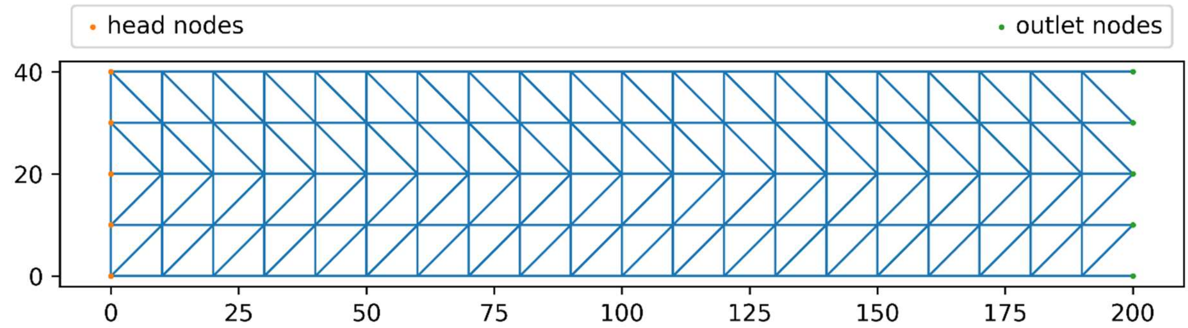
Demonstration of edge-betweenness-centrality for imaging network flow structure



The above shows a simple directed network. Flow on edges is directed from left to right and from the domain boundaries to the centre: flow cannot go right to left nor from the centre to the boundary. The graph may be thought of as several long channels, connected by shorter cross-channel segments. Several demonstrations follow that demonstrate the impact of network structure and weighting on the flow from the head nodes to the outlet nodes. In each, the upper image shows the weight (low weights are 'shorter' paths), while the lower image shows the EBC for flow from the head nodes to the outlet nodes.

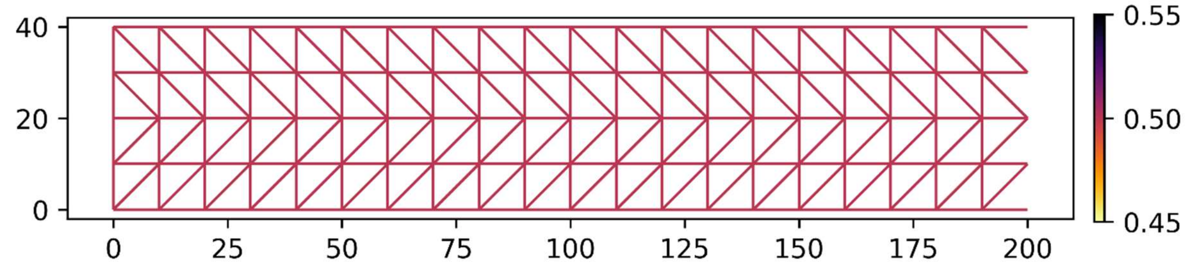
Betweenness centrality of an edge e is the sum of the fraction of all-pairs shortest paths that pass through e

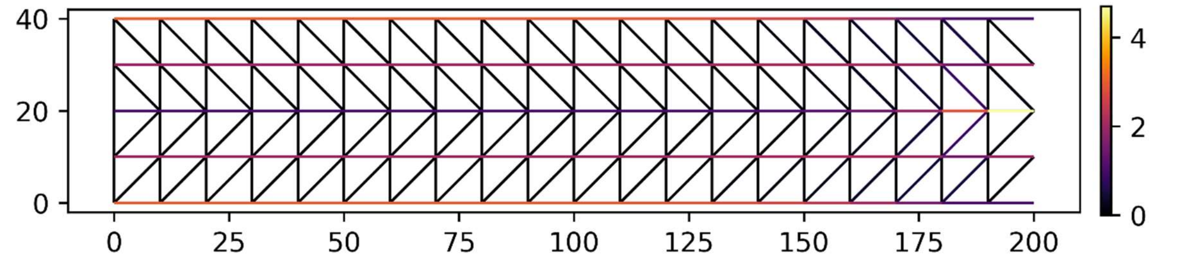
$$c_B(e) = \sum_{s,t \in V} \frac{\sigma(s,t|e)}{\sigma(s,t)}$$

where V is the set of nodes, $\sigma(s,t)$ is the number of shortest (s,t)-paths, and $\sigma(s,t|e)$ is the number of those paths passing through edge e. For the graph structure there are a total of 12 viable head to outlet node-pairs, 6 above the centre, and 6 below. For each node-pair, there are numerous shortest paths, potentially including multiple equal-weighted routes between these node-pairs.

