the statistic features of  $\Gamma_{\underline{\theta}}^F$  and  $\Gamma_{\underline{\delta}}^F$  can be practically useful when estimating  $K_{\underline{\theta}}^F$  and  $K_{\underline{\delta}}^F$  solely based on  $\varepsilon$  and  $N^2$ .

# 3 Statical features of turbulent mixing and salt finger mixing

Figure 2 suggests water properties vary greatly for the five projects, and Table 2 lists the proportions of patches for each mixing type. For the MIXET projects in the western equatorial Pacific, the Tu distribution in spring (MIXET1) shows a distinct shape from the autumn one (MIXET2). In spring, Tu shows double peaks at -30° and 110°, suggesting mixing is alternately dominated by weak and energetic turbulence, although the salt finger contribution accounts for 4.1% of the total patches and cannot be neglected. However, the autumn distribution is obviously unimodal, peaking at ~45°; and the dominant mixing types are first weak turbulence and secondly salt finger (~51.5% and 11.3%, respectively), with negligible energetic turbulence and diffusive convection. For the BBTRE projects, although they are conducted at different years, the

operating seasons are similar: one in late-summer and the other early-autumn (Southern Hemisphere), so the seasonal variation cannot be studied. Their Tu distributions are similarly bimodal, with a leading peak at 70° and a weak one at -40°, suggesting the waters are mostly salt finger-favorable (although only about 5.9% is confirmed to be salt finger) and stable (33.3%), with rare energetic turbulence and neglectable diffusive convection-favorable contribution. For the NATRE, salt finger overwhelms the others, occupying more than 21% of the total patches; weak and energetic turbulence together hold 13.3%, with the diffusive convection favorable still being negligible (1%). For these five projects, although almost half of the patches are salt finger favorable, only 9.7% of them shows For the BBTREs and NATRE, although a large proportion of the patches have  $45^{\circ} < Tu < 90^{\circ}$  and hence are salt finger favorable, most of them have elevated Re<sub>b</sub>; thus, we infer these mixing events as hybrids of salt finger and turbulence but dominated by turbulence. These patches are excluded from analysis to highlight the difference between turbulent and salt finger mixing. Only a few patches are chosen as effective salt finger events. Therefore, as concluded in the following Section 5.3, it is turbulent mixing that dominates the observed microstructures, in line with the results based on the NATRE (St. Laurent and Schmitt, 1999). For these five projects, only 9.7% of patches show clear salt finger features. Weak turbulence has a higher percentage (32.0%), followed by 6.6% of energetic turbulence. Diffusive convection occurs less than 0.5% of the total patches, and is therefore negligible. Although dominated by turbulent mixing, the rest of the patches, more than 50%, are hybrids of different mixing types, and are excluded from the analysis.

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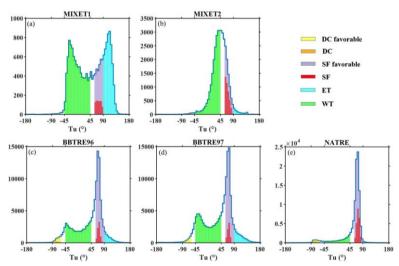


Figure 2: Histograms of patch-averaged Tu for different projects. Different Tu ranges of mixing types are marked by different colors: yellow for diffusive convection favorable (DC favorable; -90°<Tu<-60°), light purple for salt finger favorable (SF favorable;

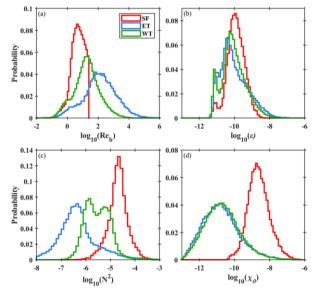
 $60^{\circ}$ <Tu<<90°), cyan for energetic turbulence (ET), and green for weak turbulence (WT). The red and orange bars denote the actual patch numbers number of patches of salt finger (SF) and diffusive convection (DC) selected by two more criteria,  $Re_b$ <25 and  $|\chi_0|/|\epsilon| \ge 5$ , respectively.

Table. 2. Proportions of patches with energetic turbulence, weak turbulence, salt finger, and diffusive convection and hybrid mixing types (turbulence and salt finger, or turbulence and diffusive convection) to the total patch-number of patches for each project, and the sums for all the projects, Patches with hybrid mixing types are excluded from analysis.

		•	Proportion (%)	•	_
	energetic turbulence	weak turbulence	salt finger	diffusive convection	excluded hybrid
MIXET1	29.56	47.05	4.11	0.06	19.22
MIXET2	2.48	51.48	11.32	0.16	<u>34.56</u>
BBTRE96	6.55	33.31	5.91	0.53	<u>53.70</u>
BBTRE97	8.67	38.19	4.56	0.08	48.50
NATRE	1.10	12.21	21.95	1.09	63.65
All	6.60	32.00	9.70	0.46	51.24

We compare the statistical differences of Re<sub>b</sub>,  $\varepsilon$ ,  $N^2$ , and  $\gamma_{\theta}$  for energetic turbulence, weak turbulence and salt finger by considering all the patches from the five projects (Fig. 3). The salt finger patches are featured with the weakest turbulence intensity compared with weak and energetic turbulence patches, whose median Re<sub>b</sub> are 5.0, 18.2 and 132.7, respectively. The median Re<sub>h</sub> of energetic turbulence is slightly smaller than that reported in Mashayek et al. (2017) but close to the result of Ijichi and Hibiya (2018). Since the samples given here are from five different projects, their Re<sub>b</sub> distributions are actually 225 different: For MIXET projects, the median Reb of energetic turbulence is small, only about 50; while the rest projects generally have a median Re<sub>b</sub> around 200 for energetic turbulence. The variations of  $\varepsilon$  for different mixing types differ little, mostly ranging from  $3\times10^{-12}$  to  $3\times10^{-8}$  W kg<sup>-1</sup>. Although the median  $\varepsilon$  for energetic turbulence is not obviously different from those for weak turbulence and salt finger (7.8×10<sup>-11</sup>, 7.9×10<sup>-11</sup> and 1.1×10<sup>-10</sup> W kg<sup>-1</sup>, respectively), it should be noted that most large  $\varepsilon$  values are induced by energetic turbulence. Distributions of  $\chi_{\theta}$  of weak turbulence and energetic turbulence differ little, but  $\chi_{\theta}$  of salt finger is clearly greater, in terms of variation ranges (salt finger:  $3\times10^{-11}$ - $10^{-7}$  °C<sup>2</sup> s<sup>-1</sup>; weak 230 turbulence and energetic turbulence: 10<sup>-13</sup>-10<sup>-7</sup> °C<sup>2</sup> s<sup>-1</sup>) and median values (salt finger: 1.8×10<sup>-9</sup> °C<sup>2</sup> s<sup>-1</sup>; energetic turbulence and weak turbulence: 1.5×10<sup>-11</sup> °C<sup>2</sup> s<sup>-1</sup>). Earlier studies considered the doubly stable regime as no mixing or excluded it from analysis (Inoue et al., 2007); however, besides some slight differences of proportion in large  $\chi_{\theta}$  and  $\varepsilon_{\epsilon}$  energetic turbulence and weak turbulence share very similar distributions of  $\chi_{\theta}$  and  $\varepsilon$  (Figs. 3b, d), suggesting the doubly stable regime does not 235 mean an absence of turbulence and should be dominated by weak turbulence. Stratification also presents different features for different mixing types. The energetic turbulence has the weakest stratification with a median of  $6.1 \times 10^{-7}$  s<sup>-2</sup>, only 1/5 of that for weak turbulence. And salt finger presents the strongest stratification (1.9×10.5 s<sup>-2</sup>). Clearly, the identified patches Inserted Cells

with energetic turbulence, weak turbulence and salt finger have distinct turbulent features, verifying the validity of the chosen criteria.



