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Dissipation ratio and eddy diffusivity of turbulent and salt finger mixing derived from microstructure measurements

Note: The reviewers' original comments are in black, and our responses are in blue.

Responses to Reviewer #1

This manuscript takes microstructure measurements from a variety of different ocean basins with different propensities for salt finger double diffusive convection, and discusses the dissipation ratio (mixing efficiency Γ) and the turbulent diffusivities of heat, salt and buoyancy. However, I do not think that the salt-finger cases are treated properly, and so I recommend against this manuscript being published in *Ocean Science*.

This paper divides the observations into different sets based on the propensity to exhibit salt-fingering behavior, as measured by the Turner angle. So far so good. But then the mixing efficiency, Γ, is estimated differently depending on which class of observations the measured data falls into. If the data comes from a doubly-stable regime, then the Oakey formula (which appears in between equations (2) and (3) of the manuscript)

$$\frac{\chi_\theta N^2}{2\varepsilon\theta_z^2}$$

is used, whereas if the data is from a water column that has warm salty seawater overlying cooler fresher seawater, then a different formula is used, namely, from their Equations (4) and (5),

$$\Gamma_\theta^{\mathrm{F}} = \left(\frac{R_\rho - 1}{R_\rho}\right)\left(\frac{r^{\mathrm{F}}}{1-r^{\mathrm{F}}}\right) \text{ and } \Gamma_S^{\mathrm{F}} = \frac{R_\rho - 1}{1-r^{\mathrm{F}}}.$$

We know from the careful study of St. Laurent and Schmitt (1999) that in the North Atlantic Central Water where R_ρ is about 2 and so is susceptible to salt-fingering, the detection of salt fingers is very difficult. Their conclusion is that most of the time the observed microstructure is due to ordinary turbulent mixing which has the same turbulent diffusivity for all conserved scalar quantities. Hence, in such locations, it is not appropriate to assume that salt fingers account for all the observed microstructure, as the present manuscript assumes. This is the reason I recommend that the present manuscript should not be published in *Ocean Science*.

There is a way of using the microstructure observations while recognizing that they are the sum of contributions from both (1) isotropic turbulence and (2) salt fingering. This method appeared in section 3 of McDougall and Ruddick (1992), and it is quite different to what is used in the present manuscript.

Reference:

McDougall, T. J. and B. R. Ruddick, 1992: The use of ocean microstructure to quantify both turbulent mixing and salt-fingering. Deep-Sea Research, 39, 1931-1952.

Responses: We sincerely appreciate the reviewer's valuable comments on our manuscript. Here, we put our views on the reviewer's concerns, and hope they can satisfy the reviewer. The reviewer's concerns are addressed in detail as follows.

Firstly, the reviewer has concerns about our estimates of the dissipation ratio, Γ, using different formulas for turbulent mixing and salt-finger mixing, respectively. We are sorry for causing this confusion due to our inappropriate expression. Indeed, the dissipation ratio Γ has the same definition for turbulent mixing and salt-finger mixing as follows,

$$\Gamma = \frac{\chi_\theta N^2}{2\varepsilon\theta_z^2}. \quad (1)$$

Based on the production-dissipation balance for TKE (Osborn,1980)

$$(1 - R_f)K_\rho N^2 - R_f\varepsilon = 0, \quad (2)$$

and the production-dissipation balance for thermal variance (Osborn and Cox, 1972)

$$2K_\theta\theta_z^2 - \chi_\theta = 0, \quad (3)$$

we can get

$$\Gamma = \frac{\chi_\theta N^2}{2\varepsilon\theta_z^2} = \left(\frac{R_f}{1 - R_f}\right)\frac{K_\theta}{K_\rho}. \quad (4)$$

Combining the expressions of buoyancy N^2 and buoyancy flux $K_\rho N^2$ together (St. Laurent and Schmitt, 1999),

$$N^2 = g\alpha\theta_z(1 - 1/R_\rho), \quad (5)$$

$$K_\rho N^2 = g[\alpha(K_\theta\theta_z) - \beta(K_S S_z)] = g\alpha(K_\theta\theta_z)(1 - 1/r). \quad (6)$$