Where the edge directionality and the weight parameter represent flow conditions, the EBC will represent the cumulative occurence of least-cost flow pathways for the sets of node-pairs provided, and therefore EBC-selected subgraphs will optimise for the most efficient flow pathways.

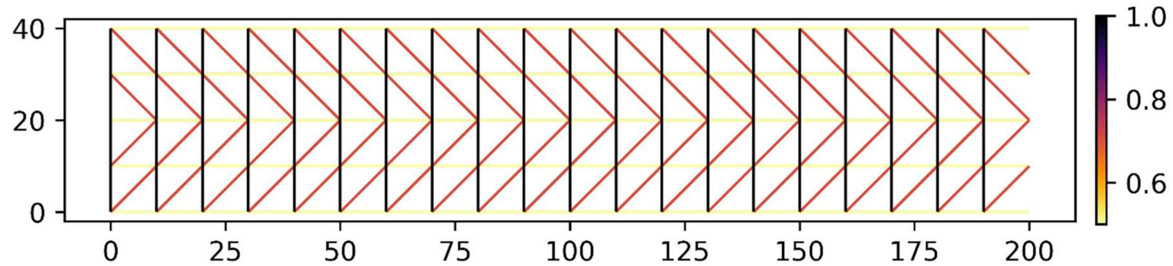
1) Equal weight

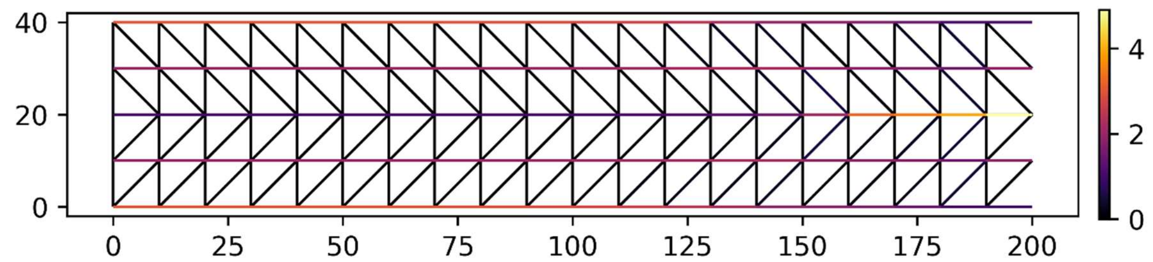




With equal weight, EBC reflects only the directional network structure. For both the top and bottom boundaries, EBC reduces down-flow from 3 at the head node to 1 at the outlet node as shortest paths move gradually to use the central line. On the central line, EBC increases slowly from 1 at the head node to (almost) 5 at the outlet node, where paths from all head nodes must converge. The intermediate lines have EBC of 2 because paths-in from the outer edges are balanced by paths-out to the central line.