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Figure 3: Probability-normalized histograms of  $log_{10}(Re_{\theta})$  (a),  $log_{10}(\epsilon)$  (b),  $log_{10}(N^2)$  (c), and  $log_{10}(\chi_{\theta})$  (d) for different mixing types: SF (salt finger), ET (energetic turbulence) and WT (weak turbulence). Data of the five projects are taken as the whole collection.

A normalized occurrence frequency is calculated to quantify the vertical variation of each mixing type (Fig. 4). Taking energetic turbulence as an example, we first divide the number of energetic turbulence patch number of patches in each depth bin toby the total number of energetic turbulence patch number of patches in the whole project; then, to eliminate the vertical variation of observation frequency, we divide the results by the total patch-number of patches within the same depth bin. This occurrence frequency is eventually normalized between 0 and 1 using its maximum. Consistent with some observations in the upper thermocline (Schmitt et al., 2005; van der Boog et al., 2021), salt finger is mostly prevailing in the upper 500-1000 m for all projects, with their occurrence frequencies reaching 1. For the MIXET projects and NATRE, the occurrence frequencies of salt finger gradually become weak and near zero with depth increasing to the seafloor. However, for the BBTRE projects, salt finger sharply disappears between 1000 and 2000 m and re-occurs at deeper depth (see Figure 10). The depth-colored T-S diagrams suggest the vertical transition of different water masses is responsible for the sudden disappearing of salt finger (Fig. 5). It is clear to see that both  $\theta$  and S decrease with depth in most water columns, providing the basic precondition for salt finger. However, this tendency changes obviously between 1000 and 2000 m. At this depth

range, θ changes little, but S increases drastically by at least 0.5; this prevents the occurrence of salt finger. This depth is just where the fresher Antarctic Intermediate Water transits to the North Atlantic Deep Water. Consequently, the occurrence frequency of salt finger is severely weakened at this depth. On the contrary to salt finger, energetic turbulence generally becomes more prevailing with increasing depth for most projects. The remarkably weak background stratification may contribute a lot to the flourish of energetic turbulence at depth, where even a weak perturbation can fully develop.

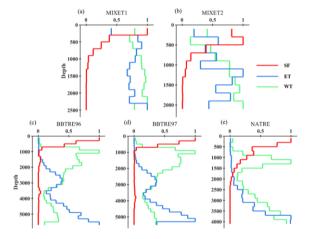
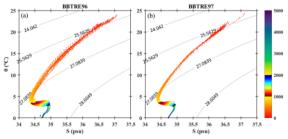


Fig. 4. Vertical variations of normalized occurrence frequency of salt finger (SF), energetic turbulence (ET) and weak turbulence (WT) for the five projects. The depth range is from 100 m to the deepest measurements, with a bin size of 200 m.



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Fig. 5. T-S diagrams for BBTRE96 and BBTRE97. Color indicates patch depth, and the contour indicates isopycnic.

# 265 4 $\Gamma$ variation of turbulence and salt finger

## 4.1 Γ variation of turbulence

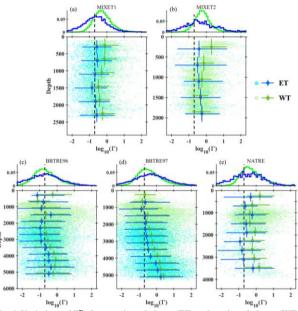
## 4.1.1 Vertical variation

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We explore the variation of  $\Gamma^T$  first. Figure 6 suggests  $\Gamma^T$  varies in distinct manners for different projects. Results for the MIXET projects suggest  $\Gamma^T$  in the western equatorial Pacific is significantly seasonally variable. In spring (MIXET1),  $\Gamma^T$  of energetic turbulence varies between  $2.5 \times 10^{-2}$  and 1.7 ( $10^{th}$ - $90^{th}$  percentiles) with a median of 0.23, smaller than that of weak turbulence ranging between  $1.4 \times 10^{-1}$  and 2.8 and peaking at 0.52.  $\Gamma^T$  in autumn is significantly elevated (MIXET2), and the medians and variation ranges for energetic turbulence and weak turbulence are [0.41 and from  $3.4 \times 10^{-2}$  to 8.8] and [0.58 and from  $1.7 \times 10^{-1}$  to 2.3], respectively. For the BBTRE projects,  $\Gamma^T$  of weak turbulence varies little between different years, with most patches varying between  $10^{-2}$  and 10, although the median value in 1997 (0.35) was greater than that in 1996 (0.20).  $\Gamma^T$  of energetic turbulence is larger in 1997 than that in 1996, with median values of 0.48 and 0.20, respectively. Estimates from the NATRE also suggest  $\Gamma^T$  largely scatters between  $10^{-2}$  and 10 for most patches; their median  $\Gamma^T$  values are 0.71 and 0.33 and 0.50 for energetic turbulence; and are 0.41 and 0.50 for weak turbulence, respectively. To summarize, besides the BBTRE and energetic turbulence of the MIXET projects showing a median value close to 0.2, the rest estimates are all clearly greater than 0.2.  $\Gamma^T$  for the five projects mostly vary within three orders of magnitude from  $10^{-2}$  to 10, in line with other observations (Ijichi and Hibiya, 2018; Vladoiu et al., 2021; Li et al., 2023).

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Fig. 6. Variations of  $\Gamma^T$  of energetic turbulence (ET) and weak turbulence (WT). Each panel consists of two sub-panels, with the upper one showing probability-normalized histogram of  $\Gamma^T$ , and the lower one being  $\Gamma^T$ -depth scatters; the median value of each depth bin is marked by a larger, darker dot overlying a cross marker, with horizontal bar indicating the  $10^{th}$  to  $90^{th}$  percentile range and vertical bar indicating the depth-bin range. The median  $Re_b$  values are compared between energetic turbulence and weak turbulence at each depth bin, and the median  $\Gamma^T$  corresponding to the larger  $Re_b$  is marked by a red dot. The conventional value of  $\Gamma^T$ , namely 0.2, is represented by the dashed black line.

For different projects,  $\Gamma^T$  varies with depth in different way. For the MIXET1,  $\Gamma^T$  of both energetic turbulence and weak turbulence fluctuate around their statistical median values weakly. For the MIXET2, the depth-median  $\Gamma^T$  of energetic turbulence varies between 0.2 and 0.7 alternately, with a slightly increasing trendincrease with depth. However,  $\Gamma^T$  of weak turbulence shows a clear decreasing from 2.5 at 300 m to 0.6 at 1400 m; then, it slightly increases to 0.8 at 1900 m. The  $\Gamma^T$  of weak turbulence for the BBTRE96 fluctuates around 0.2 in the upper 300 m, then it increases to ~1 at 4400 m and then decreases to ~0.6 at 5200 m. The scenario for energetic turbulence shares a similar picture.  $\Gamma^T$  of weak turbulence for the BBTRE97 departs little from 0.2 at depths above 1800 m, then monotonically increasing to ~2.3 at the deepest depth around 5200 m.  $\Gamma^T$  of energetic turbulence varies in a similar way in vertical, except the depth where trends change is 3000 m. For NATRE,  $\Gamma^T$  of energetic turbulence firstly decreases from 0.6 to 0.1 at 2300 m, then increases to 0.8 at 3500 m. As for weak turbulence,  $\Gamma^T$  stays around 0.8 between 600 and 3000 m and then increases beyond unity at 3500 m. In term of general trend

by linear fitting  $\Gamma^T$  with depth, the five projects show two distinct vertical patterns of  $\Gamma^T$ : One is the <u>vertically-downward</u>-decreasing pattern represented by the MIXET projects, and the other is the <u>vertically-downward</u>-increasing one suggested by the rest projects over the midlatitude of the Atlantic. <u>Vertically-Downward</u> increasing  $\Gamma^T$  was also reported by Ijichi and Hibiya (2018). Their data collection sites spread over mid-to-high latitudes of the Pacific and Southern Ocean.  $\Gamma^T$  also presented a clear <u>vertically increasing trenddownward increase</u> in the upper 500 m of the South China Sea north of 10°N (Li et al., 2023). Combining all these observational results, we suggest  $\Gamma^T$  in the equatorial area should be treated differently, since it may decrease in the <u>vertical with depth</u>, contrary to the <u>vertically increasing trenddownward-increase</u> away from the equator.