K_ρ is derived as $K_\rho = K_\theta\frac{1-1/r}{1-1/R_\rho}$ and is substituted in Eq. (4), then, we finally obtain

$$\Gamma = \frac{\chi_T N^2}{2\varepsilon\theta_z^2} = \left(\frac{R_f}{1 - R_f}\right)\frac{K_\theta}{K_\rho} = \left(\frac{R_f}{1 - R_f}\right)\left(\frac{R_\rho - 1}{R_\rho}\right)\left(\frac{r}{r - 1}\right). \quad (7)$$

Here, we stress that Eq. (7) is applicable to both turbulent mixing and salt finger mixing. Some variables referred above are listed in Table R1.

For turbulent mixing only, $K_S = K_\theta = K_\rho$, and $r = R_\rho$, resulting in $\left(\frac{R_\rho-1}{R_\rho}\right)\left(\frac{r}{r-1}\right) = 1$. Then, Eq. (7) leads to

$$\Gamma^{\mathrm{T}} = \frac{\chi_T N^2}{2\varepsilon\theta_z^2} = \frac{R_f}{1 - R_f}. \quad (8)$$

Since R_f = B/P, Γ can be written as $\Gamma = \mathrm{B}/\varepsilon$ for steady and homogeneous turbulence, i.e., P=B+ ε. As a result, Γ can be considered as the fraction of energy available to turbulent mixing, which helps mix different density waters essentially and gain the background potential energy. This is also the reason some literatures called Γ mixing efficiency when turbulent mixing prevails, although it is somewhat misleading since Γ can be greater than unity. Based on Eq. (2), we can get

$$K_\theta^T = K_S^T = K_\rho^T = \frac{R_f}{1 - R_f}\frac{\varepsilon}{N^2} = \Gamma^{\mathrm{T}}\frac{\varepsilon}{N^2}, \quad (9)$$

where superscript "T" indicates turbulent mixing.

However, for salt finger mixing only, $\lim\limits_{P\to 0}\frac{R_f}{1-R_f} = -1$. Then, Eq. (7) yields

$$\Gamma^{\mathrm{F}} = \frac{\chi_T N^2}{2\varepsilon\theta_z^2} = -\frac{K_\theta}{K_\rho} = -\left(\frac{R_\rho - 1}{R_\rho}\right)\left(\frac{r}{r - 1}\right). \quad (10)$$

Based on Eq. (2), we can get

$$K_\rho^{\mathrm{F}} = \frac{R_f}{1 - R_f}\frac{\varepsilon}{N^2} = -\frac{\varepsilon}{N^2}. \quad (11)$$

Using Eq. (10), K_θ can be obtained as follows,

$$K_\theta^{\mathrm{F}} = \left(\frac{R_\rho - 1}{R_\rho}\right)\left(\frac{r}{1 - r}\right)\frac{\varepsilon}{N^2} = \Gamma_\theta^{\mathrm{F}}\frac{\varepsilon}{N^2}. \quad (12)$$

Note that $\Gamma_\theta^{\mathrm{F}}$ is the same as the expression of dissipation ratio in Eq. (10). Along with flux ratio, $r = \frac{K_\theta}{K_S}R_\rho$, K_S can be written as follows,

$$K_S^{\mathrm{F}} = \frac{R_\rho - 1}{1 - r}\frac{\varepsilon}{N^2} = \Gamma_S^{\mathrm{F}}\frac{\varepsilon}{N^2}, \quad (13)$$

where superscript "F" indicates salt Finger mixing.

