Dear Editor,

We would like to thank you and the reviewers for the careful evaluation of our manuscript entitled "Bayesian inference based on algorithms: MH, HMC, Mala and Lip-Mala for Prestack Seismic Inversion." We are grateful for the constructive comments and suggestions, which helped us to substantially improve the clarity, structure, and impact of the paper.

Following your recommendation

"*Both reviewers recommend to restructure the paper and make it more readable. Please comply with the requests… Justifications, innovation and benchmarking remain inadequate*."

we have thoroughly revised the manuscript. The main changes are:

1. Re-structured manuscript and improved readability.
   We reorganized the paper to follow a clearer and more standard structure: the inverse problem and Bayesian framework are now presented systematically, followed by a dedicated methods section and then the synthetic and real-data applications. We also revised the text for clarity and improved cross-referencing between sections, figures, and tables.

2. Expanded introduction, justification, and innovation.
   The Introduction now contains a more detailed review of inversion methods used to estimate elastic properties from seismic amplitudes, as well as recent advances in MCMC for geophysical inversion. We explicitly highlight the gap in the literature and clarify our contributions, emphasizing the systematic benchmarking of MH, HMC, MALA and Lip-MALA for prestack seismic inversion in synthetic and real 1D and 2D settings.

3. Clearer Bayesian framework and inversion workflow.
   We restructured the section on the seismic inversion problem to clearly define model parameters, data, prior, likelihood, and posterior. We added an explicit step-by-step inference workflow and a new figure summarizing the Bayesian inversion process.

4. Improved presentation of the algorithms and their differences.
   We reorganized the methods section to give a coherent explanation of MH, HMC, MALA and Lip-MALA, added a schematic diagram that contrasts their mechanisms, and clarified the notation and terms used in the equations. The comparative Table summarizing advantages and limitations now cites the relevant literature.

5. Enhanced benchmarking and discussion of tuning and convergence.
   We now report, for each algorithm and case study, acceptance rates, computational times, and effective sample sizes, and discuss the impact of tuning parameters on convergence. The Results, Discussion and Conclusions sections explicitly interpret these metrics, highlighting the trade-offs in efficiency, robustness, and accuracy among the four algorithms.

Below, we respond point-by-point to the reviewers' comments. Reviewer comments are quoted in *italics*, followed by our responses.

We hope that the revised version meets your expectations. We would like to thank you again for the opportunity to improve our work.

Response to Reviewer 1

We sincerely thank the reviewer for the careful reading of our manuscript and the constructive comments. We have carefully considered each point and revised the manuscript accordingly. Below we respond point by point.

Comment 1

"For the introduction, which inversion methods have already been used for estimating elastic properties from seismic amplitude problems? Could you summarize the previous research, and then explain why you introduced the Markov Chain Monte Carlo (MCMC) method? We know that MCMC methods have already been used in other geophysical problems, but for your specific case, we want to know the latest progress. This is a key point for your introduction."

Response:
We agree that the Introduction needed a clearer overview of previous inversion methods and recent progress in MCMC for problems similar to ours.

In the revised Introduction, we have:

- Added a more systematic review of inversion techniques used to estimate elastic properties from prestack seismic amplitudes, including:
    - deterministic least-squares and gradient-based methods,
    - global optimization methods such as simulated annealing and genetic algorithms,
    - and early Bayesian approaches for AVO and prestack inversion.
- Explicitly discussed recent applications of MCMC in geophysics, with emphasis on:
    - Bayesian full-waveform inversion (acoustic and elastic),
    - MALA-type and other gradient-based samplers in high-dimensional settings,
    - and previous use of MCMC for seismic amplitude inversion.
- Clearly stated the current gap: existing studies usually focus on one algorithm or mainly on synthetic examples, and do not provide a systematic benchmark of MH, HMC, MALA and Lip-MALA for prestack seismic inversion targeting VP, VS and density.
- Highlighted our specific contributions: a comparative study of these four algorithms on (i) synthetic 1D data, (ii) real 1D data, and (iii) a real 2D case, including detailed analysis of convergence, uncertainty quantification, and computational cost.

We believe these additions address your request for a clearer context and justification for using MCMC in our particular setting.

Comment 2

"The framework and organization of this manuscript are messy. For example, in Part 2 The Seismic Inversion Problem, you use a sub-title '2.1 Bayesian Inference Framework for Seismic Data,' but there is no 2.2 or 2.3. The section The Seismic Inversion Problem is hard to follow. What are your input parameters and inversion parameters? In the Bayesian Inference Framework, how is the posterior distribution of unknown parameters defined? I don't see a clear inference process about your inversion problem."

Response:
We appreciate this comment and have thoroughly restructured the section on the inversion problem to improve clarity and readability.

In the revised Section 2, we have:

- Reorganized the content into three main subsections:
    1. 2.1 Bayesian Formulation of the Inverse Problem
        - 2.1.1 Prior Distribution Specification
        - 2.1.2 Likelihood Function and Data Errors
        - 2.1.3 Posterior Distribution and MCMC Sampling
    2. 2.2 Model Parameterization and Forward Modelling
        - 2.2.1 Logarithmic Parameterization of VP, VS, and ρ
        - 2.2.2 AVO Forward Modeling Formulation
    3. 2.3 Inference Workflow and Uncertainty Quantification
        - Step-by-step description of the inversion process (from prior definition to posterior analysis).
- Clearly defined the model parameters as the vector of elastic properties (typically the logarithms of VP, VS, and density) at the grid points or layers, and the data vector as the prestack seismic amplitudes at different angles or offsets.
- Explicitly stated the prior distribution (Gaussian with given mean and covariance), the likelihood function (Gaussian errors with specified covariance), and the resulting posterior distribution derived from Bayes' theorem.
- Introduced a new workflow description (and corresponding Figure, referenced in this section) that shows the inference process step by step:
    1. Define prior and forward model.
    2. Acquire data and set data error statistics.
    3. Construct the posterior distribution.
    4. Sample the posterior using one of the four MCMC algorithms.
    5. Analyze posterior samples to obtain estimates and uncertainty measures.