The full-depth statistics of the five projects disagree about whichwhether,  $\Gamma^T$  is larger for energetic turbulence andor, weak turbulence. However, when comparing  $\Gamma^T$  values of energetic turbulence and weak turbulence in the same depth bin,  $\Gamma^T$  of energetic turbulence is mostly smaller than that of weak turbulence. Considering that  $Re_b$  is reported to deeply modulate the variation of  $\Gamma^T$  (Mashayek et al., 2017; Monismith et al., 2018), and that energetic turbulence and weak turbulence have clearly different  $Re_b$  distributions (Fig. 3), we found energetic turbulence with smaller  $\Gamma^T$  generally has larger  $Re_b$  than weak turbulence, indicating a negative correlation between  $\Gamma^T$  and  $Re_b$ .

## 4.1.2 Relation between $\Gamma^{T}$ , Re<sub>b</sub> and $R_{OT}$

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We then investigate the relations between  $\Gamma^{T}$  and Re<sub>b</sub> for energetic turbulence and weak turbulence (Fig. 7). For the MIXET1,  $\Gamma^{T}$  of weak turbulence first decreases from 3.5 to 0.5 with Re<sub>b</sub> increasing from 0.1 to 1, suggesting a relation of  $\Gamma^{T} \propto Re_{b}^{-1}$ , and then it weakly increases to 0.7 with Re<sub>b</sub> reaching 100; and a weak decreasing in line with  $\Gamma^{T} \propto Re_b^{-1/2}$  can be observed for  $Re_b > 100$ . For energetic turbulence,  $\Gamma^T$  generally decreases with  $Re_b$ , indicating  $\Gamma^T \propto Re_b^{-1/2}$ ; this relation is consistent with the observations in the western Mediterranean Sea (Vladoiu et al., 2021). The pattern for the MIXET2 is similar to that for the MIXET1, although  $\Gamma^T$  of weak turbulence decreases in a smaller rate when  $Re_b$  is small and indicates  $\Gamma^T \propto Re_b^{-1/2}$ . Excluding the bins with few data points,  $\Gamma^T$  of weak turbulence for the BBTRE96 shows a weak increasing trendingrease from 0.2 to 0.3 as Re<sub>b</sub> grows from 10 to  $10^3$  and a weak decreasing trenddecrease with Re<sub>b</sub> exceeding  $10^3$ .  $\Gamma^T$  and Re<sub>b</sub> of weak turbulence for the BBTRE97 show similar relationships as those for the BBTRE96. The weak turbulence trendsyariations for the BBTRESs are the same as the estimates reported by Ijichi et al. (2020); and its shape is similar to the upper bound of the nonmonotonic  $\Gamma^{T} \sim \text{Re}_b$  relation proposed by Mashayek et al. (2017). It is notable that the scenario for energetic turbulence is distinct;  $\Gamma^T$  generally decreases from 5 to less than 0.1 with Re<sub>b</sub> between 10 and 2.5×10<sup>4</sup> for the BBTRE96, forming a fitting slope steeper than -1/2 but flatter than -1.  $\Gamma^{T}$  of energetic turbulence for the BBTRE97 also shows a similar decreasing trenddecrease with Re<sub>b</sub>. Except for the bins with few samples when Re<sub>b</sub><1 and Re<sub>b</sub>>10<sup>4</sup>,  $\Gamma^{T}$  of weak turbulence for the NATRE generally increases from 0.5 to 0.7, while  $\Gamma^{T}$  of energetic turbulence monotonically decreases from ~1 at Re<sub>b</sub>=10<sup>2</sup> to ~0.1 at Re<sub>b</sub>=10<sup>4</sup>, suggesting  $\Gamma^{\text{T}} \propto Re_{\text{b}}^{-1/2}$ .

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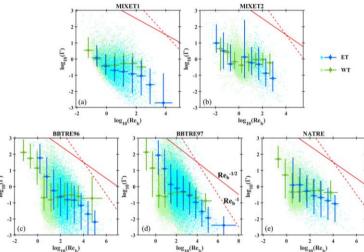
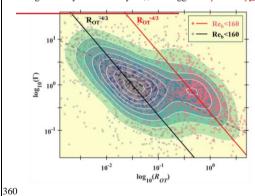


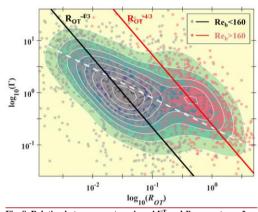
Fig. 7. Relations between  $\Gamma^{T}$  and  $Re_b$  for energetic turbulence (ET) and weak turbulence (WT). Overlying the light-color scatters of individual patches,  $Re_b$  -binned median values are marked by large darker dots; and the bin size and the  $10^{th}$ - $90^{th}$  percentile range of  $\Gamma^{T}$  are denoted by the horizontal and vertical bars, respectively. The solid and dashed red lines mark  $\Gamma^{T} \propto Re_b^{-1/2}$  and  $\Gamma^{T} \sim Re_b^{-1}$ , respectively.

Although  $\Gamma^T$  generally decreases with  $\operatorname{Re}_b$  in most cases of the five projects, the decreasing rate varies with projects and  $\operatorname{Re}_b$  ranges. There are several cases showing  $\Gamma^T$  stays constant or even increases with  $\operatorname{Re}_b$ . These suggest  $\Gamma^T$  is not solely modulated by  $\operatorname{Re}_b$ ; and there may be other factors that influence  $\Gamma^T$  in a comparable or even dominating role relative to  $\operatorname{Re}_b$ .  $R_{OT}$  is reported as such a parameter that regulates  $\Gamma^T$  more strongly than  $\operatorname{Re}_b$ ,  $\Gamma^T \simeq R_{OT}^{-4/3}$  (Ijichi and Hibiya, 2018).  $R_{OT}$  is the ratio of the Ozmidov scale  $L_O$  to the Thorpe scale  $L_T$ ,  $R_{OT} = L_O/L_T$  with  $L_O = \varepsilon^{1/2}/N^{3/2}$  and  $L_T = (\delta_T^2)^{\frac{1}{2}}$ , where the Thorpe displacement  $\delta_T$  is the depth difference of a water parcel between the original and sorted potential temperature profiles of an overturn. Overturns are identified by the cumulative Thorpe displacement  $\sum \delta_T$  (Mater et al., 2015; Ijichi and Hibiya, 2018). Because the vertical resolution of temperature profiles is 1 m or 0.5 m, overturns with vertical size of O(1) m or smaller cannot be identified. Additionally, the identified overturns with size smaller than 10 m or greater than 400 m are excluded from analysis, because the former contain too few data points and the latter are possibly the vertical structures of different water masses instead of genuine turbulent overturns. We also estimate the overturn-averaged Tu,  $\Gamma^T$  and  $Re_b$ . Due to the coarse vertical resolution of temperature profiles used in our study, only a few overturns meet the identification criteria for each project; as a result, the overturns of the five projects are taken as one collection (total overturn-number of overturns is 3862).