Now, the $\Gamma_\theta^{\mathrm{F}}$ and Γ_S^{F} in our manuscript are actually two **artificial** "mixing efficiencies", to make the estimations of K_θ^{F} and K_S^{F} for salt finger mixing analogical to the Osborn relation for turbulent mixing. Identical "analogical" Osborn relation for salt finger mixing was developed in Schmitt et al. (2005). Here, we call the terms $\left(\frac{R_\rho-1}{R_\rho}\right)\left(\frac{r}{1-r}\right)$ and $\frac{R_\rho-1}{1-r}$ before "ε/N^2" as $\Gamma_\theta^{\mathrm{F}}$ and Γ_S^{F}, respectively. Investigating the statistic features of $\Gamma_\theta^{\mathrm{F}}$ and Γ_S^{F} can be practically useful when estimating K_θ^{F} and K_S^{F} solely based on ε and N^2.

Table R1. Variables and their definitions.

Variable	Definition
P	Shear production term in the TKE equation
B	Buoyancy flux term in the TKE equation
χ_θ	Dissipation rate of thermal variance
N^2	Buoyancy frequency squared
ε	Dissipation rate of TKE
θ_z	Vertical gradient of temperature
S_z	Vertical gradient of salinity
R_f	Flux Richardson number, R_f=B/P
K_S, K_θ, K_ρ	Eddy diffusivities of salt, heat, buoyancy
α, β	Expansion coefficient due to heat, contraction coefficient due to salinity
R_ρ	Density ratio, $R_\rho=\alpha\theta_z/\beta S_z$
r	Density flux ratio, $r=\alpha K_\theta\theta_z/\beta K_S S_z=K_\theta/K_S\cdot R_\rho$. For salt finger, $r=\frac{R_\rho\Gamma}{R_\rho\Gamma+R_\rho-1}$

As suggested by the reviewer and Reviewer #2, we realized that the corresponding text, section 2.3 in the manuscript, was misleading; also, it was mostly a collection of some published literatures, not needed for the manuscript. Therefore, we reorganized section 2.3 in the revision as follows,

"Dissipation ratio Γ is defined as

$$\Gamma=\frac{\chi_\theta N^2}{2\varepsilon\theta_z^2} \quad (1)$$

for turbulent mixing and salt-finger mixing (Oakey, 1985). Based on the production-dissipation balances for TKE and thermal variance (Osborn and Cox, 1972; Osborn,1980), and introducing R_ρ and the density flux ratio $r=\alpha K_\theta\theta_z/\beta K_S S_z=K_\theta/K_S\cdot R_\rho$, we get

$$\Gamma=\frac{\chi_T N^2}{2\varepsilon\theta_z^2}=\left(\frac{R_f}{1-R_f}\right)\frac{K_\theta}{K_\rho}=\left(\frac{R_f}{1-R_f}\right)\left(\frac{R_\rho-1}{R_\rho}\right)\left(\frac{r}{r-1}\right), \quad (2)$$

which is applicable to both turbulent mixing and salt finger mixing (St. Laurent and Schmitt, 1999).

For turbulent mixing only, $K_S=K_\theta=K_\rho$. Then, Eq. (2) leads to

$$\Gamma^{\mathrm{T}}=\frac{\chi_T N^2}{2\varepsilon\theta_z^2}=\frac{R_f}{1-R_f}, \quad (3)$$

and

$$K_\theta^{\mathrm{T}}=K_S^{\mathrm{T}}=K_\rho^{\mathrm{T}}=\Gamma^{\mathrm{T}}\frac{\varepsilon}{N^2}, \quad (4)$$

where superscript "T" indicates turbulent mixing.