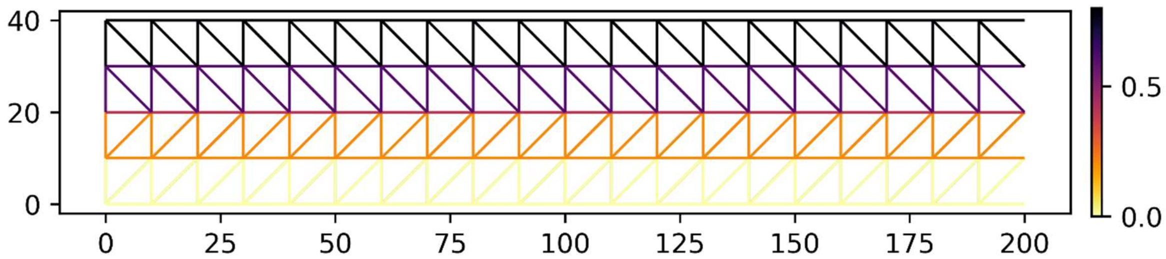
2) 'Herringbone' weight

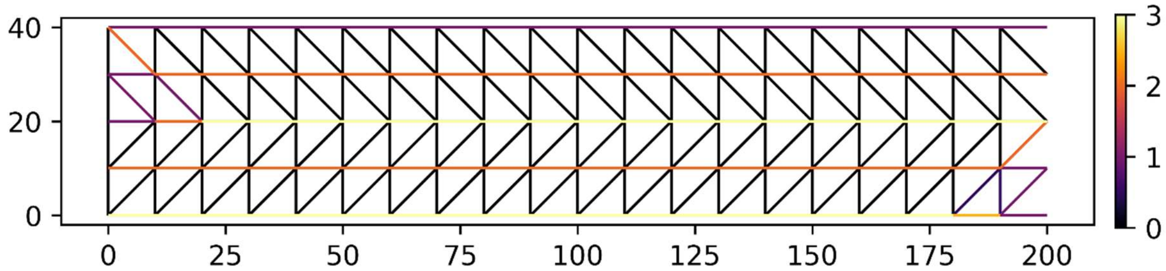




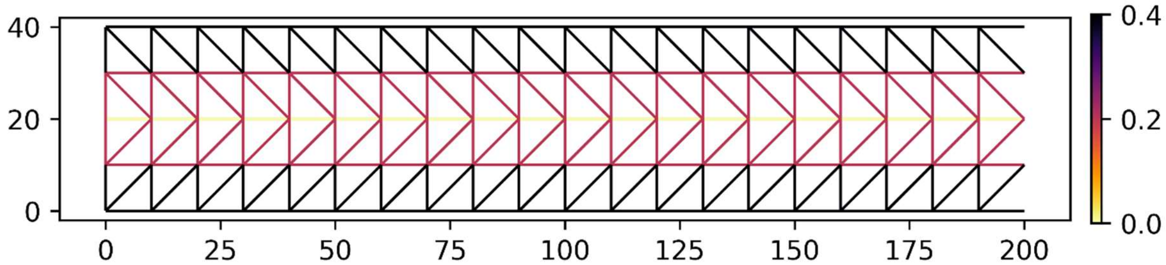
Increasing the across-flow weights relative to the down-flow weights shows a similar pattern to equal weight, but with a somewhat earlier transition to the central line as flow-orthogonal edges are abandoned in favour of diagonal edges.