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Figure 8 shows the overturn-based relation between  $\Gamma^T$  and  $R_{OT}$ . Since most overturns are identified at depth, with only one fifth shallower than 1000 m but more than one third at depth below 2000 m, the overturn-based  $\Gamma^T$  is clearly greater than 0.2, with a median value of 0.91. In Fig. 8, although overturns are evenly scattered in the  $R_{OT}\Gamma^T$  space, the probability density shows they concentrate around two sites mostly, one with  $R_{OT}$  and  $\Gamma^T$  of (0.03, 1.19) and the other (0.56, 0.53). These two clusters are well distinguished by  $Re_b$ , with the first location corresponding to  $Re_b$ <160 (median value is 25) and the other to  $Re_b$ >160 (median value is 835). For both clusters, the contours of probability density tilt at slopes of -4/3, confirming  $\Gamma^T \sim R_{OT}^{-4/3}$  is valid for each cluster. However, the general trend between  $R_{OT}$  and  $\Gamma^T$  for the whole data collection is much flatter, with a slope of only about -1/2. Comparing  $Re_b$  of the two clusters, it is easy to find that  $Re_b$  grows exponentially with  $R_{OT}$ . Therefore, the general variation of  $\Gamma^T$  with the growth of  $R_{OT}$  is not only influenced by  $R_{OT}$ , but also partly affected by  $Re_b$ . Supposing  $\Gamma^T$  is mostly modulated by these two parameters, and considering the decrease—trend of  $\Gamma^T$  with  $R_{OT}$  is significantly weakened by  $Re_b$ , this suggests a positively positive relation between  $\Gamma^T$  and  $Re_b$ .





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Fig. 8. Relation between overturn-based  $\Gamma^T$  and  $R_{OT}$ , overturns from the five projects are considered. The shading describes the distribution of probability density, with yellow indicating minimum probability density and blue representing maximum one. The overturns are correspondingly divided into two clusters: the gray dots have  $Re_b < 160$ , and the pink ones,  $Re_b > 160$ . The black and red lines represent  $\Gamma^T_{\Delta} \propto R_{OT}^{-4/3}$ , crossing the centers of the two clusters. The graywhite dashed line is the general relation between  $\Gamma^T$  and  $R_{OT}$  of the whole data collection.

Figure 9 shows the variation of median value of  $\Gamma^T$  jointly binned by  $Re_b$  and  $R_{OT}$ . Note that most parts of the  $Re_b$ - $R_{OT}$  space are null, with all the data gathered around a band originating from large  $Re_b$  and  $R_{OT}$  to small  $Re_b$  and  $R_{OT}$ . This confirms that  $Re_b$  and  $R_{OT}$  are positively correlated in general. As for the median  $\Gamma^T$ , although its value is scattered, its general pattern indicates  $\Gamma^T$  grows fastest along a direction from small  $Re_b$  and large  $R_{OT}$  to large  $Re_b$  and small  $R_{OT}$ , suggesting that  $\Gamma^T$  is indeed positively correlated with  $Re_b$  and negatively correlated with  $R_{OT}$ . Assuming  $\Gamma^T \propto R_{OT}^{-4/3} \cdot Re_b^c$ , we substitute the median values of  $\Gamma^T$ ,  $Re_b$  and  $R_{OT}$  in Fig. 9 into this relation to fit the exponent c. The fitting results suggest  $c \approx 1/2$  and a relation of  $\Gamma^T \approx 10^{-3} \cdot R_{OT}^{-4/3} \cdot Re_b^{1/2}$ . The isolines of this relation are shown in Fig. 9, which can well capture the main variation trend-of  $\Gamma^T$  with  $Re_b$  and  $R_{OT}$ . Based on the microstructure measurements collected from the upper layer in the South China Sea, Li et al. (2023) presented a relation of  $\Gamma^T \approx aR_{OT}^{-4/3} \cdot Re_b^{1/2}$ , but a is around 0.02 in that region, one magnitude larger than the value presented here. This is because  $Re_b$  have much smaller magnitude in the upper South China Sea, with most  $Re_b$  varying between  $10^{-1}$  and  $10^3$ . Therefore, compared with the results in Li et al. (2023), the larger  $Re_b$  in this study lead to a relatively smaller a. On the other hand, the significant variation of a may suggest some other parameters can influence  $\Gamma^T$  besides  $Re_b$  and  $R_{OT}$ .

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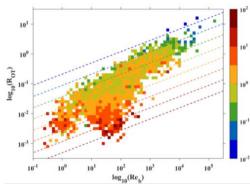
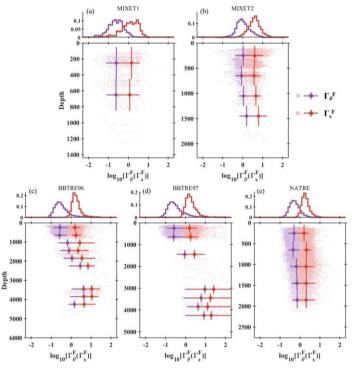


Fig. 9. Variation of median  $\Gamma^{T}$  binned by  $Ro\tau$  and  $Re_b$ , based on overturn estimates of the five projects. The The color bar refers to the median  $\Gamma^{T}$ , and the colored dashed lines indicate the isolines of  $10^{-3} \cdot Ro\tau^{-4/3} \cdot Re_b^{1/2}$ .

# 4.2 $\Gamma$ variation of salt finger

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 $\Gamma^F$  has been widely used to distinguish salt finger from turbulence, since its value is reported to be larger than the conventional  $\Gamma^T$  value of 0.2 (St. Laurent and Schmitt, 1999). However, the full-depth observations presented in either this study or previous ones indicate 0.2 is an underestimate of  $\Gamma^T$ , the difference of dissipation ratio between turbulent mixing and salt finger mixing in the deep water needs to be examined. Figure 10 presents the variations of  $\Gamma_\theta^F$  with depth. Compared with  $\Gamma^T$  varying over three orders of magnitude, both  $\Gamma_\theta^F$  and  $\Gamma_S^F$  are less variable and change by two orders in magnitude or as small as one order. The median  $\Gamma_\theta^F$  for all samples from the five projects is 0.47, slightly smaller than the  $\Gamma^F$  observed in the diurnal thermocline of the Arabian Sea (0.65; Ashin et al., 2023), in the Kuroshio Extension Front (~1; Nagai et al., 2015), and in the thermocline of the western tropical Atlantic (~1.2; Schmitt et al., 2005). The median  $\Gamma_\theta^F$  for the five projects are distinct: 0.25, 0.29 and 0.28 for the MIXET1, BBTRE96 and BBTRE97, similar to the conventional  $\Gamma^T$  value of 0.2; 0.52 for NATRE, distinguishable from 0.2 but close to their observed  $\Gamma^T$  (Fig. 6); 0.98 for the MIXET2, significantly larger than 0.2 and different from their observed  $\Gamma^T$  (Fig. 6). This suggests the dissipation ratio difference between turbulence and salt finger is complex.



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Fig. 10. Variations of  $\Gamma_{\theta}^{F}(\Gamma_{S}^{F})$  of salt finger for the five projects. Each panel consists of two sub-panels, with the upper one showing the probability-normalized histograms of  $\Gamma_{\theta}^{F}$  and  $\Gamma_{S}^{F}$ , and the lower one being their vertical variations. The median value of each depth bin is marked by a larger, darker dot overlying a cross marker, with horizontal bar indicating the  $10^{th}$  to  $90^{th}$  percentile range and vertical bar indicating the depth-bin range.

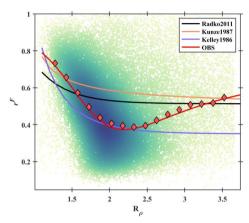
Vertically,  $\Gamma_{\theta}^{F}$  for MIXET1 keeps nearly constant as 0.25. While  $\Gamma_{\theta}^{F}$  for MIXET2 first decreases from 1 to 0.7 in the upper 700 m and then slightly increases to 2 at 1500 m.  $\Gamma_{\theta}^{F}$  present a similar vertical trend for both BBTRE96 and BBTRE97:  $\Gamma_{\theta}^{F}$  is small and stays as a constant within the upper 800 m, with a median of 0.28; with depth increasing to 3000 m, it significantly increases over orders of magnitude, and the median value reaching ~10; it is weakened at deeper depth. Note that the relatively small median  $\Gamma_{\theta}^{F}$  for the BBTRE projects is mainly caused by the dominant patches with small  $\Gamma_{\theta}^{F}$  values in the upper 800 m; and  $\Gamma_{\theta}^{F}$  at depth is actually very large and significantly greater than 0.2 or the observed  $\Gamma^{T}$ . The scenario for the NATRE is similar to that for the MIXET2, whose depth-median  $\Gamma_{\theta}^{F}$  remains nearly consistent around ~0.5, although a very weak increasing positive trend exists.