However, for salt finger mixing only, with $\lim_{P\to 0}\frac{R_f}{1-R_f}=-1$ (St. Laurent and Schmitt, 1999), Eq. (2) yields

$$\Gamma^{\mathrm{F}}=\frac{\chi_T N^2}{2\varepsilon\theta_z^2}=-\frac{K_\theta}{K_\rho}=-\left(\frac{R_\rho-1}{R_\rho}\right)\left(\frac{r}{r-1}\right), \quad (5)$$

which cannot be used directly to estimate the salt finger induced eddy diffusivities. And they are estimated separately by introducing R_ρ and $r^{\mathrm{F}}=R_\rho\Gamma^{\mathrm{F}}/\left(R_\rho\Gamma^{\mathrm{F}}+R_\rho-1\right)$ (St. Laurent and Schmitt, 1999; Schmitt et al., 2005; Inoue et al., 2007),

$$K_\theta^{\mathrm{F}}=\left(\frac{R_\rho-1}{R_\rho}\right)\left(\frac{r}{1-r}\right)\frac{\varepsilon}{N^2}=\Gamma_\theta^{\mathrm{F}}\frac{\varepsilon}{N^2}, K_S^{\mathrm{F}}=\frac{R_\rho-1}{1-r}\frac{\varepsilon}{N^2}=\Gamma_S^{\mathrm{F}}\frac{\varepsilon}{N^2}. \quad (6)$$

Note that all these equations are written into forms analogical to the Osborn relation for turbulent mixing. $\Gamma_\theta^{\mathrm{F}}$ and Γ_S^{F} are two artificial "mixing efficiencies", which are actually $\left(\frac{R_\rho-1}{R_\rho}\right)\left(\frac{r}{1-r}\right)$ and $\frac{R_\rho-1}{1-r}$ before "ε/N^2" for K_θ^{F} and K_S^{F} estimation. $\Gamma_\theta^{\mathrm{F}}$ is the same as Γ^{F}, while Γ_S^{F} are further derived based on R_ρ and r^{F}, $\Gamma_S^{\mathrm{F}}=\Gamma^{\mathrm{F}}\cdot R_\rho/r^{\mathrm{F}}$. Investigating the statistic features of $\Gamma_\theta^{\mathrm{F}}$ and Γ_S^{F} can be practically useful when estimating K_θ^{F} and K_S^{F} solely based on ε and N^2."

We hope the reviewers find it improved, to be accurate and readable.

Secondly, the reviewer thinks this study concludes that salt fingers account for all the observed microstructures in the North Atlantic Central Water and other locations where the data were collected. We understand the reviewer's concern. However, we do not intend to examine the relative contributions of turbulent mixing and salt finger mixing in shaping the observed microstructures. Our focus is to investigate the necessity of differentiating mixing types and to refine their dissipation ratios on eddy diffusivity estimation. Therefore, based on their unique features, we only choose and analyze a part of the microstructure patches, which are overwhelmingly dominated by either turbulent mixing or salt finger mixing, and excluded the patches suspecting to be hybrids of different mixing types. In the manuscript, the statistical features of the "pure" turbulent patches and "pure" salt finger patches are examined separately in sections 4.1 (5.1) and 4.2 (5.2), respectively. We do not explore the relative contributions of these two mixing types. Taking the NATRE project conducted in the North Atlantic Central Water as an example, we chose patches with $|Tu|<45°$ or $|Tu|>90°$ as "pure" turbulent mixing, since double diffusion is prohibited in these situations. Meanwhile, we chose those with $60°<Tu<90°$, $Re_b<25$ and $|\chi_\theta|/|\varepsilon|\geq 7$ as "pure" salt finger mixing, owing to the fact that all these criteria efficiently lower the possibility of the occurrence and intensity of turbulent mixing. Note that here we do not use $60°<Tu<90°$ as the sole criterion, since, as the reviewer suggested that although the North Atlantic Central Water where R_ρ is about 2 and so is susceptible to salt-fingering, the detection of salt fingers is very difficult (St. Laurent and Schmitt, 1999). Among all the patches, only about 35% of them meet the above two criteria and are further analysed in terms of "pure" turbulent and salt-finger mixing, respectively (Fig. R1e, Table. R2). For the rest 65% of the microstructure patches of hybrid mixing types, although we excluded them from our analysis, we can reasonably infer that they are mostly dominated by turbulent mixing, as their Re_b values exceed the typical range for salt finger. Besides, among the chosen patches, "pure" salt fingers are mostly confined in the upper 500 m (Fig. R2). Therefore, although it is not our focus, our results implicitly suggest turbulent mixing is undoubtedly the dominant contribution to the observed microstructure strength, in line with the conclusion drawn by St. Laurent and Schmitt (1999). The microstructures of the chosen five projects analysed in this study are all dominated by turbulent mixing (Fig. R1). Our concern here is the dissipation ratios and hence the eddy diffusivities separately induced by those chosen "pure" turbulent patches and "pure" salt finger patches. They are indeed a small fraction of the total observed microstructure patches, and their specific contributions to the total microstructure field are not covered by this manuscript.