We believe that this reorganization and explicit definition of parameters and distributions addresses your concerns about the "messy" structure and lack of a clear inference process.

Comment 3

"Part 3 looks like a rough method explanation. Could you add a diagram to express their differences and show the research process, for example, step by step? Many terms in the equations are not explained. It is hard to read. In Table 1, the advantages and limitations of those methods are summarized, but from which references or studies? There are no references."

Response:
Thank you for pointing this out. We have substantially revised the methods section to make it more understandable and better documented.

In the revised Section 3, we have:

- Reorganized the section into a clear algorithm-oriented structure:
  - 3.1 Algorithm Overview and Comparative Framework
  - 3.2 Metropolis–Hastings (MH)
  - 3.3 Hamiltonian Monte Carlo (HMC)
  - 3.4 Langevin Algorithms (MALA and Lip-MALA)
  - 3.5 Algorithm Comparison and Tuning Considerations
- Added a new schematic diagram (now Figure [X]) that:
  - contrasts the main mechanisms of MH, HMC, MALA and Lip-MALA,
  - highlights where gradient information is used,
  - and summarizes the research process step-by-step for each algorithm.
- Carefully explained the terms and notation used in the equations:
  - proposal distributions,
  - gradient of the log-posterior,
  - step sizes, mass matrices, and integration lengths for HMC,
  - and stochastic differential equation terminology in the Langevin-based methods.
- Updated Table 1 (algorithm comparison) so that the advantages, limitations, and recommended acceptance rates are now explicitly supported by references, including:
  - Roberts et al. (1997) for MH,
  - Roberts and Rosenthal (1998) for MALA,
  - Beskos et al. (2013) for HMC,
  - and Izzatullah et al. (2021) for Lip-MALA.

We believe these changes make the methods section much clearer and properly grounded in the relevant literature, as you suggested.

Comment 4

"In my view, these methods (e.g., MH, HMC, MALA, and Lip-MALA) are challenging to tune, mainly because the step size (or proposal scale) strongly affects convergence and stability. Please discuss this issue and, if possible, provide comments on their computational efficiency or convergence behavior in your study."

Response:
We fully agree that tuning is a crucial and challenging aspect of MH, HMC, MALA and Lip-

MALA. We have substantially enhanced the discussion of tuning, convergence, and computational efficiency throughout the manuscript.

Specifically:

- In Section 3 we added dedicated subsections on tuning considerations:
  - For MH, we discuss the impact of proposal variance on acceptance rate and mixing, and mention the theoretical optimal acceptance rate in high dimensions.
  - For HMC, we describe the role of the step size and number of leapfrog steps, and how they affect energy conservation, stability, and acceptance rate.
  - For MALA and Lip-MALA, we explain how the step size must be scaled with dimension, the role of the Metropolis correction, and how the Lipschitz-based adaptation in Lip-MALA aims to improve stability and robustness.
- In a new comparative subsection (Section 3.5), we summarize the tuning challenges and provide qualitative guidelines for choosing and adjusting the algorithms in practice for seismic inversion.
- In the Results section, we now report quantitative diagnostics for each algorithm and test case, including:
  - acceptance rates and wall-clock computational times (Tables [4] and [8]),
  - minimum effective sample sizes (mESS) and related convergence indicators (Tables [5] and [9]).
- In the Discussion and Conclusions, we explicitly interpret these metrics, emphasizing:
  - the trade-off between computational cost and sampling efficiency,
  - the relative robustness of MALA and Lip-MALA compared to MH and HMC in our problem,
  - and the practical implications for choosing an algorithm depending on available computing resources and the required accuracy/uncertainty characterization.

We hope this expanded analysis of tuning and convergence responds to your request and clarifies how these issues manifest in our particular inversion study.

Response to Reviewer 2

(We did not quote the full set of comments here, but we briefly summarize how we addressed the issues highlighted by Reviewer 2 and echoed in the editor's decision letter.)

Overall structure and readability.
Reviewer 2, like Reviewer 1, emphasized that the manuscript should be restructured and made more readable. In response, we reorganized the manuscript into a more conventional and transparent structure (Introduction – Bayesian inverse problem – Methods – Forward modelling – Results – Discussion – Conclusions), cleaned up the numbering of sections, figures and tables, and improved transitions between sections.

Justification, innovation and benchmarking.
Reviewer 2 also pointed out that the justification, innovation and benchmarking were not sufficiently emphasized. We have strengthened the Introduction and the Discussion to:

- clarify the novelty of systematically comparing MH, HMC, MALA and Lip-MALA for prestack seismic inversion of VP, VS and density,
- highlight the originality of including synthetic, 1D real and 2D real case studies under a unified framework,
- and explicitly frame our work as a benchmarking study with practical recommendations for algorithm choice.

Clarity of the Bayesian formulation and results interpretation.
To improve clarity, we restructured the Bayesian framework section (now Section 2), added a workflow diagram, and more carefully defined all symbols and distributions. In the Results and Discussion sections, we interpreted the numerical findings more thoroughly, especially regarding posterior uncertainty, correlation structures, and the relative performances of the algorithms.

We thank Reviewer 2 for these suggestions, which have helped us improve both the structure and the scientific message of the paper.