The "effective" salt dissipation ratio  $\Gamma_S^F$  tends to be obviously larger than  $\Gamma_\theta^F$  (Fig. 10). With the overall median  $\Gamma_S^F$  of 1.87, the median values of  $\Gamma_S^F$  for the five projects are 1.35 (MIXET1), 3.98 (MIXET2), 1.67 (BBTRE96), 1.71 (BBTRE97), and 1.83 (NATRE), floating around the value reported in the thermocline of the western tropical Atlantic of ~2.8 (ref). Schmitt et al., 2005).  $\Gamma_S^F$  is strongly positively proportional to  $\Gamma_\theta^F$ , with the median values of  $\Gamma_S^F/\Gamma_\theta^F$  for the five projects being 5.1, 3.7, 6.3, 6.9, and 3.8, respectively. Thus, a general relation of  $\Gamma_S^F \approx 5\Gamma_\theta^F$  can be inferred. Due to this correlation,  $\Gamma_S^F$  presents vary similar vertical variation as  $\Gamma_\theta^F$ .

Note that Γ<sub>S</sub><sup>F</sup>/Γ<sub>θ</sub><sup>F</sup> is equivalent to R<sub>ρ</sub>/r<sup>F</sup><sub>τ</sub>(Eq. (6)). Since R<sub>ρ</sub> is relatively easy to calculate, as a result, it is an alternate way to infer the hard-to-measure r<sup>F</sup>. R<sub>ρ</sub> and r<sup>F</sup> are the key parameters to estimate the dissipation ratios of heat and salt for salt finger (Section 2.3). Therefore, many studies tried to explore the relation of R<sub>ρ</sub> and r<sup>F</sup> based on theoretical derivations, laboratory experiments and numerical simulations (Kelley, 1986; Kunze, 1987; Radko and Smith, 2012). Here, the R<sub>ρ</sub>-r<sup>F</sup> diagram colored by probability density for the five projects indicate the salt finger patches are rather scattered (Fig. 11). However, the median r<sup>F</sup> binned by R<sub>ρ</sub> shows a clear nonmonotonic variability. For R<sub>ρ</sub> increasing from 1 to 2.4, r<sup>F</sup> decreases from ~0.8 to 0.4; then, it gradually increases to 0.55 with R<sub>ρ</sub> approaching 3.7. This correlation between R<sub>ρ</sub> and r<sup>F</sup> can be well fitted by

$$r^{\rm F} = \frac{0.79 \cdot R_{\rho}^2 - 2.96 \cdot R_{\rho} + 3.18}{R_{\rho}^2 - 3.26 \cdot R_{\rho} + 3.46} \tag{6}$$

Compared with other correlation curves (Kelley, 1986; Kunze, 1987; Radko and Smith, 2012), all of them present a  $r^F$  decreasing trenddecrease for  $R_\rho$  smaller than 2, although the variation range and rate differ. The most obvious discrepancy between them is that  $r^F$  tends to regain a larger value with  $R_\rho$  exceeding 2.4 in our study, while all the other curves decrease little to asymptote to a constant value. The observational result presented here falls in the area outlined by the existing results. For our results, the salt finger patches with  $R_\rho$ <2.5 are abundant and mostly concentrated to indicate a negative correlation between  $R_\rho$  and  $r^F$ . It needs to be mentioned that patches with  $R_\rho$ >2.5 are much rare and sparsely distributed, making the increasing trendincrease of  $r^F$  in larger  $R_\rho$  range need to be treated carefully.



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Fig. 11. Relation between  $R_{\rho}$  and  $r^{\rm F}$ . Salt finger patches from all the five projects are considered. Dots are colored by probability density, with darker color indicating larger probability density. The median  $r^{\rm F}$  binned by  $R_{\rho}$  are marked by red diamonds with black edge, and the red curve is the fitting curve. The black, orange and purple curves are adopted from Radko and Smith (2011), Kunze (1987) and Kelley (1986), respectively.

We also investigate the relation between observed  $\Gamma_{\theta}^{F}$  and  $Re_{b}$  (Fig. 12), which differs considerably between different projects. For the MIXET1, a nearly linear decreasing trenddecrease of  $\Gamma_{\theta}^{F}$  (in logarithmic scale) from ~1 to ~0.1 can be easily observed for all patches with  $Re_{b}$  between 0.3 and 25, indicating  $\Gamma_{\theta}^{F} \propto Re_{b}^{-1/2}$ .  $\Gamma_{\theta}^{F}$  for MIXET2 with  $Re_{b} < 2.5$  are also well fitted as  $\Gamma_{\theta}^{F} \propto Re_{b}^{-1/2}$ , but  $\Gamma_{\theta}^{F}$  for  $Re_{b} > 2.5$  tends to remain a constant of 0.7. For the BBTRE projects, when  $Re_{b} < 3$ ,  $\Gamma_{\theta}^{F}$  decreases at a larger rate than the MIXET projects,  $\Gamma_{\theta}^{F} \propto Re_{b}^{-1}$ , and  $\Gamma_{\theta}^{F}$  stays almost unchanged when  $Re_{b}$  exceeds 3.  $\Gamma_{\theta}^{F}$  for the NATRE stays as a constant of 0.7 with most  $Re_{b}$  ranging from 1 to 25. Due to the strong correlation between  $\Gamma_{s}^{F}$  and  $\Gamma_{\theta}^{F}$ , the dependence of  $\Gamma_{s}^{F}$  on  $Re_{b}$  is similar to that of  $\Gamma_{\theta}^{F}$ , although variation rates are different for some projects. Taking all the projects together,  $\Gamma_{\theta}^{F}$  and  $\Gamma_{s}^{F}$  decrease with  $Re_{b}$  in general; however, the decreasing rate of decrease varies greatly with projects and different  $Re_{b}$  bands, indicating  $\Gamma_{\theta}^{F}$  and  $\Gamma_{s}^{F}$  may also be modulated by variables other than  $Re_{b}$ .

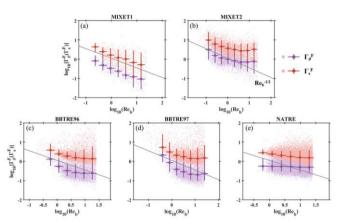


Fig. 12. Relations between  $\Gamma \rho^F(\Gamma_S^F)$  and  $Re_b$  for the five projects. Overlying the light-color scatters of individual patches, the  $Re_b$ -binned median values are marked by darker large dots. The bin size and  $10^{th}$ - $90^{th}$  percentile range are denoted by the horizontal and vertical bars, respectively. The gray line in each panel marks  $\Gamma \rho^F(\Gamma_S^F)_{\rho} \propto Re_b^{-1/2}$ .