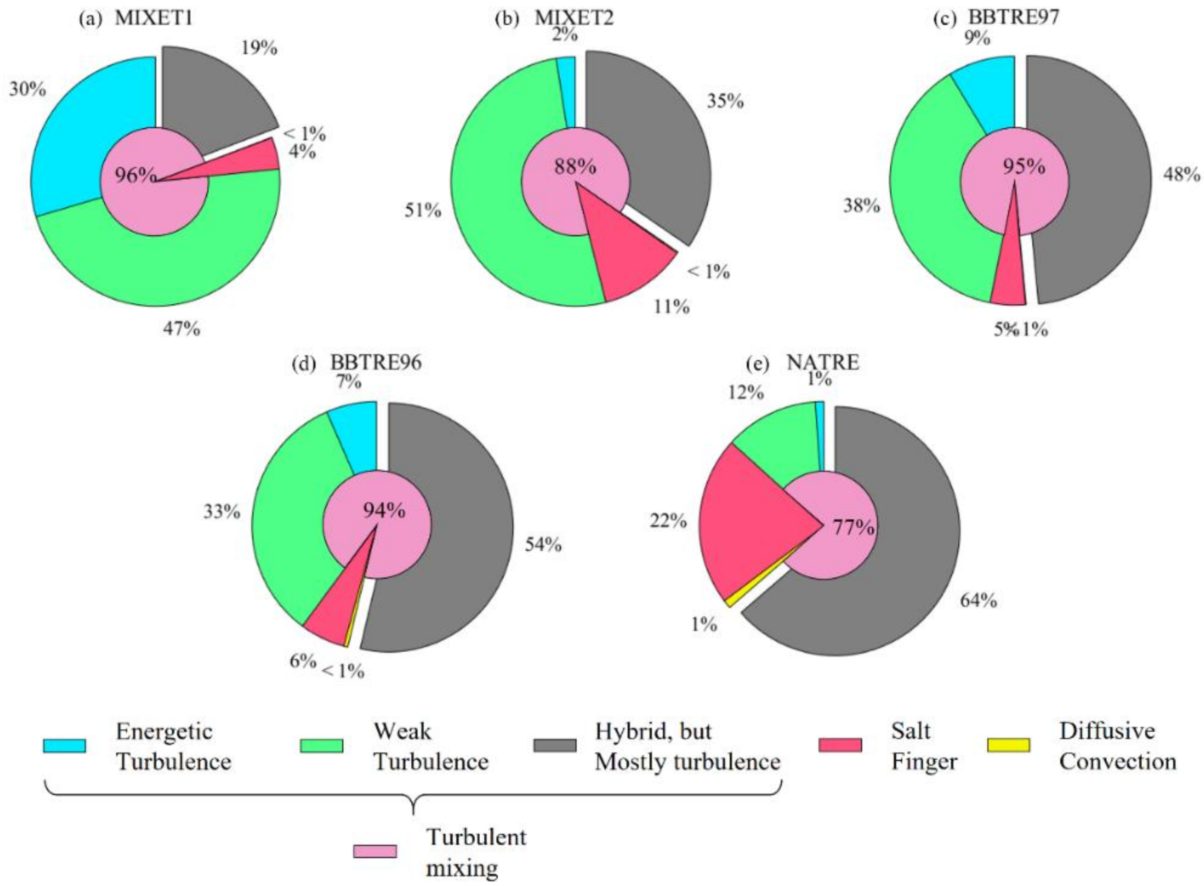

Fig. R1. Proportions of patches with different mixing types for each project: “pure” energetic turbulence, “pure” weak turbulence, “pure” salt finger, “pure” diffusive convection and hybrid (turbulence and salt finger, or turbulence and diffusive convection). Patches with hybrid mixing types are excluded from the analysis.