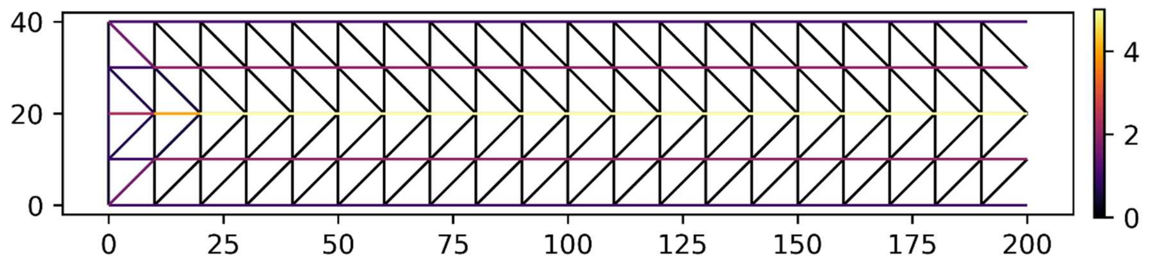
3) Y-axis weighting



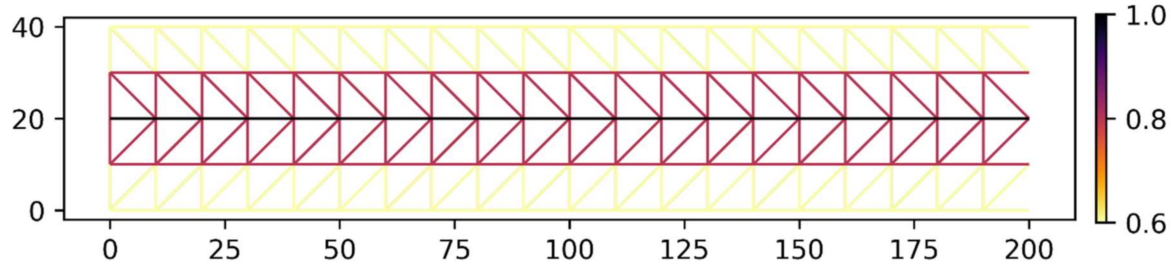


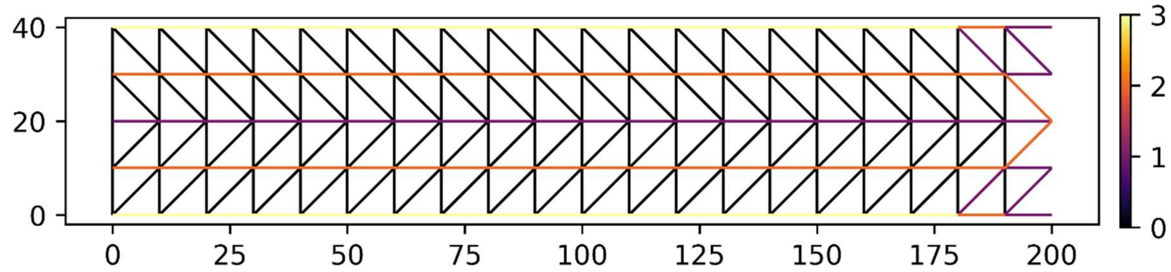
Applying a higher-weight based on Y-coordinate reduces the EBC of the entire upper boundary to 1, because shortest paths are redirected to the lower-weight central line as soon as possible. Similarly, in the lower portion, shortest paths are redirected to the lower boundary.





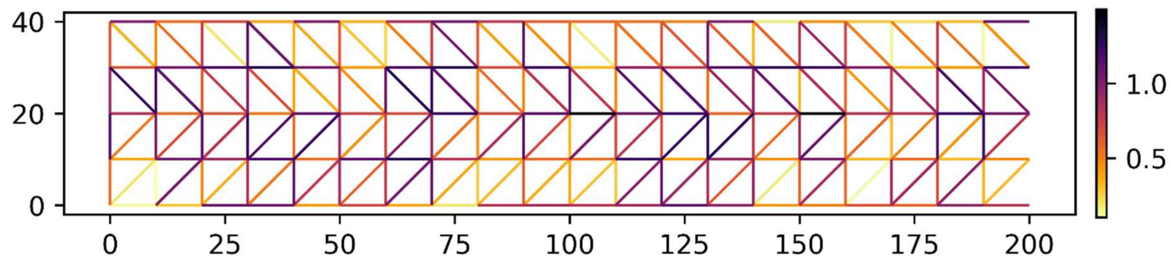
Reducing weighting towards the centre makes the central line the dominant route for shortest paths, with EBC of ~5 from 20 km onwards.

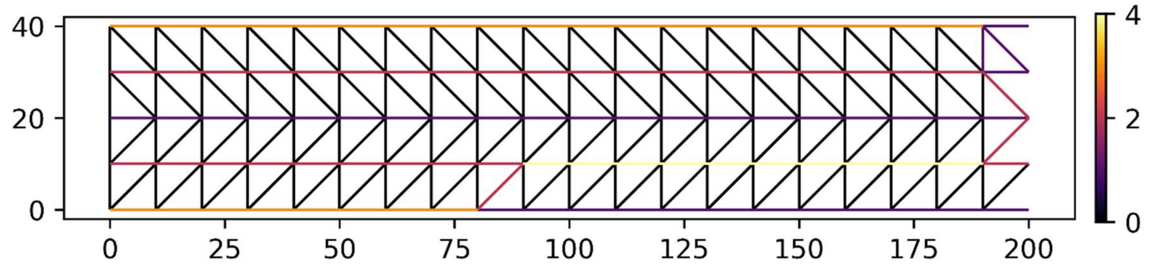




Reducing weighting towards the boundaries mitigates against inward migration of shortest paths which do not cross over to the central until required to after 180 km.

4) Random weighting





A random weighting leads to a more complex flow structure that is sensitive to details in the weightings. For example, on the lower boundary, a change in the weighting from low-weighted between 40-80 km to high weighted from 80-140 km, leads to a diversion of shortest-paths over to the intermediate line. In contrast, the upper segment has lower weights along the upper boundary and has EBC = 3 up until 180 km.