# 5 Eddy diffusivities induced by turbulence and salt finger

# 450 5.1 Eddy diffusivities induced by turbulence

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Since  $\Gamma^T$  deviates from the conventionally used constant of 0.2 in the Osborn relation,  $K_\rho^T$  (also  $K_\theta^T$  and  $K_s^T$ ) based on  $\Gamma^T$  differs from  $K_c$  based on 0.2 ( $K_c$ =0.2 $\epsilon$ / $N^2$ ) to different extents (Fig. 13). For the MIXET1, since  $\Gamma^T$  is only slightly larger than 0.2 in general, the magnitudes of  $K_\rho^T$  and  $K_c$  differ slightly, with mean  $K_c$ =2.1×10<sup>-6</sup> m<sup>2</sup> s<sup>-1</sup> and mean  $K_\rho^T$ =4.6×10<sup>-6</sup> m<sup>2</sup> s<sup>-1</sup>. Vertically, both  $K_\rho^T$  and  $K_c$  decrease in the upper 1200 m and increase at deeper depth. Obvious differences between  $K_\rho^T$  and  $K_c$  occur at depth ranges shallower than 1200 m and deeper than 2000 m, where the mean ratio of  $K_\rho^T$  to  $K_c$  are 2.7 and 2.3, respectively. For the MIXET2, the magnitude difference between  $K_\rho^T$  and  $K_c$  is larger, with the mean values being 1.3×10<sup>-6</sup> m<sup>2</sup> s<sup>-1</sup> and 3.9×10<sup>-6</sup> m<sup>2</sup> s<sup>-1</sup>, respectively. Compared with  $K_c$  that stays nearly constant in the upper 1700 m,  $K_\rho^T$  first decreases in the upper 700 m and then stays around 2×10<sup>-6</sup> m<sup>2</sup> s<sup>-1</sup> between 700 and 1700 m. For the BBTRE96, except for several depth bins, the difference between mean  $K_\rho^T$  and mean  $K_c$  in the upper 3700 m is small; and they share similar vertical increasing rates downward increases and similar depth-averaged median values around 2.0×10<sup>-5</sup> m<sup>2</sup> s<sup>-1</sup>, with  $K_\rho^T$  is about 2.5 times of  $K_c$ . Although both increase at depths deeper than 3700 m,  $K_\rho^T$  is nearly 4.7 times larger than  $K_c$ ; and the mean values for  $K_\rho^T$  and  $K_c$  are 5.0×10<sup>-4</sup> and 1.1×10<sup>-4</sup> m<sup>2</sup> s<sup>-1</sup>, respectively. For the BBTRE97,  $K_\rho^T$  and  $K_c$  share the same vertical decreasing downward negative trend and magnitude in the upper 1000 m, with mean values close to 2.6×10<sup>-5</sup> m<sup>2</sup> s<sup>-1</sup>. Beneath 1000 m, although sharing similar increasing positive trend,  $K_\rho^T$  becomes larger and larger than  $K_c$  with depth. At depth between 1000 and 3700

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465 m,  $K_{\rho}^{\mathrm{T}}/K_{c}\approx2.7$  with median  $K_{\rho}^{\mathrm{T}}$  around  $8.3\times10^{-5}$  m<sup>2</sup> s<sup>-1</sup>, while the corresponding values for depths deeper than 3700 m are  $K_{\rho}^{\mathrm{T}}/K_{c}\approx8.8$  and mean  $K_{\rho}^{\mathrm{T}}\approx1.2\times10^{-3}$  m<sup>2</sup> s<sup>-1</sup>. For the NATRE,  $K_{\rho}^{\mathrm{T}}$  is always larger than  $K_{c}$  at all depth ranges; and the mean values of  $K_{\rho}^{\mathrm{T}}$  and  $K_{c}$  are  $4.4\times10^{-5}$  and  $1.0\times10^{-5}$  m<sup>2</sup> s<sup>-1</sup>, respectively. Vertically,  $K_{c}$  generally fluctuating around its mean value for the whole water column.  $K_{\rho}^{\mathrm{T}}$  also shows no clear vertical trendvariation in the upper 2700 m, but it increases significantly from  $2.6\times10^{-5}$  m<sup>2</sup> s<sup>-1</sup> at 2700 m to  $1.2\times10^{-4}$  m<sup>2</sup> s<sup>-1</sup> at 3900 m. As a result,  $K_{\rho}^{\mathrm{T}}$  is 13.7 times larger than  $K_{c}$  at 3900 m. For the five projects, taking  $\Gamma^{\mathrm{T}}$  as a constant of 0.2 underestimates the actual eddy diffusivity induced by turbulence, and this underestimate may become more severe as  $\Gamma^{\mathrm{T}}$  increases with depth.

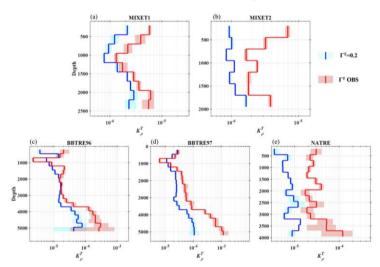
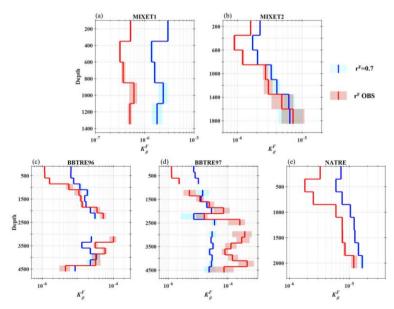


Fig. 13. Vertical profiles of depth-bin mean  $K_{\rho}^{T}(K_{c})$  based on energetic turbulence and weak turbulence patches for the five projects. The blue curve is  $K_{c}$  estimates by using  $\Gamma^{T}$ =0.2, and the red curve is  $K_{\rho}^{T}$  based on the measured  $\Gamma^{T}$ . The colored shadings correspond to 95% bootstrapped confidence intervals. To exclude the influence of extreme values, we only consider patches with  $\Gamma^{T}$  withinbetween its upper and lower quartiles for each depth bin. The depth-bin size is 250 m.

### 5.2 Eddy diffusivities induced by salt finger

For salt finger-induced eddy diffusivities, some studies estimated their values by taking a constant  $r^F$  around 0.7 (0.75 in Schmitt et al., 2005; 0.6 in St. Laurent and Schmitt 1999). Here,  $K_{\theta}^F$  derived from the observed  $r^F$  is compared with the  $r^F$ =0.7 estimate,  $K_{\theta}^F_c$  (Fig. 14). Depending on the deviation of the observed  $r^F$  from 0.7, the five projects are distinct in terms of the difference between  $K_{\theta}^F$  and  $K_{\theta}^F_c$ . For the MIXET1,  $K_{\theta}^F$  and  $K_{\theta}^F_c$  both vary little with depth. But the magnitude of  $K_{\theta}^F_c$  is significantly greater than that of  $K_{\theta}^F$ , with mean values being 2.2×10<sup>-6</sup> and 4.6×10<sup>-7</sup> m² s<sup>-1</sup>, respectively. This is in line with