Table. R2. Proportions of patches with different mixing types for each project: “pure” energetic turbulence, “pure” weak turbulence, “pure” salt finger, “pure” diffusive convection and hybrid (turbulence and salt finger, or turbulence and diffusive convection), and those for all projects. Patches with hybrid mixing types are excluded from the analysis.

	Proportion (%)				
	energetic turbulence	weak turbulence	salt finger	diffusive convection	excluded hybrid
MIXET1	29.56	47.05	4.11	0.06	19.22
MIXET2	2.48	51.48	11.32	0.16	34.56
BBTRE96	6.55	33.31	5.91	0.53	53.70
BBTRE97	8.67	38.19	4.56	0.08	48.50
NATRE	1.10	12.21	21.95	1.09	63.65
All	6.60	32.00	9.70	0.46	51.24

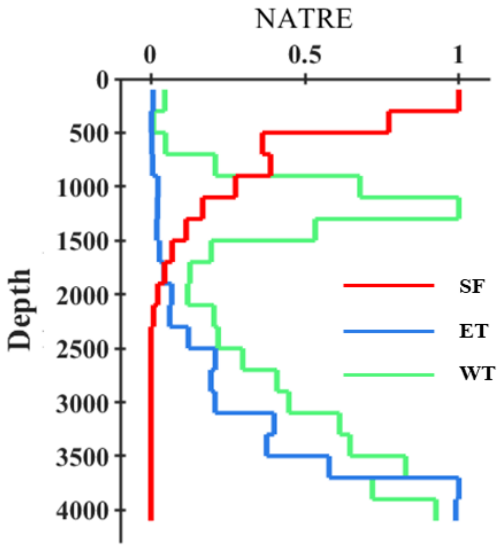

Fig. R2. Vertical variation of normalized occurrence frequency of salt finger (SF), energetic turbulence (ET) and weak turbulence (WT) for NATRE. The occurrence frequency of each mixing type is normalized by its maximum, only reflecting its own vertical variation of prevalence, and cannot be compared with others.

We are sorry that some expressions in the manuscript may have caused confusion. The major part is the first paragraph in section 3 (Lines 164-179), where we only introduced the small part of the chosen patches and didn't state the larger part of excluded ones clearly, would make the reader think we overemphasized the prevalence of salt finger, especially for the NATRE project. This paragraph has been revised to eliminate any misunderstanding statements, by adding two explanations in the revision: "For the BBTREs and NATRE, although a large proportion of the patches have 45°<Tu<90° and hence are salt finger favorable, most of them have elevated Re_b; thus, we infer these mixing events as hybrids of salt finger and turbulence but dominated by turbulence. These patches are excluded from analysis to highlight the difference between turbulent and salt finger mixing. Only a few patches are chosen as effective salt finger events. Therefore, it is turbulent mixing that dominates the observed microstructures, in line with the results based on the NATRE (St. Laurent and Schmitt, 1999)." We also add the text "Although dominated by turbulent mixing, the rest of the patches, more than 50%, are hybrids of different mixing types, and are excluded from the analysis." The phrase "The salt finger-induced eddy diffusivities become more comparable or even stronger than the turbulent diffusivities with depth" in the Abstract may also be misleading, which is rewritten now: "The salt finger-induced eddy diffusivities also increase with depth, with some being comparable to or even stronger than the mean turbulent ones."

By reworking/reorganizing the aforementioned text, we hope the revised manuscript has addressed the reviewer's concerns clearly and meet the publication standard of *Ocean Science*.