the fact that the mean value of the measured  $r^{F}$  for the MIXET1 is only 0.37, about one half of 0.7. For MIXET2, with the median  $r^{F}$  elevated to 0.63,  $K_{\theta}^{F}_{c}$  is only slightly larger than  $K_{\theta}^{F}$ . And they both increase with depth form  $O(10^{-6})$  m<sup>2</sup> s<sup>-1</sup> at 100 m to  $O(10^{-5})$  m<sup>2</sup> s<sup>-1</sup> at 1850 m. The median values of  $K_{\theta}^{F}$  and  $K_{\theta}^{F}$  are  $4.4 \times 10^{-6}$  m<sup>2</sup> s<sup>-1</sup> and  $3.2 \times 10^{-6}$  m<sup>2</sup> s<sup>-1</sup>, respectively. The difference between MIXET1 and MIXET2 indicates a strong seasonal variation of salt finger in the tropical Pacific. For both BBTRE96 and BBTRE97,  $K_{\theta}^{F}_{c}$  is significantly larger than  $K_{\theta}^{F}$  in the upper layer with magnitudes around  $O(10^{-5})$  and  $O(10^{-6})$ m<sup>2</sup> s<sup>-1</sup>, respectively, and this difference turns small as they both increase to 2×10<sup>-5</sup> m<sup>2</sup> s<sup>-1</sup> with depth increasing to 2000 m. At deeper depths, although salt finger disappears at some depth ranges,  $K_{\theta}^{F}_{c}$  varies little around  $2.5 \times 10^{-5}$  m<sup>2</sup> s<sup>-1</sup>.  $K_{\theta}^{F}$  is generally larger than  $K_{\theta_c}^F$  between 2400 m and 3400 m with  $K_{\theta_c}^F/K_{\theta_c}^F$  varying between 3 and 10, and this ratio drops to less than 2 for depths deeper than 3400 m. For NATRE, both  $K_{\theta}^{F}$  and  $K_{\theta}^{F}_{c}$  present clear vertical increasing trendsdownward increases, and  $K_{\theta_{F}}^{F}$  is dominantly greater than  $K_{\theta_{F}}^{F}$ . The difference between  $K_{\theta_{F}}^{F}$  and  $K_{\theta_{F}}^{F}$  is reduced with increasing depth, due to the fact that  $K_{\theta}^{F}$  increases much faster in the vertical with depth from about  $2 \times 10^{-6}$  m<sup>2</sup> s<sup>-1</sup> at upper 500 m to  $1.5 \times 10^{-5}$  m<sup>2</sup> s<sup>-1</sup> at 2400 m. For all the projects,  $K_{\theta}^{F}$  is generally smaller than  $K_{\theta}^{F}_{c}$  since  $r^{F}$  is mostly smaller than 0.7; and this phenomenon is most obvious in the upper layer (upper 1000 m of the BBTRE96, BBTRE97, and NATRE). At deeper depths,  $K_{\theta}^{F} > K_{\theta}^{F}_{c}$  can be observed in projects like the BBTRE96, BBTRE97. All these indicate rF is highly variable regionally and vertically. We also explore vertical variation of  $K_s^F$ , which is very similar to that of  $K_{\theta}^F$  but with a larger magnitude (Fig. 15), as the result of  $\Gamma_s^F$ being larger than and strongly proportional to  $\Gamma_{\theta}^{F}$ .



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Fig. 14. Vertical profiles of depth-bin mean  $K_0^F(K_0^{F_c})$  based on salt finger patches for the five projects. The blue curves are  $K_0^{F_c}$  estimated with  $r^F$ =0.7, and the red ones are  $K_0^F$  based on the measured  $r^F$ . The colored shades correspond to 95% bootstrapped confidence intervals. To exclude the influence of extreme values, we only consider patches with  $\Gamma^F$  within  $\Gamma_0^F$  between its upper and lower quartiles for each depth bin. The depth-bin size is 250 m, and depth bins with patch number of patches smaller than 10 are excluded.

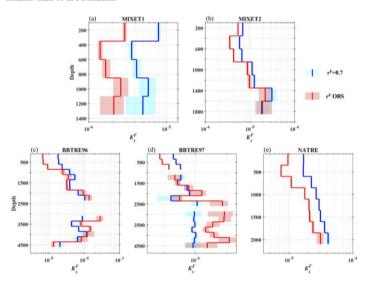


Fig. 15. Same as Fig. 14, but for  $K_S^F$ .

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Next, we examine vertical variation of the ratio of  $K_S^F$  to  $K_\theta^F$  for the five projects (Fig. 16). For the MIXET1,  $K_S^F/K_\theta^F$  generally decreases from 5.3 at upper 400 m to 4 at 1400 m, with an averaged value of 4.5. The averaged  $K_S^F/K_\theta^F$  drops to 3.9 for the MIXET2, and it varies between 3.7 and 4.5 except the small values shallower than 400 m and beneath 1600 m. The BBTRE projects share similar vertical structure of  $K_S^F/K_\theta^F$ : It has the maximum value of 6 in the upper 800 m, then sharply decreases to 2.5 at 1350 m and keeps at this value until reaching 4600 m.  $K_S^F/K_\theta^F$  for the NATRE first increases from 3.0 to 4.7 in the upper 800 m, and then sharply decreases to 3 at 1100 m and remains unchanged. From the five projects,  $K_S^F/K_\theta^F$  generally increases with depth at the upper 1000 m with an average value about 5; then, it sharply drops to around 3 and stays at this value at deeper depths. This ratio is reported to be 2.3 in the western tropical Atlantic (Schmitt et al., 2005), slightly smaller than the result presented here. Van de Boog et al. (2021) presented a global map of  $K_S^F$  and  $K_\theta^F$  based on Argo data and an empirical method; and their results indicate  $K_S^F/K_\theta^F$  vary between 1.3 and 7.8 for  $R_\theta$  ranging from 1 to 4. These earlier works do not show the vertical variation of  $K_S^F/K_\theta^F$  due to indirect methods used.

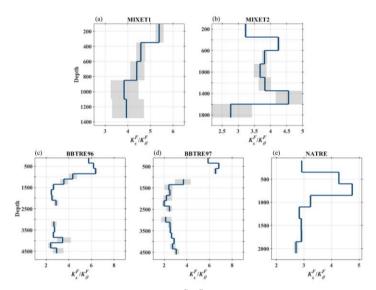


Fig. 16. Vertical profiles of depth-bin mean  $K_0^F/K_S^F$  based on salt finger patches for the five projects. The dark blue curves are the mean  $K_0^F/K_S^F$ , and the gray shadings are 95% bootstrapped confidence intervals. To exclude the influence of extreme values, we only consider patches with  $\frac{\Gamma^T}{\Gamma^T}$  within  $\frac{\Gamma^F}{\Gamma^T}$  between its upper and lower quartiles for each depth bin. The depth-bin size is 250 m; and depth bins with  $\frac{\Gamma^T}{\Gamma^T}$  number of patches smaller than 10 are excluded.

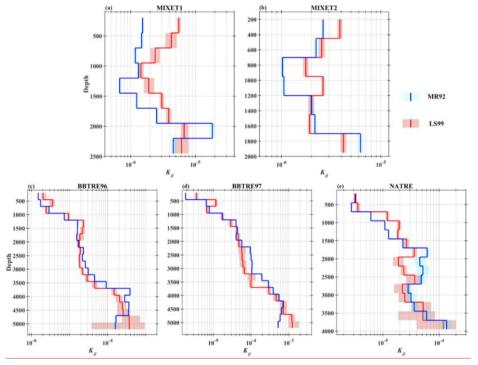
## 5.3 "Total" eddy diffusivities under superposed salt finger and turbulence

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We examine the "total" eddy diffusivities contributed by both salt finger and turbulence by combining the patches with weak turbulence, energetic turbulence and salt finger. Two different methods are used to estimate the "total" eddy diffusivities. The first is from McDougall and Ruddick (1992) (hereinafter MR92). MR92 does not need to differentiate salt finger and turbulent patches; it estimates the total eddy diffusivities by (i) evaluating the departure of observed  $\Gamma$  (Eq. (1)) to a preset reasonable turbulent  $\Gamma^T$  (e.g.,  $\Gamma^T$ =0.265) and (ii) introducing a "salt flux enhancement factor",  $M_0$ , scaled by  $R_0$  and r (more details are given in McDougall and Ruddick (1992)). Here, r is treated specifically depending on the mixing type, that is,  $r^T$ = $R_0$  for turbulence and  $r^F$ = $\frac{R_0\Gamma}{R_0\Gamma+R_0-1}$  for salt finger (St. Laurent and Schmitt, 1999). The second is from St. Laurent and Schmitt (1999) (hereinafter LS99), which differentiates turbulence and salt finger firstly, then estimates their eddy diffusivities separately, and finally obtains the total ones as  $K_0$ =  $P^T$ - $K_0^T$ +  $P^F$ - $K_0^F$  and  $K_3$ =  $P^T$ - $K_2^T$ +  $P^F$ - $K_3^F$ , where  $P^T$  and  $P^F$  are the number proportions of turbulence and salt finger patches to their sum, respectively. Fig. 17 shows the "total"  $K_0$  estimated by these two methods. Compared with the BBTREs and NATRE, the results based on MR92 and LS99 present

larger differences for MIXETs, which may be due to the fewer patches and more scattered Γ<sup>T</sup> and Γ<sup>F</sup>. Nonetheless, it is obvious that both estimates have similar magnitude and vertical trend for all the five projects. Comparing the total K<sub>θ</sub> with K<sub>θ</sub><sup>T</sup> and K<sub>θ</sub><sup>F</sup> (Figs. 13, 14), we can see K<sub>θ</sub>, especially for the LS99 result, is obviously closer to K<sub>θ</sub><sup>T</sup> for all the five projects, confirming that turbulence dominates the observed microstructures. Note that K<sub>θ</sub> in the upper 500 m for the BBTREs and NATRE are significantly lower than K<sub>θ</sub><sup>T</sup>, seemingly indicating a strong weakening of K<sub>θ</sub> due to the prevalence of salt finger.
However, the effect of salt finger is actually overestimated, since the dominant hybrid mixing patches at this depth range are all excluded, which should be dominated by turbulence, as indicated by the elevated Re<sub>b</sub>. The total K<sub>δ</sub> is not shown since it is very similar to the situation of K<sub>θ</sub>, and the only notable difference is K<sub>δ</sub> is not significantly weakened by salt finger in the upper 500 m for BBTREs and NATRE, owing to K<sub>δ</sub><sup>F</sup> is clearly greater than K<sub>θ</sub><sup>F</sup> and much closer to K<sub>δ</sub><sup>T</sup> (Fig. 15).



545 Fig. 17. Vertical profiles of depth-bin averaged total *K*<sub>0</sub> based on turbulence and salt finger patches for the five projects. The blue curves are results based on MR92, and the red ones are based on LS99. The shades correspond to 95% bootstrapped confidence intervals. The depth-bin size is 250 m, and depth bins with number of patches smaller than 10 are excluded.

## 6 Summary

The Osborn relation is widely used to estimate vertical eddy diffusivity in practice, assuming a constant dissipation ratio of Γ<sup>T</sup>=0.2 without identifying underlying mixing mechanisms. The dissipation ratios of heat, salinity and density are equal for turbulent mixing; however, they differ for salt finger-induced mixing. As a result, the eddy diffusivities derived from a constant dissipation ratio would inevitably depart from the actual values. In this study, we differentiated turbulent mixing and salt finger mixing, quantified their dissipation ratios and eddy diffusivities, and examined their relations based on the datasets from "Microstructure Database".

We evaluated the variation of  $\Gamma^T$  and its relations with  $Re_b$  and  $R_{OT}$ . The observed  $\Gamma^T$  scatters over orders of magnitude, typically from  $10^{-2}$  to 10. The significant difference between the five projects suggests  $\Gamma^T$  is highly variable with space and time. Vertically,  $\Gamma^T$  in the western equatorial Pacific presents a weak decreasing trenddecrease downwards, while it increases obviously in the midlatitude in the Atlantic. Although a negative relation between  $\Gamma^T$  and  $Re_b$  was supported by most of the projects, further investigation of the relations of  $\Gamma^T$  with  $Re_b$  and  $R_{OT}$  suggested  $\Gamma^T \propto R_{OT}^{-4/3} \cdot Re_b^{1/2}$ . This indicates  $\Gamma^T$  is modulated by more than one variable, and explains why different relations between  $\Gamma^T$  and  $Re_b$  have been reported (i.e., Mashayek et al., 2017; Jiichi and Hibiya, 2018).

We compared  $K_{\rho}^{T}$  estimated using observed  $\Gamma^{T}$  with  $K_{c}$  estimated using  $\Gamma^{T}$ =0.2.  $K_{\rho}^{T}$  is clearly larger than  $K_{c}$ . For the MIXET projects with <u>verticallydownward</u> weak decreasing  $\Gamma^{T}$ ,  $K_{\rho}^{T}$  shares similar vertical structure of  $K_{c}$ , with magnitude elevated by about two or three times. For the rest projects whose  $\Gamma^{T}$  increases significantly with depth,  $K_{\rho}^{T}$  generally presents a much more obvious <u>increasing trendincrease</u> than  $K_{c}$ , and  $K_{\rho}^{T}$  can be larger than  $K_{c}$  by an order of magnitude. This suggests the intensity of bottom-enhanced mixing may be underestimated when assuming  $\Gamma^{T}$ =0.2.

For salt finger, two "effective" dissipation ratios for heat  $(\Gamma_{\theta}^F)$  and salt  $(\Gamma_{s}^F)$  are derived, and two "artificial" Osborn relations are used to calculate corresponding eddy diffusivities.  $\Gamma_{\theta}^F$  spans about two orders of magnitude. Both the magnitude and vertical structure of  $\Gamma_{\theta}^F$  are distinct for the five projects.  $\Gamma_{s}^F$  is strongly related to  $\Gamma_{\theta}^F$ , and they share similar vertical structures,  $\Gamma_{s}^F \approx 5\Gamma_{\theta}^F$ . Data from some projects indicate a negative relation between  $\Gamma_{\theta}^F$  ( $\Gamma_{s}^F$ ) and  $Re_b$ , while the others suggest no clear relation. Unlike the existing results indicating  $r^F$  decreases then asymptotes to a constant value with increasing  $R_{\theta}$ , our results suggest  $r^F$  decreases sharply with  $R_{\theta}$  when it is smaller than 2.4 and grows to a larger value with  $R_{\theta}$  when it exceeds 2.4.

We examined salt finger-induced  $K_{\theta}^{F}$  and  $K_{S}^{F}$ . Although salt finger becomes rarer with depth,  $K_{\theta}^{F}$  and  $K_{S}^{F}$  increase clearly in the vertical with depth, and  $K_{S}^{F}$  is greater than  $K_{\theta}^{F}$ . In the upper 1000 m,  $K_{S}^{F}$  is significantly greater than  $K_{\theta}^{F}$  by about five times for most projects; but below 1000 m,  $K_{S}^{F}/K_{\theta}^{F}$  generally stays around 3.  $K_{\theta}^{F}$  and  $K_{S}^{F}$  estimated using the observed  $r^{F}$  are generally smaller than those using  $r^{F}$ =0.7 due to most observed  $r^{F}$  being smaller than 0.7, but varying more sharply in the vertical.

Compared with eddy diffusivity induced by turbulence,  $K_{\theta}^{F}$  is smaller than  $K_{\theta}^{T}$  in the upper 1000 m, but they become more and more comparable with increasing depth.  $K_{S}^{F}$  is close to or even larger than  $K_{S}^{T}$  at all depths for all the projects. In general, although salt finger events are much rare than turbulence at depth (so they may beare incapable of largely altering the background mixing intensity shaped by turbulence), they can play a crucial role in local, short-period mixing events, which is worth to be investigated and properly parameterized in numerical models.

## 585 Data Availability

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The microstructure datasets used in this study are available at http://microstructure.ucsd.edu. And the ETOPO 2022 bathymetry data used in Fig. 1 is from https://www.ncei.noaa.gov/products/etopo-global-relief-model.

Author contributions

The study was conceived and designed by all co-authors. Data preparation, material collection, and analysis were performed by JL. JL prepared the manuscript with contributions from all co-authors.

Competing interests

The contact author has declared that none of the authors has any competing interests.

Acknowledgments

We thank the Climate Process Team for publicly sharing the "Microstructure Database". This work is supported by the National Natural Science Foundation of China (Grants 42076012, 42376012), the Postdoctoral Fellowship Program of CPSF (Grant GZC20241610)—and), the China Postdoctoral Science Foundation (Grant 2024M753046) and the Fundamental Research Funds for the Central Universities (Grant 202413031).

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